ULTIMATE BEARING CAPACITY OF AXISYMMETRIC AND PLANE-STRAIN FOUNDATIONS RESTED OVER SATURATED/UNSATURATED STRATUM USING LIMIT ANALYSIS

A Thesis

Submitted for the Degree of

Doctor of Philosophy

In the Faculty of Engineering

By

SURYA DEV PRASAD

Under the supervision of

Dr. Manash Chakraborty

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY
BANARAS HINDU UNIVERSITY
VARANASI-221005
INDIA

17061010 **2023**

DECLARATIONS BY THE CANDIDATE

I, **SURYA DEV PRASAD**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Manash Chakraborty** from **July 2017 to July 2023** at the **Department of Civil Engineering, IIT(BHU), Varanasi**. The matter embodied in this has not been submitted for the award of any other/diploma. I declared that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc, reported in journals, book, magazines, reports dissertations, thesis, etc., or available at websites and included them in this thesis and cited as my own work.

Date: 10/7/2023

Place: Varanasi

(Surya Dev Prasad)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my knowledge.

Dr. Manash Chakraborty

Supervisor

Asst. Professor

Dept. of Civil Engineering

IIT(BHU) Varanasi,

U.P, India-22100

Supervisor
Department of Civil Engineering
Indian Institute of Technology (BHU)
Varanasi-221005

CERTIFICATE

It is certified that the work contained in the thesis titled "**ULTIMATE BEARING CAPACITY OF AXISYMMETRIC AND PLANE-STRAIN FOUNDATIONS RESTED OVER SATURATED/UNSATURATED STRATUM USING LIMIT ANALYSIS**" has been carried out under my supervision and this work has not ben submitted elsewhere for a degree.

It is further certified that the student has been fulfilled all the requirements of Comprehensive Examination, Candidacy, and State of the Art (SOTA) for the award of Ph.D degree.

Dr. Manash Chakraborty
Supervisor
Asst. Professor
Dept. of Civil engineering
IIT(BHU)
Varanasi, U.P, India-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: **ULTIMATE BEARING CAPACITY OF AXISYMMETRIC AND PLANE-STRAIN FOUNDATIONS RESTED OVER SATURATED/UNSATURATED STRATUM USING LIMIT ANALYSIS**

Name of the Student: **Mr. Surya Dev Prasad**

Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology, (BHU), Varanasi, all rights under copyright that may exist in and for the above thesis for the award of the Doctor of Philosophy.

Date: 10/7/23

Place: Varanasi

(Surya Dev Prasad)

Note: Howerver, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for the author's personal use provided that the source and the Institute's copyright notice are indicated.

ACKNOWLEDGMENT

First and foremost, I would like to express my deepest gratitude to Lord Shiva for blessing me with countless opportunities, knowledge, and unwavering support, enabling me to complete this thesis successfully. Though my name appears on the cover, the accomplishment of this dissertation would not have been possible without the assistance and guidance of numerous individuals, to whom I owe my heartfelt appreciation.

I am immensely grateful to my supervisor, **Dr. Manash Chakraborty**, from the Civil Engineering Department at IIT (BHU), Varanasi, for his exceptional guidance, continuous monitoring, and unwavering encouragement throughout the entire duration of this research work. I am indebted to Dr. Manash Chakraborty for his invaluable inspiration and helpful recommendations, which significantly contributed to the completion of my research.

I extend my sincere appreciation to the members of my RPEC, Dr. Abhishek Mudggal as the internal expert and Dr. Joysurya Basu as the external expert, for their valuable assistance, insightful suggestions, and constant encouragement during the entirety of my research work.

Furthermore, I would like to express my gratitude to Prof. Sasankasekhar Mandal, Head of the Department of Civil Engineering, Indian Institute of Technology (BHU), Varanasi, for providing the necessary facilities for my research. My heartfelt regards go to Prof. Arun Prasad, Dr. Bala Ramudu Paramkusam, Dr. Suresh Kumar, Dr. Supriya Mohanty, and all the faculty members of the Civil Engineering Department for their unconditional support throughout my academic journey.

Special gratitude is extended to Dr. Dhirendra Pal of Precision Instruments for his innovative ideas and continuous guidance throughout the course of my work. I am also thankful to my friends and colleague, Dr. Nitesh Bonal, Mrs. Shivani Dhiriyan, Dr. Parul Rawat, Mr. Amit Kumar Ram, Mr. Saurav Sarkar, Mr. Abhay Kumar, Mr. Amit Singh, Mr. Ankit Tiwari, Dr. Hitesh Upreti. Dr. Shambhavi Mishra and Dr. Gaurav Tiwari for their thought-provoking discussions, unwavering support, cooperation, and assistance in various ways.

In particular, I would like to express my heartfelt appreciation to some senior Mr. Manish Kumar Mandal, Dr. Bablesh Kumar Jha, Dr. Rahul Singh, Dr. Sujeet Kumar, Dr. Abhishek Kumar, Dr. Satyajeet Mondal, Mr. Nitesh Gupta for their constant encouragement, love, and moral support. I take immense pleasure in sharing the credit for my research work with all my teachers who have played a significant role at different stages of my academic career.

Lastly, I am eternally grateful to my entire family for their unwavering faith, patience, encouragement, blessings, and unconditional love. My deepest appreciation goes to my father, Mr. Raja Ram Prasad, and my late mother, Late Humeli Prasad, for their unwavering motivation, belief in me, and continuous support. A special thanks to my special family for always holding my hand through the ups and downs, providing unwavering support and love.

I am sincerely grateful to each and every individual mentioned above, as well as those who have supported me in various ways, for their invaluable contributions to the completion of this thesis.

Surya Dev Prasad

CONTENTS

LIST OF FIGURES

LIST OF NOTATIONS

a	=	Square of the radius of the Mohr circle of any point for the current stress state
a_{11}^{e}	=	Local matrix defined in Equation (7.17)
a_{12}^{e}	=	Local matrix defined in Equation (7.17)
a_{21}	=	Local matrix defined in Equation (7.25)
a_{23}	=	Local matrix defined in Equation (7.25)
a_{31}	=	Local matrix defined in Equation (7.27)
a_{eq}^{e}	=	Local coefficient matrix of element equilibrium defined in Equation (3.5)
a_{eq}^{dc}	=	Local coefficient matrix of discontinuity equilibrium defined in Equation (3.8)
a_{eq}^{bd}	=	Local coefficient matrix of stress boundary condition defined in Equation (3.10)
$\boldsymbol{A_{11}}$	=	Global matrix defined in Equation (7.43a)
$\boldsymbol{A_{12}}$	=	Global matrix defined in Equation (7.43a)
$\boldsymbol{A_{21}}$	=	Global matrix defined in Equation (7.43a)
$\boldsymbol{A_{23}}$	=	Global matrix defined in Equation (7.43c)
$\boldsymbol{A_{31}}$	=	Global matrix defined in Equation (7.43c)
$\boldsymbol{A^{e}}$	=	Area of triangular element
$\boldsymbol{A_{eq}}$	=	Global matrix defined in Equation (3.32a)
b_{eq}^{e}	=	Vector associated with element equilibrium defined in Equation (3.5)
b_{eq}^{dc}	=	Vector associated with element equilibrium defined in Equation (3.8)
b_{eq}^{bd}	=	Vector associated with stress boundary condition defined in Equation (3.10)
b	=	Half width of the strip footing
B	=	Width of strip footing /diameter of the circular pile
$\boldsymbol{B}_{eq}$	=	Global vector defined in Equation(3.23b)

c' = Effective cohesion

c = Cohesion of soil

c_o = Cohesion of soil at the ground surface

c_b = Cohesion at the level of pile base

c_u = Undrained shear strength of the clayey strata

$\boldsymbol{C_1}$ = Global vector defined in Equation (7.43a)

$\boldsymbol{C_2}$ = Global vector defined in Equation (7.43a)

$\boldsymbol{C_3}$ = Global vector defined in Equation (7.43a)

d = Square of the radius of the Mohr circle at failure as defined in Equation (3.5)

$\boldsymbol{d_x}$ = Search direction vector as defined in Equation (3.30)

D = Vertical extent of domain below the pile

D_c = Total number of stress/velocity discontinuities

D_f = Disturbance factor

$\mathcal{D}_i$ = Inner diameter of ring footing

$\mathcal{D}_o$ = Outer diameter of ring footing

e = Load eccentricity

E = Total number of elements in the domain

F_1 = Form of the modified MC* yield circle

F_2 = Form of the modified MC yield envelope

GSI = Geological Strength Index

$g(\boldsymbol{\sigma})$ = Set of inequality contraints defined in Equation (3.23c)

h = Thickness of upper soil

h_{opt} = Optimum thickness of the upper layer

h_w = Water table depth

h_{wcr} = Critical water table depth

H = Length of the circular pile (Chapter3)/ Vertical extent of domain

$\boldsymbol{I}$ = Unit matrix

k_s = Saturated hydraulic conductivity

K_r = Relative hydraulic conductivity

* MC=Mohr-Coulomb

K_M = Mualem's variable relating the vG-SWRC model parameters m and n

h_m = Matric suction head

l = Side length of an element

L_g = Horizontal extent of the domain measured

L_s = Length of boundary edge between two boundary nodes

L_0 = Complete loading condition

L_1 = Inner half loading condition

L_2 = Middle half loading condition

L_3 = Outer half loading condition

m = Rate at which the cohesion increases linearly with depth (Chapter3)/ nondimentional strength parameter of OHB* criterion (Chapter 6)/ vG parameter (Chapter 7)

m_b = Broken rock mass strength parameter of GHB** criterion as defined in Equation (6.1b)

m_i = Value of m for intact rock mass

n = Total number of unknown stress variables (Chapter 3)/ Pore size spectrum number (Chapter 7)

N_b = Number of nodes where the boundary stress/velocities are prescribed

N = Number of nodes

N_c = Bearing capacity factor due to cohesion

N_i = Shape function of i^{th} node of triangular element

N_{qs} = Bearing capacity factor due to surcharge pressure for unsaturated soil

N_s = Total number of nodes along the footing–soil interface

N_γ = Bearing capacity factor due to unit weight of soil

N_q = Bearing capacity factor due to surcharge

$N_{\gamma s}$ = Bearing capacity factor due to unit weight for unsaturated soil

N_σ = Non-dimensional bearing capacity factor for rock mass

$N_{\sigma 0}$ = Nondimensional bearing capacity factor for the weightless rock mass

* OHB= Ordinary Hoek-Brown

* GHB= Generalized Hoek-Brown

p = Number of sides of a polygon required to linearize the MC yield criterion

p_u = Ultimate bearing capacity of footing

P_{1e} = Rate of work done by the body forces

P_{2e} = Power dissipation in an element due to plastic deformation

P_{3d} = Power dissipation along a velocity discontinuity

P_{ext} = Rate of the external work done defined in Equation (7.28)

P_{int} = Internal power dissipation defined in Equation (7.29)

q = Normal surcharge (Chapter 3)/discharge flow rate (Chapter 7)

q_s = Surcharge pressure on unsaturated soil

Q = Vertical compressive downward load of magnitude

Q_u = Maximum vertical compressive downward load of magnitude

$\bar{r}$ = Horizontal distance between the centroid of the triangle element and axis of the footing

r_0 = Outer radius of ring footing

r_i = Inner radius of ring footing

r_s = Mean radius of boundary edge L_s

$\boldsymbol{s}$ = Slack variable

s = Nondimentional strength parameter of OHB criterion

S_r = Residual degree of saturation

S_e = Effectivel degree of saturation

S = Degree of saturation

t_1 and t_2 = Prescribed shear stresses at boundary edge

t_{cr} = critical time

u = Horizontal velocity (unknown)

u_a = Pore air pressure

u_w = Pore water pressure

u_{ij}^{+}, u_{ij}^{-} = Non-negative velocity discontinuity

v = Vertical velocity (unknown)

Δu = Normal velocity jump

Δv = Tangential velocity jump

v_n = Velocity specified for the footing nodes

y = Distance above water table

Z_{inf} = Zone of influence

$\dot{\lambda}$ = Non-negative plastic multiplier rate

λ = Lagrangian multiplier for the inequality constraints

υ = Lagrangian multiplier for the equality constraints

$\dot{\varepsilon}_{xx}$ = Plastic normal strain rates in x direction

$\dot{\varepsilon}_{zz}$ = Plastic normal strain rates in z direction

$\dot{\gamma}_{xz}$ = Plastic shear strain rates in xz direction

γ = Unit density of soil

η = Efficiency factor

μ^k = Barrier parameter at k^{th} iteration

δ = Interface friction angle between footing and adjoining soil mass

Ψ = Angle made by the footing-soil interface with +r-axis

$\overleftrightarrow{\sigma}$ = Stress invariant by permuting $\overline{\sigma}$ with a small term ε

$\overline{\sigma}$ = Octahedral/Deviatoric Stress

σ_n = Normal stress acting on a plane

σ_{nt} = Shear stress acting along a plane

σ_r = Normal stress in r direction

σ_z = Normal stress in z direction

σ_m = Hydrostatic Stress

σ_{ci} = Uniaxial compressive strength of intact rock

σ_1 = Major principal stress

σ_3 = Minor principal stress

σ^s = Suction stress

σ_θ = Hoop stress

σ' = effective normal stress

σ_2 = intermediate principal stress

θ = Lode angle (Chapter 3) / circumferential direction(Chapter 3) /volumetric water content (Chapter 7)

θ_r = Residual volumetric water content

θ_s = Saturated volumetric water content

Θ_n = Normalized water content

θ_T = Transition angle

τ_{rz} = Shear stress in r–z plane

μ^k = Barrier parameter

$f(\sigma)$ = Yield condition

$h(\sigma)$ = Set of inequality constraints defined in Equation (3.14)

$u(\sigma)$ = Set of inequality constraints defined in Equation (3.14)

β = Inclination of load

χ = Saturation-state-dependent single valued effective stress parameter

γ_w = Unit weight of water

δ = Interface friction angle between the footing base and soil

ϕ = Angle of friction of soil

ϕ_u = Undrained cohesion

ϕ' = Effective angle of friction of soil

ϕ_b = Angle indicating the rate of increase in shear strength with respect to a change in matric suction

ψ = Dilation angle (Chapter 4)/ matric suction (Chapter 7)

α_b = Adhesion factors for pile base cohesion

α_s = Adhesion factors for and pile shaft-soil interface cohesion

α = Exponential term in the GHB criterion (Chapter 6)/air entry parameter (Chapter 7)

ε = Offset distance along hydrostatic axis for smoothening the apex of the yield (MC/GHB) pyramid

η = Efficiency factor accounts for the presence of sand layer over clayey stratum

ω = Angle made by the stress with +*r*-axis /velocity discontinuity line with +*x*-*axis*

Ω = Angle made by the stress with +*r*-axis /velocity discontinuity line with +*x*-*axis*

LIST OF TABLES