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PREFACE

Chapter 1 includes a glimpse of basics of dynamical systems classification and history of fractional calculus, chaos theory, synchronization, types of synchronization, Lyapunov exponent, Stability theory of fractional order systems and different synchronization methods which have used in this thesis. Each topic plays an important role during preparation of this thesis.

Chapter 2 reports an investigation on dual phase synchronization results among chaotic systems with nonlinear observer controller. The dual phase synchronization is achieved using nonlinear state observer technique and stability theory. Qi system and Newton-Leipnik system are considered during demonstration of dual phase synchronization. The nonlinear state observer technique is found to be very effective and convenient to achieve dual phase synchronization of various types of chaotic systems.

In Chapter 3, the difference synchronization and chaos control of chaotic systems with nonlinear exponential terms have been studied by using the feedback control method. The Routh-Hurwitz condition is used during chaos control and synchronization. The nonlinear ten-ring chaotic system, 3D chaotic system, new 3D chaotic system are considered to simulate the difference synchronization scheme for continuous case, and Wang, 3D Henon map and Rossler systems are considered during simulation of discrete time chaotic systems.

Chapter 4 addresses exponential synchronization between fractional order chaotic systems. The stability analysis has been done with the help of a new lemma, which is given for Lyapunov function for fractional order system. The fractional order complex chaotic systems viz., Lorenz and Lu systems are considered to illustrate the exponential synchronization. The application in communication through digital cryptography is also discussed between the sender (transmitter) and receiver using the exponential

synchronization. A well secured key system of a message is obtained in a systematic and simple way.

Chapter 5 contains the stability analysis and chaos control of Simple chaotic system. During hybrid projective synchronization the fractional order Simple chaotic system is considered as drive system and fractional order Lu chaotic system is taken as response systems. Nonlinear control method has been used to analyse the hybrid projective synchronization of fractional order simple system.

In Chapter 6, the stability analysis, chaos control and the function projective synchronization between fractional order identical satellite systems have been studied. Based on the stability theory of fractional order systems, the conditions of local stability of nonlinear three-dimensional commensurate and incommensurate fractional order systems are discussed. Feedback control method is used to control the chaos in the considered fractional order satellite system. Using the fractional calculus theory and computer simulation, it is found that the chaotic behaviour exists in the fractional order satellite system.