ON INDUCTIVE ALGEBRAS

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by

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Conclusion

In this thesis, we have studied the structure and classification of inductive algebras, which are defined as weakly closed abelian subalgebras of $\mathcal{B}(\mathcal{H})$ that are normalized by a unitary representation π of a locally compact group G. These algebras, also called π-inductive algebras, are of interest not only in abstract harmonic analysis but also in the study of operator algebras and representation theory.

We have shown that for the affine group of a finite field, each irreducible unitary representation admits a unique maximal π-inductive algebra, which is explicitly described and shown to be self-adjoint. This provides a complete classification in that setting. In the case of compact groups, we proved that all π-inductive algebras are necessarily self-adjoint, thereby significantly simplifying the classification problem. This is due to the availability of spectral-theoretic tools in the self-adjoint setting.

For the motion group of the plane $M(2) = SO(2) \ltimes \mathbb{R}^2$, we established that each irreducible unitary representation also admits a unique maximal π-inductive algebra, which again turns out to be self-adjoint. This result parallels our findings in the case of finite and compact groups, and further emphasizes the role of symmetry and group structure in determining the nature of inductive algebras.

Altogether, the results presented in this thesis offer new insights into the realization theory of representations, particularly in the context of Mackey's Imprimitivity Theorem. They also provide concrete classifications of inductive algebras in several important group settings, and serve as a foundation for further investigation into the role of such algebras in analysis and geometry.