

CHAPTER

4

Chapter 4. Modeling, Identification and IMC based PID Control of Two Input Two Output Non-minimum Phase Nonlinear Process

4.1. Introduction

Several multiple input multiple output processes such as distillation columns, absorbers, reactors etc. also involve multiple control loops and also exhibit non-minimum phase behaviour. Design of efficient control systems for such processes involve study of effect of control loop interactions and decoupling techniques in addition to mathematical modelling and identification. At the laboratory scale, the experimental Quadruple Tank Process (QTP) developed by (Johansson, 2000) serves as a good example to carry out in-depth study of a Two Input Two Output (TITO) non minimum phase process (Mehri & Tabatabaei, 2021) (Sutha et al., 2015) (Moradi & Katebi, 2005).

4.1.1 Experimental Quadruple Tank Process (QTP)

The experimental setup of Quadruple Tank Process (QTP) (Make: Apex Innovations Pvt. Ltd., Model No.: 327A) used in this work is shown in Figure 4.1. The schematic of Quadruple Tank Process (QTP) is depicted in Figure 4.2. The experimental process consists of four cylindrical tanks. Two Variable Frequency Drive (VFD) regulated positive displacement pumps P1 and P2 are used to supply water from the reservoir to the four tanks, in fully circulating mode.

Water from pump P1 flowing through a three-way valve is split into a (desired) fraction γ_1 and fed to tank 1 and tank 4. Similarly, the water from pump P2 flows through another three-way valve where it is split into another (desired) fraction γ_2 and fed to tank 2 and

tank 3. The water outlet (through adjustable valves) from tank 3 and tank 4 serves as the second inlet source for tank 1 and tank 2, respectively. Finally, the outlets of tank 1 and tank 2 drain water back to the reservoir, through respective adjustable valves. Level transmitters are used to measure the liquid levels of the two bottom tanks, tank 1 and tank 2. Signals from the level transmitters act as the two controlled variables (CV) that are sent to a Serial based dual loop PID controller, which in turn is connected to the USB port of a computer. The physical process is operated in both open loop and closed-loop modes, through the computer software. The controller output signals serve as the two manipulated variables (MV), which act as input to the two variable frequency drives (VFD) for manipulating the volumetric flowrate of water discharging through the two pumps. The technical specifications of the experimental setup are listed in Table 4.1.



Figure 4. 1 Experimental setup of the Quadruple Tank Process (QTP)

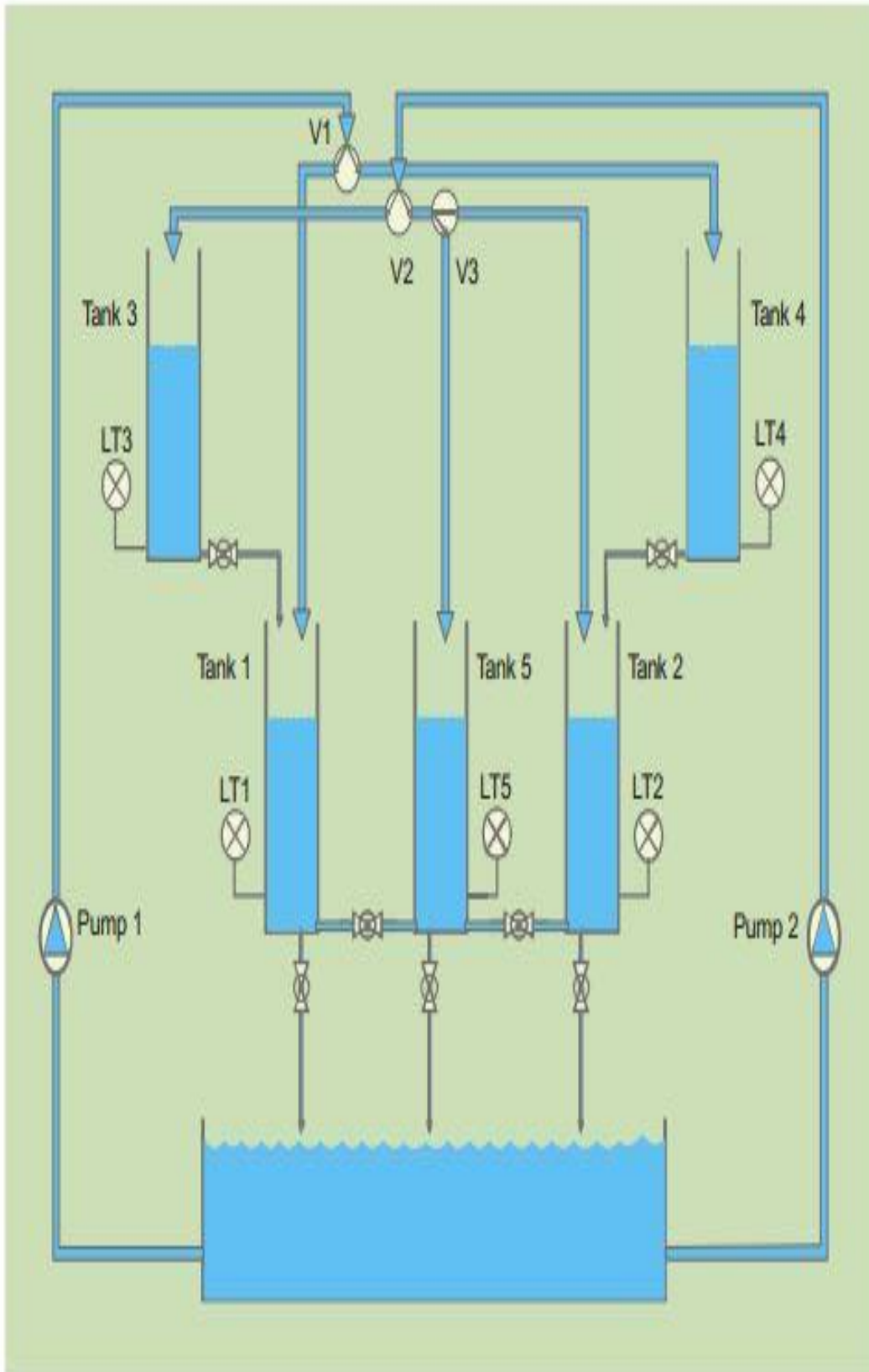


Figure 4. 2 Schematic of the Quadruple Tank Process (QTP)

Table 4. 1 Technical specifications of the experimental Quadruple Tank Process

Product	Multi variable control trainer
Product code	327A
Type of control	SCADA
Control unit	ADAM-4022T Serial based dual loop PID controller; Analog input 4, Analog output 2, Digital input 2, Digital output 2. with RS485 communication.
RS485-USB converter	Make Advantech, Model ADAM 4561, 1 Port USB to RS232/485/422 converter
Communication	USB port using RS485-USB converter
Power supply	Make Meanwell, Model NES-15-24, O/P 24 V, 0.7 A
Level transmitter	Make Wika, Model SL-1-A-MAG-ND-ZA4Z-ZZZ and output 4-20 mA, supply 10-30 Vdc, conn. 1/2"NPT (M), Range 0-25 mbar. Type Electronic, two wire, Range 0–250 mm, Output 4–20mA (5 Nos)
Dosing pump	Positive displacement dosing pump with adjustable stroke (2Nos)
Variable Frequency Drive (VFD)	Programmable, input single phase 200VAC, 1.1A, output AC 3 phase 0-230V, 0.3kVA Make Emerson Industrial Automation , Model CGK2100075, 0.75 KW , 1ph I/P , ,230 V O/P 3 ph
Diaphragm pump	Make SITC Spanish, Model 60-AD43-D95 PBM, Cap 174 LPH, with 230V, 0.5 HP, 3ph motor
Process tank	Transparent, Acrylic, with 0-100% graduated scale (5 Nos)
Supply tank	SS304
Overall dimensions	600Wx600Dx1650H mm Gross volume 1.40m ³ , Gross weight 177kg, Net weight 95 kg

4.2 Mathematical Model of Quadruple Tank Process (QTP)

The process variables and system parameters in the Quadruple Tank Process (QTP) are shown in Figure 4.3. Notations of process variables and system parameters are shown in Table 4.2

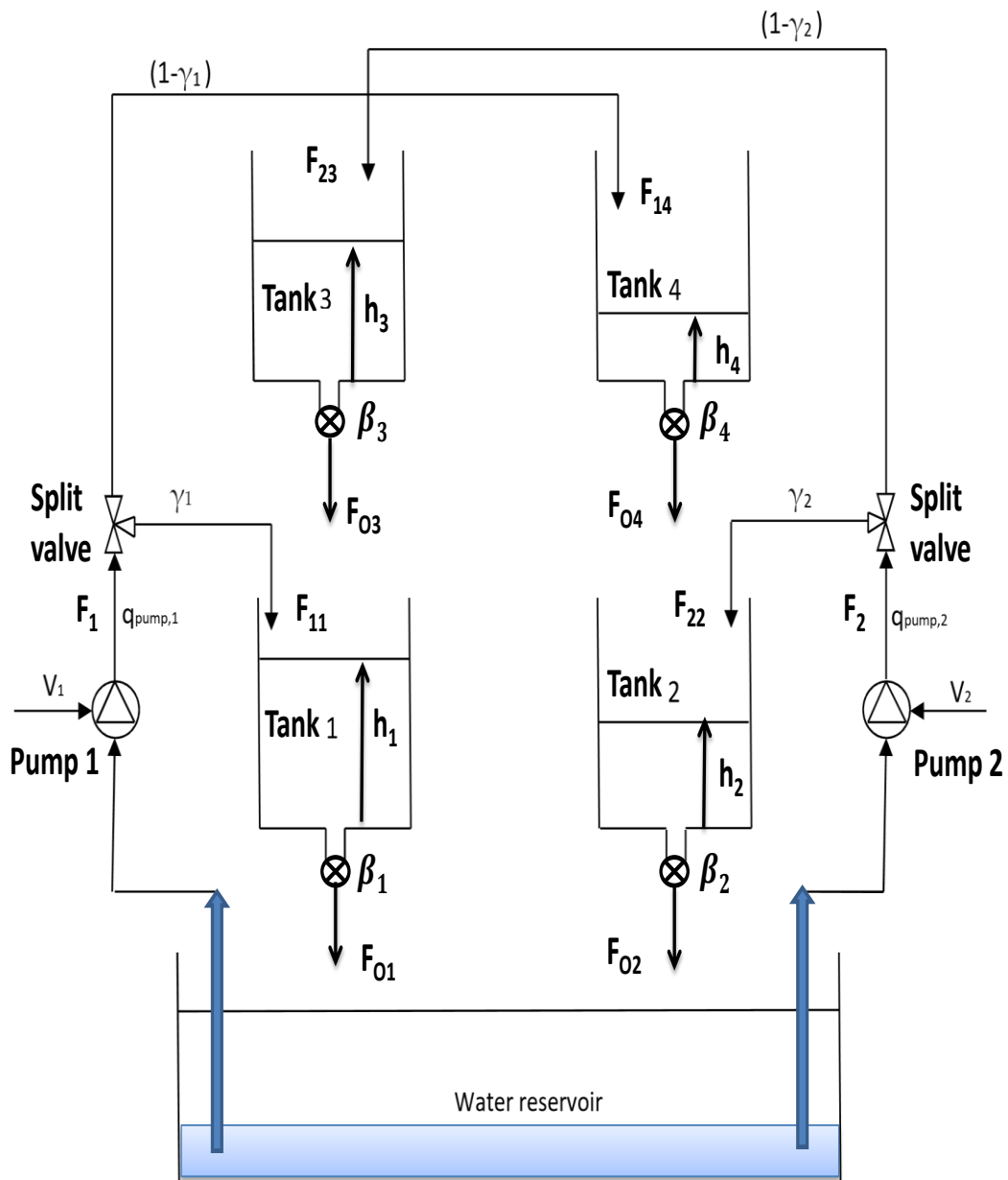


Figure 4. 3 Process Variables and System Parameters in the Quadruple Tank Process (QTP)

Table 4. 2 Process Variables and System Parameters in QTP

Process Variables	
Total volumetric flowrates of pump 1 and pump 2 respectively	F_1, F_2
Volumetric flowrate of water flowing from Pump i to tank j.	F_{ij}
Outlet volumetric flowrate discharging through tank j.	F_{Oj}
Height of liquid level in tank j	h_j
System parameters	
Adjustable split fraction of water flowing from Pump 1 to tank 1	$\gamma_1 (0 \leq \gamma_1 \leq 1)$
Adjustable split fraction of water flowing from Pump 2 to tank 2	$\gamma_2 (0 \leq \gamma_2 \leq 1)$
Nonlinear flow resistance of outlet discharge valve of tank j.	β_j
Area of tank j.	A_j
Voltage to pump 1 and pump 2 respectively	V_1, V_2

4.2.1 Assumptions

The development of Mathematical model for the Quadruple Tank Process (QTP) involves the following assumptions:

- i. Constant density

Since no heat effects are involved in the process, the density of water at the operating conditions is assumed constant.

- ii. Hydraulic relationships

Based on the Bernoulli's principle, a square root relationship between the outlet volumetric flowrate and height of level in each tank is assumed.

$$F_{O1} = \beta_1 \sqrt{h_1} \quad \text{Eq. 4. 1}$$

$$F_{O2} = \beta_2 \sqrt{h_2} \quad \text{Eq. 4. 2}$$

$$F_{O3} = \beta_3 \sqrt{h_3} \quad \text{Eq. 4. 3}$$

$$F_{O4} = \beta_4 \sqrt{h_4} \quad \text{Eq. 4. 4}$$

4.2.2 Process Operating Conditions:

The Quadruple Tank Process (QTP) is operated at fixed (constant) values of the split fractions, γ_1 and γ_2 . The inlet volumetric flowrates of water in the four tanks are calculated as:

$$F_{11} = \gamma_1 F_1 \quad \text{Eq. 4. 5}$$

$$F_{14} = (1 - \gamma_1) F_1 \quad \text{Eq. 4. 6}$$

$$F_{22} = \gamma_2 F_2 \quad \text{Eq. 4. 7}$$

$$F_{23} = (1 - \gamma_2) F_2 \quad \text{Eq. 4. 8}$$

4.2.3 Unsteady State Mass balances

The unsteady state mass balance equations for the four tanks are written as:

$$f_1(h_1, h_2, h_3, h_4, F_1, F_1) = \frac{dh_1}{dt} = \frac{\gamma_1 F_1}{A_1} + \frac{\beta_3 \sqrt{h_3}}{A_1} - \frac{\beta_1 \sqrt{h_1}}{A_1} \quad \text{Eq. 4. 9}$$

$$f_2(h_1, h_2, h_3, h_4, F_1, F_1) = \frac{dh_2}{dt} = \frac{\gamma_2 F_2}{A_2} + \frac{\beta_4 \sqrt{h_4}}{A_2} - \frac{\beta_2 \sqrt{h_2}}{A_2} \quad \text{Eq. 4. 10}$$

$$f_3(h_1, h_2, h_3, h_4, F_1, F_1) = \frac{dh_3}{dt} = \frac{(1 - \gamma_2) F_2}{A_3} - \frac{\beta_3 \sqrt{h_3}}{A_3} \quad \text{Eq. 4. 11}$$

$$f_4(h_1, h_2, h_3, h_4, F_1, F_1) = \frac{dh_4}{dt} = \frac{(1 - \gamma_1) F_1}{A_4} - \frac{\beta_4 \sqrt{h_4}}{A_4} \quad \text{Eq. 4. 12}$$

4.2.4 Steady State Solution

At steady state, all the time derivatives in equations 4.9 to 4.12 vanish, i.e.

$$f_1 = f_2 = f_3 = f_4 = 0 \quad \text{Eq. 4. 13}$$

The resulting nonlinear algebraic equations are sequentially solved to obtain the steady states for the four tanks, as shown below:

$$h_{1s} = \left(\frac{\gamma_1 F_{1s} + (1 - \gamma_2) F_{2s}}{\beta_1} \right)^2 \quad \text{Eq. 4. 14}$$

$$h_{2s} = \left(\frac{\gamma_2 F_{2s} + (1 - \gamma_1) F_{1s}}{\beta_2} \right)^2 \quad \text{Eq. 4. 15}$$

$$h_{3s} = \left[\frac{(1 - \gamma_2) F_{2s}}{\beta_3} \right]^2 \quad \text{Eq. 4. 16}$$

$$h_{4s} = \left[\frac{(1 - \gamma_1) F_{1s}}{\beta_4} \right]^2 \quad \text{Eq. 4. 17}$$

In the equations 4.14 to 4.17, the subscript 's' is used to indicate steady state operation.

4.3 Linearized State Space Model of the Quadruple-Tank Process (QTP)

The linearized state space model of a multivariable nonlinear process is represented by equations 3.96 and 3.97.

Defining the (4x1) vector of state variables (in deviation form) as:

$$\mathbf{x} = \begin{pmatrix} h_1 - h_{1s} \\ h_2 - h_{2s} \\ h_3 - h_{3s} \\ h_4 - h_{4s} \end{pmatrix} \quad \text{Eq. 4. 18}$$

Defining the (2x1) vector of input variables (in deviation form) as:

$$\mathbf{u} = \begin{pmatrix} F_1 - F_{1s} \\ F_2 - F_{2s} \end{pmatrix} \quad \text{Eq. 4. 19}$$

The (4x1) vector of time derivatives of the state variables (in deviation form) is written as:

$$\dot{\mathbf{x}} = \begin{pmatrix} \frac{d(h_1 - h_{1s})}{dt} \\ \frac{d(h_2 - h_{2s})}{dt} \\ \frac{d(h_3 - h_{3s})}{dt} \\ \frac{d(h_4 - h_{4s})}{dt} \end{pmatrix} = \begin{pmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \\ \frac{dh_3}{dt} \\ \frac{dh_4}{dt} \end{pmatrix} \quad \text{Eq. 4. 20}$$

The elements of (4x4) matrix of partial derivatives with respect to the state variables (Matrix **A**) are expressed as:

$$a_{11} = \left. \frac{\partial f_1}{\partial h_1} \right|_s = \frac{-1}{2\sqrt{h_{1s}}} \left(\frac{\beta_1}{A_1} \right) \quad \text{Eq. 4. 21}$$

$$a_{12} = \left. \frac{\partial f_1}{\partial h_2} \right|_s = 0 \quad \text{Eq. 4. 22}$$

$$a_{13} = \left. \frac{\partial f_1}{\partial h_3} \right|_s = \frac{\beta_3}{2A_1\sqrt{h_{3s}}} \quad \text{Eq. 4. 23}$$

$$a_{14} = \left. \frac{\partial f_1}{\partial h_4} \right|_s = 0 \quad \text{Eq. 4. 24}$$

$$a_{21} = \left. \frac{\partial f_2}{\partial h_1} \right|_s = 0 \quad \text{Eq. 4. 25}$$

$$a_{22} = \frac{\partial f_2}{\partial h_2} \Big|_s = \frac{-1}{2A_2\sqrt{h_{2s}}} (\beta_2) \quad \text{Eq. 4. 26}$$

$$a_{23} = \frac{\partial f_2}{\partial h_3} \Big|_s = 0 \quad \text{Eq. 4. 27}$$

$$a_{24} = \frac{\partial f_2}{\partial h_4} \Big|_s = \frac{\beta_4}{2A_2\sqrt{h_{4s}}} \quad \text{Eq. 4. 28}$$

$$a_{31} = \frac{\partial f_3}{\partial h_1} \Big|_s = 0 \quad \text{Eq. 4. 29}$$

$$a_{32} = \frac{\partial f_3}{\partial h_2} \Big|_s = 0 \quad \text{Eq. 4. 30}$$

$$a_{33} = \frac{\partial f_3}{\partial h_3} \Big|_s = \frac{-\beta_3}{2A_3\sqrt{h_{3s}}} \quad \text{Eq. 4. 31}$$

$$a_{34} = \frac{\partial f_3}{\partial h_4} \Big|_s = 0 \quad \text{Eq. 4. 32}$$

$$a_{41} = \frac{\partial f_4}{\partial h_1} \Big|_s = 0 \quad \text{Eq. 4. 33}$$

$$a_{42} = \frac{\partial f_4}{\partial h_2} \Big|_s = 0 \quad \text{Eq. 4. 34}$$

$$a_{43} = \frac{\partial f_4}{\partial h_3} \Big|_s = 0 \quad \text{Eq. 4. 35}$$

$$a_{44} = \frac{\partial f_4}{\partial h_4} \Big|_s = \frac{-\beta_4}{2A_4\sqrt{h_{4s}}} \quad \text{Eq. 4. 36}$$

Similarly, the elements of (4x2) matrix of partial derivatives with respect to the input variables (Matrix **B**) are expressed as:

$$b_{11} = \frac{\partial f_1}{\partial F_1} \Big|_s = \frac{\gamma_1}{A_1} \quad \text{Eq. 4. 37}$$

$$b_{12} = \frac{\partial f_1}{\partial F_2} \Big|_s = 0 \quad \text{Eq. 4. 38}$$

$$b_{21} = \frac{\partial f_2}{\partial F_1} \Big|_s = 0 \quad \text{Eq. 4. 39}$$

$$b_{22} = \frac{\partial f_2}{\partial F_2} \Big|_s = \frac{\gamma_2}{A_2} \quad \text{Eq. 4. 40}$$

$$b_{31} = \frac{\partial f_3}{\partial F_1} \Big|_s = 0 \quad \text{Eq. 4. 41}$$

$$b_{32} = \frac{\partial f_3}{\partial F_2} \Big|_s = \frac{(1-\gamma_2)}{A_3} \quad \text{Eq. 4. 42}$$

$$b_{41} = \frac{\partial f_4}{\partial F_1} \Big|_s = \frac{(1-\gamma_1)}{A_4} \quad \text{Eq. 4. 43}$$

$$b_{42} = \frac{\partial f_4}{\partial F_2} \Big|_s = 0 \quad \text{Eq. 4. 44}$$

4.3.1 Degrees of Freedom Analysis

The degrees of freedom analysis is useful in identifying the number of measured outputs and the number of controllers required. The total number of variables is $n_v = 6$, total number of equations is $n_e = 4$ (equations 4.9-4.12). The degrees of freedom, is $f = n_v - n_e = 2$. Therefore, the number of measured outputs and the number of controllers required is two.

4.3.2 Classification of Variables

The degrees of freedom analysis suggests that the Quadruple-Tank Process (QTP) is a Two Input Two Output (TITO) process from a control perspective. The control objective is therefore to design a suitable IMC based PID controller for maintaining the liquid levels of tank 1 and tank 2 (h_1 and h_2 in deviation form) at their desired (set point) values, by manipulating the volumetric flowrates (F_1 and F_2 in deviation form) of the two pumps. The variables and parameters involved in the Quadruple-Tank Process (QTP) are classified as shown in Table 4.3

Table 4.3 Classification of Process Variables and System Parameters in QTP

Process Outputs (in deviation form)	Process Inputs (in deviation form)		Process Parameters (j=1,2,3,4) (i=1,2)
	Controlled Variables (CV)	Manipulated variables (MV)	
h_1	F_1	Nil	β_j
h_2	F_2		A_j
			γ_i

Since the required number of measured outputs is two, the (2x1) vector of measured outputs is defined as:

$$\mathbf{y} = \begin{pmatrix} h_1 - h_{1s} \\ h_2 - h_{2s} \end{pmatrix} \quad \text{Eq. 4. 45}$$

The elements of (2x4) matrix of constant coefficients (Matrix **C**) are accordingly selected as:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{Eq. 4. 46}$$

The elements of (2x2) matrix of constant coefficients (Matrix **D**) are taken as zero (equation 3.99). The (linearized) flow resistances for each tank are expressed as:

$$R_j = \frac{2\sqrt{h_{js}}}{\beta_j} \quad \text{Eq. 4. 47}$$

The process time constants for each tank are expressed as the product of capacitance (tank area) and resistance:

$$\tau_j = A_j R_j = \frac{2A_j \sqrt{h_{js}}}{\beta_j} \quad \text{Eq. 4. 48}$$

Finally, the (4x4) **A** matrix (equations 4.21-4.36) is expressed in terms of process time constants (equation 4.48) as:

$$\mathbf{A} = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{A_3 / A_1}{\tau_3} & 0 \\ 0 & \frac{-1}{\tau_2} & 0 & \frac{A_4 / A_2}{\tau_4} \\ 0 & 0 & \frac{-1}{\tau_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_4} \end{bmatrix} \quad \text{Eq. 4. 49}$$

The (4x2) **B** matrix (equations 4.37-4.44) is written as:

$$\mathbf{B} = \begin{bmatrix} \frac{\gamma_1}{A_1} & 0 \\ 0 & \frac{\gamma_2}{A_2} \\ 0 & \frac{1-\gamma_2}{A_3} \\ \frac{1-\gamma_1}{A_4} & 0 \end{bmatrix} \quad \text{Eq. 4. 50}$$

4.4 Transfer function Model of Quadruple-Tank Process (QTP)

The multivariable process transfer function matrix, $\mathbf{G}(s)$ for the Quadruple-Tank Process (QTP) is obtained using the equation 3.106, as shown below:

$$\mathbf{G}(s) = \begin{pmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{pmatrix} = \begin{bmatrix} \frac{K_{11}}{(\tau_1 s + 1)} & \frac{K_{12}}{(\tau_1 s + 1)(\tau_3 s + 1)} \\ \frac{K_{21}}{(\tau_2 s + 1)(\tau_4 s + 1)} & \frac{K_{22}}{(\tau_2 s + 1)} \end{bmatrix} \quad \text{Eq. 4. 51}$$

Where, the process (steady state) gains are defined as:

$$K_{ii} = \gamma_i R_i \quad (i=1, 2) \quad \text{Eq. 4. 52}$$

$$K_{ij} = (1 - \gamma_i) R_j \quad (i=1, 2; j=1, 2; i \neq j) \quad \text{Eq. 4. 53}$$

The transfer function matrix, $\mathbf{G}(s)$ has two first order transfer functions and two second order transfer functions:

First order transfer functions:

The transfer function relating the liquid level in tank 1 with the volumetric flowrate of pump 1 is first order and is expressed as:

$$g_{11}(s) = \frac{y_1(s)}{u_1(s)} = \frac{K_{11}}{(\tau_1 s + 1)} \quad \text{Eq. 4. 54}$$

Similarly, the transfer function relating the liquid level in tank 2 with the volumetric flowrate of pump 2 is first order and is expressed as:

$$g_{22}(s) = \frac{y_2(s)}{u_2(s)} = \frac{K_{22}}{(\tau_2 s + 1)} \quad \text{Eq. 4. 55}$$

Second order transfer functions:

The transfer function relating the liquid level in tank 1 with the volumetric flowrate of pump 2 is second order and is expressed as:

$$g_{12}(s) = \frac{y_1(s)}{u_2(s)} = \frac{K_{12}}{(\tau_1 s + 1)(\tau_3 s + 1)} \quad \text{Eq. 4. 56}$$

Similarly, the transfer function relating the liquid level in tank 2 with the volumetric flowrate of pump 1 is second order and is expressed as:

$$g_{21}(s) = \frac{y_2(s)}{u_1(s)} = \frac{K_{21}}{(\tau_2 s + 1)(\tau_4 s + 1)} \quad \text{Eq. 4. 57}$$

4.5 Transmission Zeros of Quadruple-Tank Process (QTP)

The transmission zeros of a multivariable process transfer function matrix are obtained by setting its determinant to zero and solving the resulting polynomial in s .

$$\det[\mathbf{G}(s)] = [g_{11}(s)g_{22}(s) - g_{21}(s)g_{12}(s)] = 0 \quad \text{Eq. 4. 58}$$

Determinant of $\mathbf{G}(s)$ is obtained from equation 4.51, as:

$$\det[\mathbf{G}(s)] = \frac{\mathbf{K}_{11}\mathbf{K}_{22}}{\prod_{i=1}^4 (\tau_i s + 1)} \left[(\tau_3 s + 1)(\tau_4 s + 1) - \frac{\mathbf{K}_{12}\mathbf{K}_{21}}{\mathbf{K}_{11}\mathbf{K}_{22}} \right] = 0 \quad \text{Eq. 4. 59}$$

Upon Simplification:

$$(\tau_3 s + 1)(\tau_4 s + 1) - \frac{(1 - \gamma_1)(1 - \gamma_2)}{\gamma_1 \gamma_2} \quad \text{Eq. 4. 60}$$

$$\text{Let, } \eta = \frac{(1 - \gamma_1)(1 - \gamma_2)}{\gamma_1 \gamma_2} \quad \text{Eq. 4. 61}$$

Substituting equation 4.61 into 4.60 gives:

$$\tau_3 \tau_4 s^2 + (\tau_3 + \tau_4) s + (1 - \eta) = 0$$

Eq. 4. 62

Equation 4.62 is quadratic in s . The two transmission zeros z_1 and z_2 are obtained from the roots of quadratic equation. Equation 4.62 is re-written in standard form of a quadratic equation as shown below:

$$as^2 + bs + c = 0 \quad \text{Eq. 4. 63}$$

where, the coefficients are defined as:

$$a = \tau_3 \tau_4 \quad \text{Eq. 4. 64}$$

$$b = (\tau_3 + \tau_4) \quad \text{Eq. 4. 65}$$

$$c = (1 - \eta) = \frac{(\gamma_1 + \gamma_2) - 1}{\gamma_1 \gamma_2} \quad \text{Eq. 4. 66}$$

The discriminant of quadratic equation (equation 4.63) is:

$$d = \sqrt{b^2 - 4ac} \quad \text{Eq. 4. 67}$$

The solution of quadratic equation (transmission zeros) is given as:

$$z_1 = \frac{-b + d}{2a} \quad \text{Eq. 4. 68}$$

$$z_2 = -\frac{(b + d)}{2a} \quad \text{Eq. 4. 69}$$

Equation 4.69 suggests that the transmission zero, z_2 is always negative. Therefore, z_2 always lies on the left half plane (LHP zero). Whereas, equation 4.68 suggests that the transmission zero, z_1 may take negative, positive or zero values, depending on the value of the coefficient c (equation 4.67), which in turn depends on the process operating parameters γ_1 and γ_2 . Thus, the location of the transmission zero, z_1 on complex plane depends on the operating conditions of the process. Therefore, γ_1 and γ_2 are the crucial process operating parameters that define whether the Quadruple-Tank Process (QTP) behaves as minimum phase (LHP transmission zeros) or non-minimum phase (at least one RHP transmission zero).

Mathematically, value of the coefficient c (equation 4.67) can be either negative or positive or zero. The following three cases are possible:

i) Case A: Coefficient c (equation 4.67) is zero:

$$c = \frac{(\gamma_1 + \gamma_2) - 1}{\gamma_1 \gamma_2} = 0 \quad \text{Eq. 4. 70}$$

Upon simplification, the following condition is obtained:

$$(\gamma_1 + \gamma_2) = 1 \quad \text{Eq. 4. 71}$$

Solution of equation 4.68 gives:

$$z_1 = 0 \quad \text{Eq. 4. 72}$$

Solution of equation 4.69 gives:

$$z_2 = \frac{-b}{a} \quad \text{Eq. 4. 73}$$

Equation 4.72 suggests that one of the two transmission zeros, z_1 is located at origin and equation 4.73 suggests that the other transmission zero, z_2 is located on the left half plane (LHP).

ii) Case B: Coefficient c (equation 4.67) is negative:

$$c = \frac{(\gamma_1 + \gamma_2) - 1}{\gamma_1 \gamma_2} < 0 \quad \text{Eq. 4. 74}$$

Upon simplification, the following condition is obtained:

$$(\gamma_1 + \gamma_2) < 1 \quad \text{Eq. 4. 75}$$

Solution of equation 4.68 gives:

$$z_1 > 0 \quad \text{Eq. 4. 76}$$

Solution of equation 4.69 gives:

$$z_2 < 0 \quad \text{Eq. 4. 77}$$

Equation 4.76 suggests that one of the two transmission zeros, z_1 is located on the right half plane (RHP). Thus the Quadruple Tank Process (QTP) behaves as non-minimum

phase under the operating condition (equation 4.75). Equation 4.77 suggests that the other transmission zero, z_2 is located on the left half plane (LHP)

iii) Case C: Coefficient c (equation 4.67) is positive:

$$c = \frac{(\gamma_1 + \gamma_2) - 1}{\gamma_1 \gamma_2} > 0 \quad \text{Eq. 4. 78}$$

Upon simplification, the following condition is obtained:

$$(\gamma_1 + \gamma_2) > 1 \quad \text{Eq. 4. 79}$$

Solution of equation 4.68 gives:

$$z_1 < 0 \quad \text{Eq. 4. 80}$$

Solution of equation 4.69 gives:

$$z_2 < 0 \quad \text{Eq. 4. 81}$$

Equation 4.80 suggests that one of the two transmission zeros, z_1 is located on the left half plane (LHP). Thus the Quadruple Tank Process (QTP) behaves as minimum phase under the operating condition (equation 4.79). Equation 4.81 suggests that the other transmission zero z_2 is always located on the left half plane (LHP).

4.6 Identification of Quadruple-Tank Process (QTP)

The design parameters of Quadruple-Tank Process (QTP) experimental setup are shown in Table 4.4:

Table 4. 4 Design Parameters of Quadruple-Tank Process Experimental Setup

Design parameter	Value
Internal diameter of each tank	9.2 cm
Maximum height (H) of each tank	26.5 cm
Cross sectional area of each tank A_i	66.48 cm ²
Measured value of normalized process variable h_1	PV0 (%)
Measured value of normalized process variable h_2	PV1 (%)
Measured value of normalized process variable h_3	PV2 (%)
Measured value of normalized process variable h_4	PV3 (%)
Measured value of normalized manipulated variable (Volumetric flowrate of pump P1) F_1	OP0 (%)
Measured value of normalized manipulated variable (Volumetric flowrate of pump P2) F_2	OP1 (%)
Linear velocity of inlet fluid (under zero outflow conditions)	v_i

Identification of the Quadruple-Tank Process (QTP) was carried out by performing open loop experiments in the following two modes:

- a) Pure capacity mode: In the pure capacity mode, the outlet valves of all the four tanks were fully closed. It was ensured that there was only in-flow and no out-flow of liquid in all the four tanks. The experimental data was used to estimate the split fractions (γ_1 and γ_2)
- b) Resistance mode: In the resistance mode, the outlet valves of all the four tanks were slightly opened (at fixed positions). The experimental data was used to estimate the nonlinear flow resistances of each of the four tanks, β_j

4.6.1 Open loop Experiments (Pure Capacity Mode)

The unknown process parameters were evaluated from open loop experiments, as shown in the following steps:

4.6.1.1 Experimental Procedure

1. To begin with, the experimental setup was started and interfaced to a personal computer, in which, the values of the four state variables (h_1, h_2, h_3 and h_4) and the two input (manipulated) variables (F_1 and F_2) were recorded at different time instants.
2. The experimental setup was operated at non-minimum phase condition ($\gamma_1 + \gamma_2 < 1$) by trial and error adjustment of the two (three-way) split valves. The pump flow split fractions, γ_1 and γ_2 were fixed for all the open loop and closed loop experiments. The non-minimum phase condition ($\gamma_1 + \gamma_2 < 1$) was physically verified from the observation that the two upper tanks filled faster than the two lower ones.
3. The outlet valves of all the four tanks were fully closed. It was ensured that the experiments were conducted in pure capacity mode (there was only in-flow and no out-flow of liquid in all the four tanks).
4. Both the pumps were operated at their maximum volumetric flowrates F_1 and F_2 . This was achieved by setting the values of OP0% and OP1% to 100 in open loop/manual mode.
5. The setup was thus operated in purely capacitive mode and all the measurements were recorded. Experiment was repeated for a slightly different setting of γ_1 and γ_2 . The experimental observations of two independent experiments are shown in Table 4.4 and Table 4.5.

Table 4.5 Open loop Experiment 1 (Purely Capacitive Mode)

Time (s)	PV0 (%)	PV1 (%)	PV2 (%)	PV3 (%)	OP0 (%)	OP1 (%)
17	4.56	3.35	14.31	10.98	100	100
18	5.67	4.71	19.58	16.78	100	100
19	6.79	6.66	24.84	22.49	100	100
20	7.99	8.49	30.21	27.78	100	100
21	9.5	10.46	35.51	31.89	100	100
22	10.78	12.38	40.67	38.62	100	100
23	12.14	14.41	46.28	43.93	100	100
24	13.48	16.48	51.59	49.18	100	100
25	14.95	18.44	56.85	54.54	100	100
26	16.28	20.51	62.28	60.15	100	100
27	17.75	22.53	67.59	65.77	100	100
28	19.47	24.51	72.83	71.45	100	100
29	20.78	26.54	78.15	77.18	100	100
30	22.27	28.43	83.38	83.03	100	100
31	23.66	30.53	88.67	88.99	100	100
32	25.01	32.56	94.63	94.91	100	100

Table 4. 6 Open loop Experiment 2 (Purely Capacitive Mode)

Time (sec)	PV0 (%)	PV1 (%)	PV2 (%)	PV3 (%)	OP0 (%)	OP1 (%)
17	4.39	3.54	4.82	4.88	100	100
25	9.89	11.3	17.71	21.92	100	100
34	15.91	20.93	31.74	38.16	100	100
43	23.76	33.12	50.2	59.29	100	100
52	31.13	44.58	66.89	79.59	100	100
61	38.52	56.11	83.45	99.69	100	100
72	46.2	68.6	100	100	100	100
82	55.1	81.84	100	100	100	100
90	62.9	94.64	100	100	100	100
99	70.12	100	100	100	100	100
108	77.47	100	100	100	100	100
119	84.72	100	100	100	100	100
128	93.56	100	100	100	100	100
138	100	100	100	100	100	100

4.6.1.2 Estimation of Linear Velocity

The unsteady state mass balance equations (equation 4.9 to 4.12) are modified for the special case of zero outflow, to obtain the linear velocities, as shown below:

$$v_1 = \frac{dh_1}{dt} = \frac{\gamma_1 F_1}{A} \quad \text{Eq. 4. 82}$$

$$v_2 = \frac{dh_2}{dt} = \frac{\gamma_2 F_2}{A} \quad \text{Eq. 4. 83}$$

$$v_3 = \frac{dh_3}{dt} = \frac{(1-\gamma_2)F_2}{A} \quad \text{Eq. 4. 84}$$

$$v_4 = \frac{dh_4}{dt} = \frac{(1-\gamma_1)F_1}{A} \quad \text{Eq. 4. 85}$$

$$A_1 = A_2 = A_3 = A_4 = A \quad \text{Eq. 4. 86}$$

All the four tanks have the same cross sectional area. Adding equation 4.80 and 4.85 gives the total volumetric flowrate of water discharging through pump P1, as shown below:

$$F_1 = A(v_1 + v_4) \quad \text{Eq. 4. 87}$$

Similarly, adding equation 4.83 and 4.84 gives the total volumetric flowrate of water discharging through pump P2, as shown below:

$$F_2 = A(v_2 + v_3) \quad \text{Eq. 4. 88}$$

The pump flow split fraction γ_1 is calculated from equations 4.82 and 4.87, as shown below:

$$\gamma_1 = \frac{v_1}{(v_1 + v_4)} \quad \text{Eq. 4. 89}$$

Similarly, the pump flow split fraction γ_2 is calculated from equations 4.83 and 4.88, as shown below:

$$\gamma_2 = \frac{v_2}{(v_2 + v_3)} \quad \text{Eq. 4. 90}$$

4.6.1.3 Estimation of Split Fractions

Based on the data of experiment 1 (Table 4.5), the open loop response of liquid level in the four tanks (operated in pure capacity mode) is shown in Figure 4.4. The linear velocity of each tank is estimated from the linear fit of open loop response, as shown in Figures 4.5 to 4.8. The split fractions (γ_1 and γ_2) are finally estimated (using equation 4.89 and 4.90), as shown in Table 4.7.

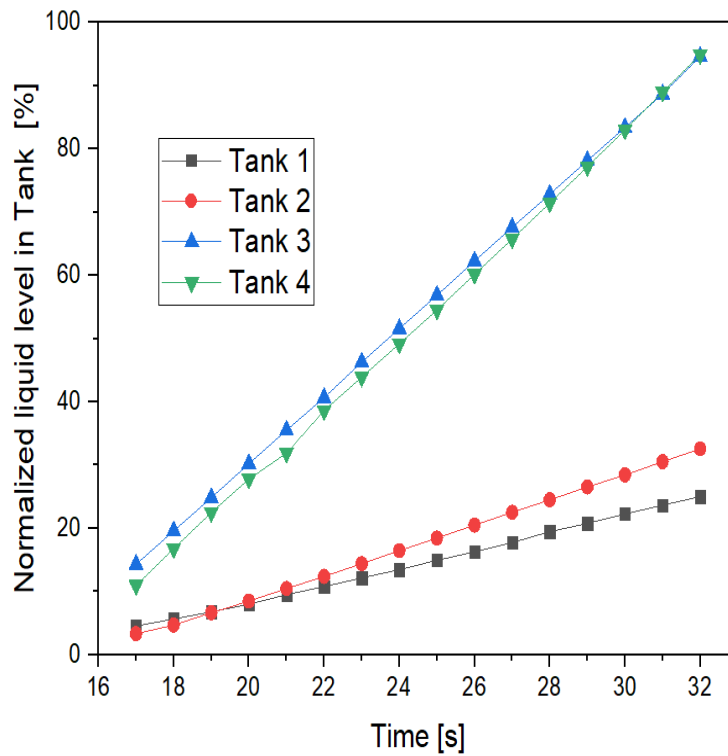


Figure 4. 4 Open loop Response of Quadruple Tank Process (Pure Capacity Mode)

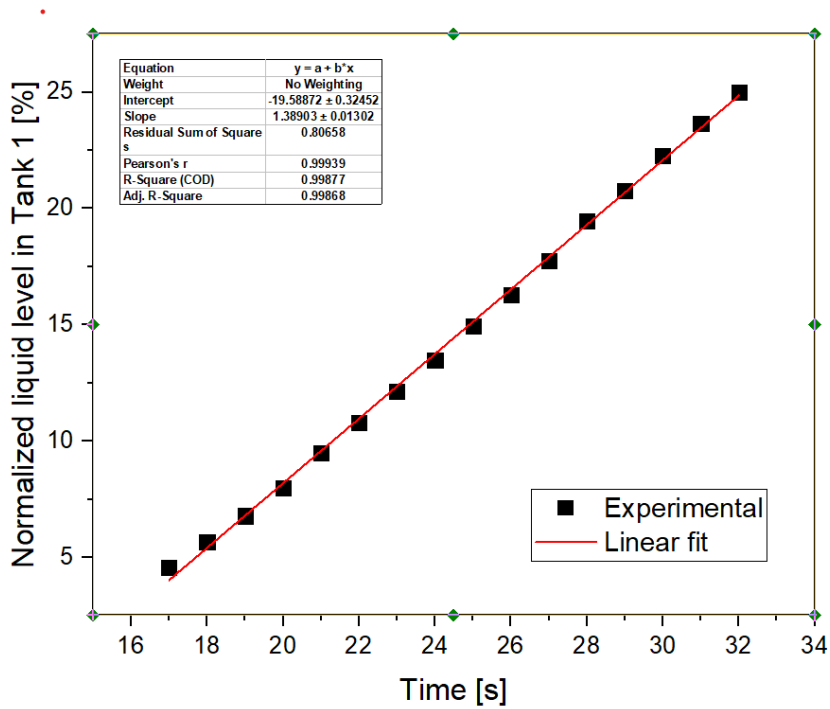


Figure 4. 5 Estimation of Linear Velocity of Tank 1

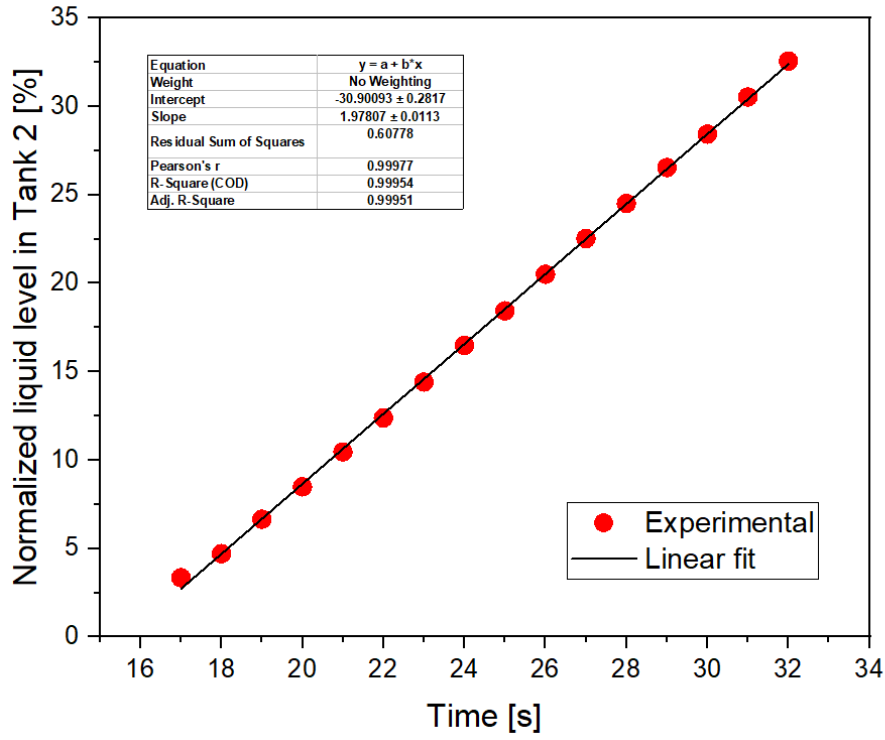


Figure 4. 6 Estimation of Linear Velocity of Tank 2

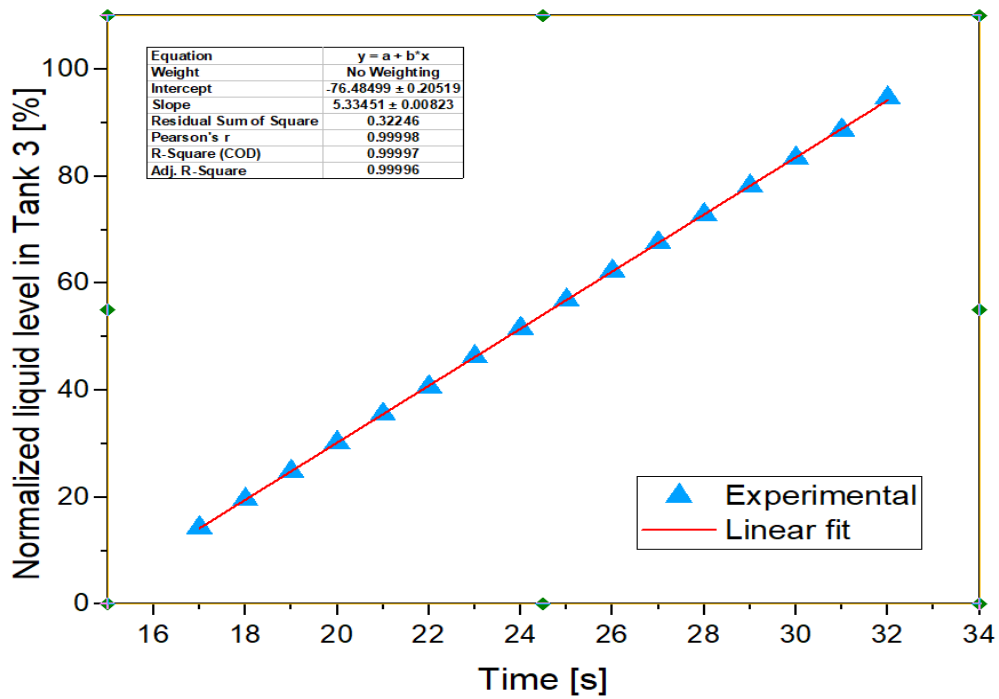


Figure 4. 7 Estimation of Linear Velocity of Tank 3

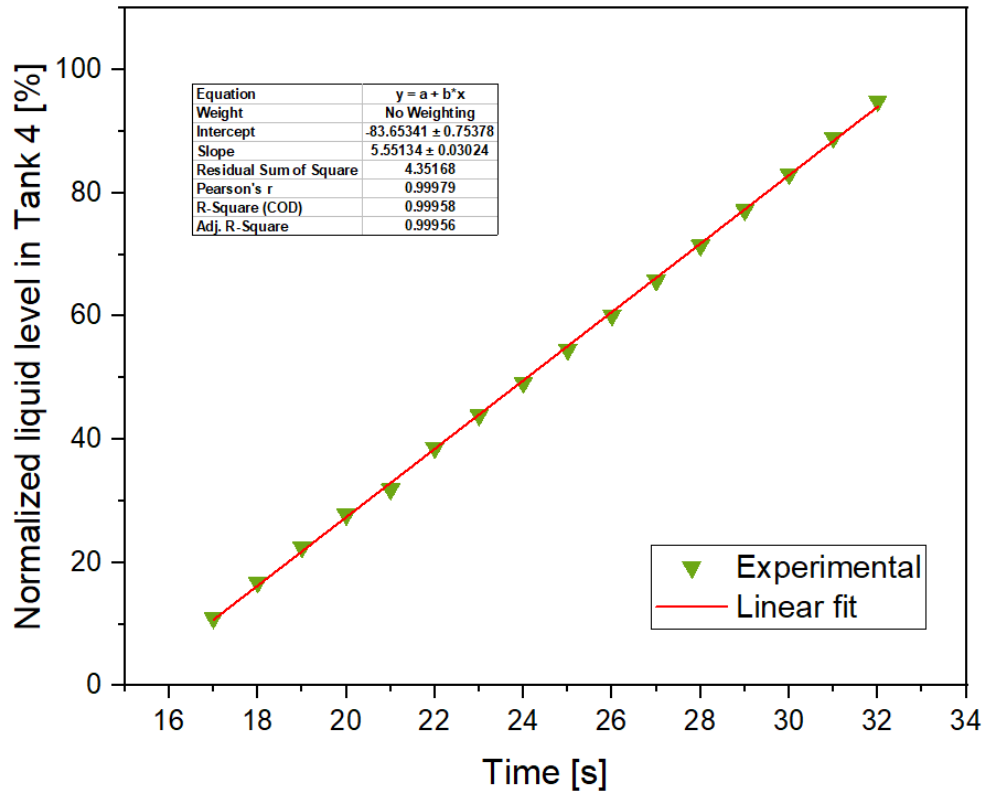


Figure 4. 8 Estimation of Linear Velocity of Tank 4

Table 4. 7 Estimation of split fractions

Tank No.	Units	Tank 1	Tank 2	Tank 3	Tank 4
Area, A	[cm ²]	66.48	66.48	66.48	66.48
Maximum Height, H	cm	26.50	26.50	26.50	26.50
Intercept on ordinate (Figure 4.5-4.8)	[%]	-19.59	-30.90	-76.49	-83.65
Slope (Figure 4.5-4.8)	[%/s]	1.39	1.98	5.34	5.55
Inlet linear velocity	[cm/s]	0.37	0.52	1.41	1.47
Inlet volumetric flowrate	[cm ³ /s]	24.47	34.85	93.99	97.79
Gamma1	[-]	0.20			
Gamma 2	[-]	0.27			

4.6.2 Open loop Experiments (Resistance Mode)

4.6.2.1 Experimental Procedure

1. The experimental setup was operated at non-minimum phase condition ($\gamma_1 + \gamma_2 < 1$) at the fixed values of γ_1 and γ_2 (Table 4.7).
2. Both the pumps were operated at their maximum volumetric flowrates F_1 and F_2 . This was achieved by setting the values of OP0% and OP1% to 100 in open loop/manual mode.
3. The outlet valves of all the four tanks were slightly opened (at fixed positions) in such a way that the tank levels were maintained at 80-90% of their maximum heights, under steady state condition.
4. After the initial steady state was achieved, a sequence of negative and positive step changes was introduced in the volumetric flowrate (F_1) of Pump P1, while keeping the volumetric flowrate (F_2) of pump P2 constant.
5. The open loop transient response (between the initial and final steady states) of liquid levels in all the four tanks was recorded.
6. Steps 4 was repeated by introducing a sequence of negative and positive step changes in the volumetric flowrate (F_2) of pump P2, while keeping the volumetric flowrate (F_1) of pump P1 constant.

The steady state experimental data is presented in Table 4.8 and open loop step response is shown in Figure 4.9

Table 4. 8 Steady State Experimental Data of Quadruple Tank Process

Split fractions		Forcing function: Inlet volumetric flowrate (LPH)		Steady state liquid level (cm)			
γ_1	γ_2	F_{1s}	F_{2s}	h_{1s}	h_{2s}	h_{3s}	h_{4s}
0.20	0.27	55.57	55.57	21.7	20.0	22.9	21.1
0.20	0.27	44.26	55.57	20.6	15.3	22.9	14.6
0.20	0.27	32.94	55.57	19.3	10.9	22.9	8.7
0.20	0.27	21.63	55.57	18.4	7.2	22.9	4.4
0.20	0.27	32.94	55.57	19.4	10.9	22.9	8.6
0.20	0.27	44.26	55.57	20.7	15.2	22.9	14.3
0.20	0.27	55.57	55.57	22.0	20.4	22.9	21.4
0.20	0.27	55.57	38.60	13.3	18.5	11.5	21.4
0.20	0.27	55.57	21.63	6.7	16.6	4.0	21.4
0.20	0.27	55.57	32.94	10.5	18.0	8.2	21.4
0.20	0.27	55.57	44.26	15.7	19.1	14.2	21.4
0.20	0.27	55.57	55.57	21.9	20.8	23.0	21.4

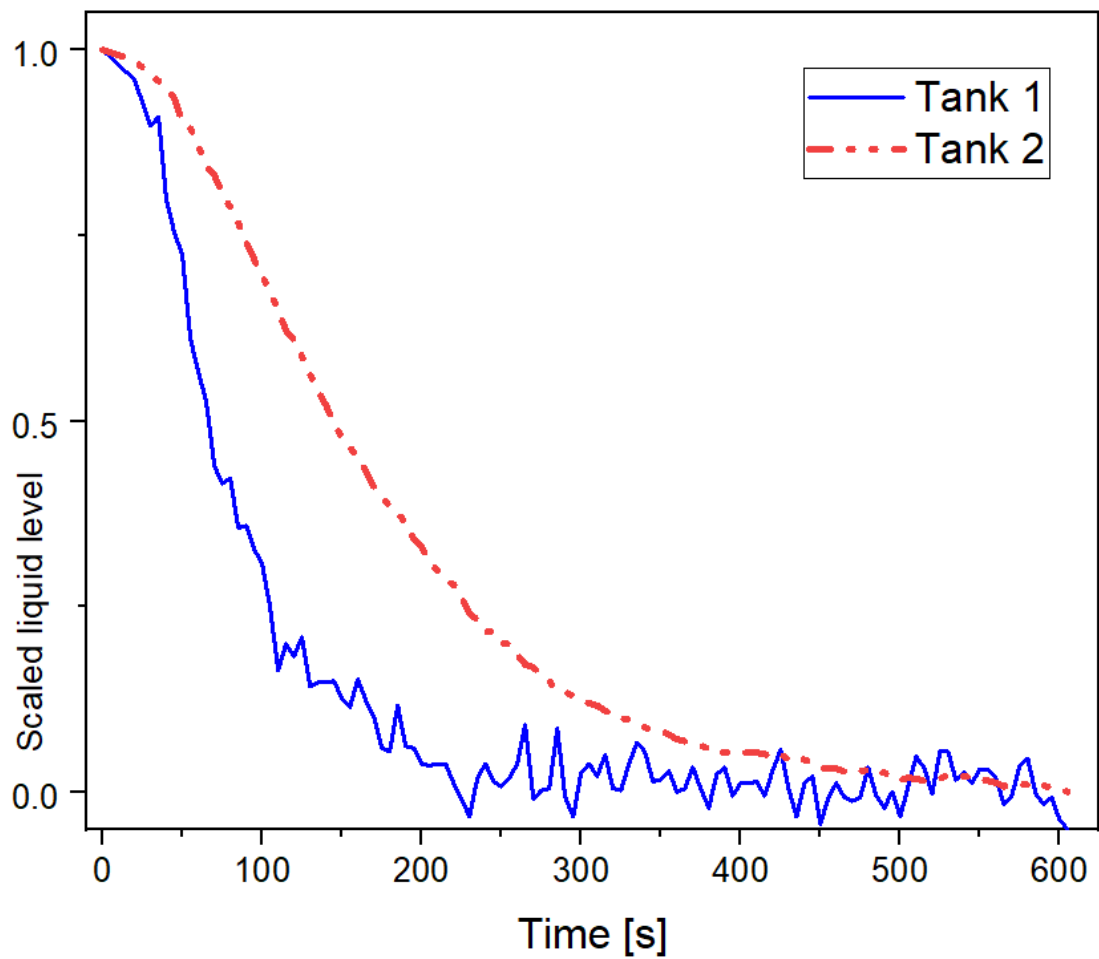


Figure 4. 9 Open loop step response of liquid level in Tank 1 and Tank 2

4.6.2.2 Estimation of Nonlinear Flow Resistances

The steady state input-output curves are the plots of square root of steady state height (ordinate) against the steady state volumetric flowrate (abscissa). Based on the steady state experimental data (Table 4.8) and the steady state relationships (equation 4.14 to 4.17), the state input-output curves for the Quadruple Tank Process (QTP) are plotted as shown in Figures 4.10 - 4.13.

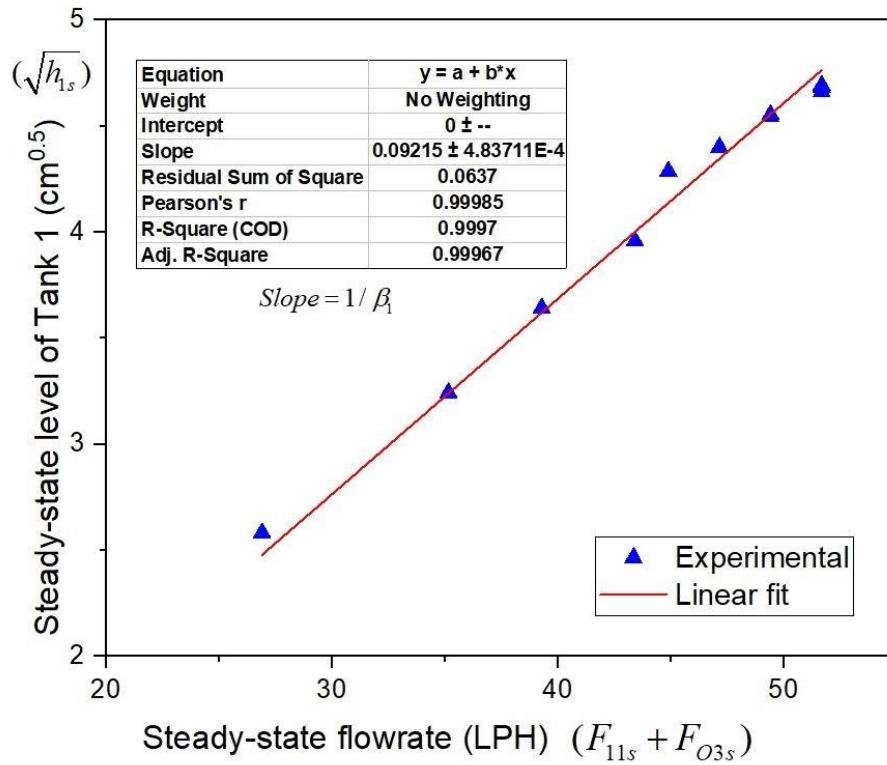


Figure 4. 10 Steady State Input-Output Curve for Tank 1

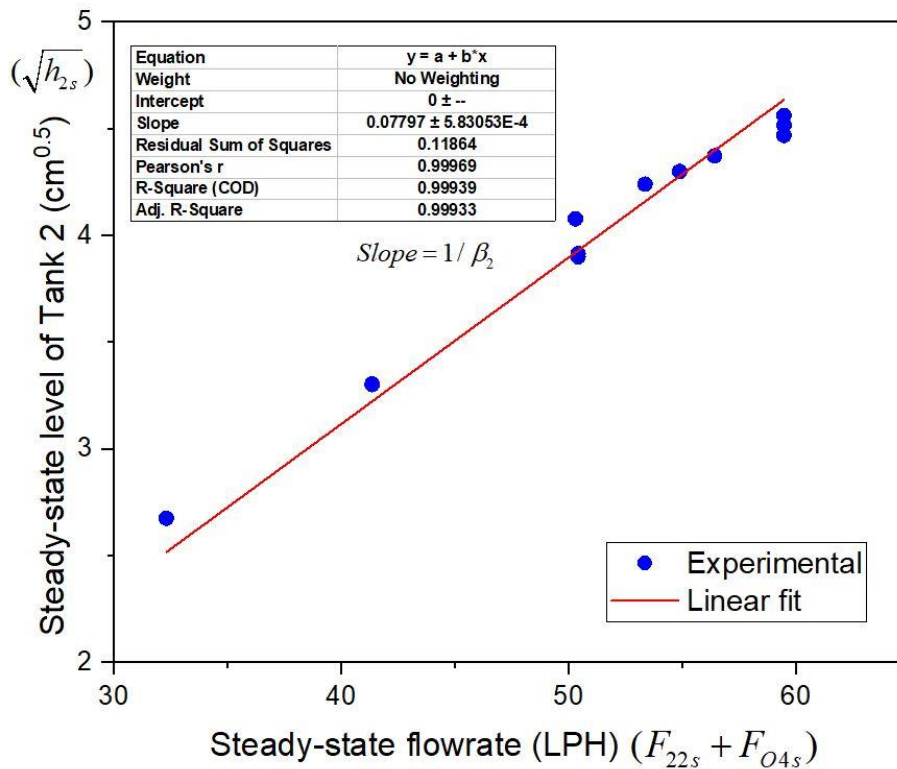


Figure 4. 11 Steady State Input-Output Curve for Tank 2

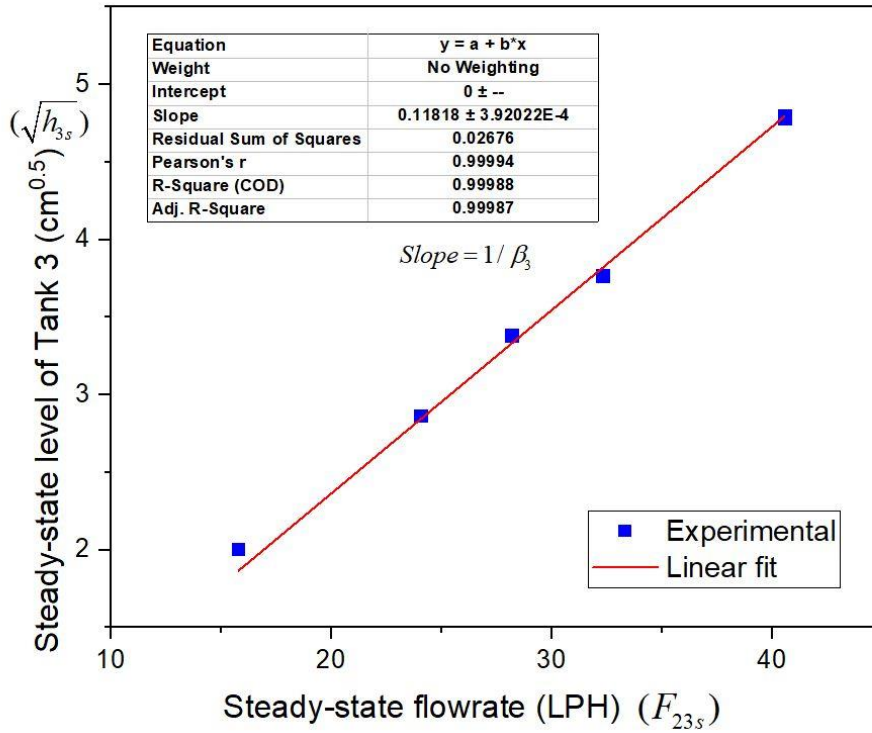


Figure 4. 12 Steady State Input-Output Curve for Tank 3

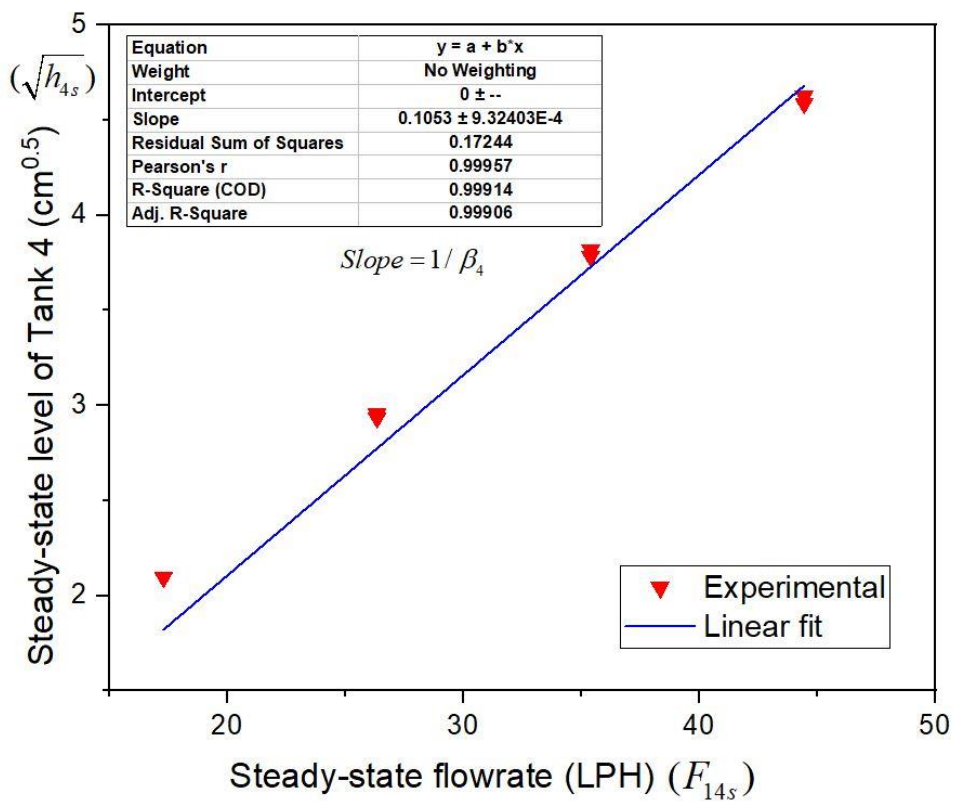


Figure 4. 13 Steady State Input-Output Curve for Tank 4

The nonlinear flow resistances for each tank are estimated from the slope of linear fit of steady state input-output curves, as shown in Table 4.9.

Table 4. 9 Estimation of Nonlinear Flow Resistance

Tank	Slope of linear fit of steady state input-output curve (Figure 4.10 to 4.13)	Nonlinear flow resistance, β_i
Tank 1	0.09	10.85
Tank 2	0.08	12.83
Tank 3	0.12	8.46
Tank 4	0.11	9.50

4.6.2.3 Estimation of Time Constant and Steady State Gain

Based on the steady state experimental data (Table 4.8) and the steady state relationships (equation 4.47, 4.48, 4.52 and 4.53), the time constant and steady state gain are estimated for each tank, as shown in Tables 4.10 and 4.11 respectively.

Table 4. 10 Estimation of Time Constants corresponding to different Steady States

Steady state inlet volumetric flowrate (LPH)		Time constant (sec)			
F_{1s}	F_{2s}	τ_1	τ_2	τ_3	τ_4
55.57	55.57	54.70	45.50	73.01	62.94
44.26	55.57	54.31	41.34	73.01	54.75
32.94	55.57	53.28	36.11	73.01	44.05
21.63	55.57	53.11	30.53	73.01	33.81
32.94	55.57	53.33	36.02	73.01	43.45
44.26	55.57	54.57	41.03	73.01	53.74
55.57	55.57	55.43	46.47	73.01	63.99
55.57	38.60	44.03	45.44	52.58	63.99
55.57	21.63	32.46	44.35	33.03	63.99
55.57	32.94	39.05	45.38	44.23	63.99
55.57	44.26	47.00	45.79	56.87	63.99
55.57	55.57	55.19	47.43	73.35	63.99

Table 4. 11 Estimation of Steady State Gain Corresponding to Different Steady States

Steady state inlet volumetric flowrate (LPH)		Steady state gain			
F_{1s}	F_{2s}	K_{11}	K_{12}	K_{21}	K_{22}
55.57	55.57	0.16	0.62	0.55	0.17
44.26	55.57	0.16	0.61	0.50	0.16
32.94	55.57	0.16	0.60	0.43	0.14
21.63	55.57	0.16	0.60	0.37	0.11
32.94	55.57	0.16	0.60	0.43	0.14
44.26	55.57	0.16	0.62	0.49	0.15
55.57	55.57	0.17	0.63	0.56	0.17
55.57	38.60	0.13	0.50	0.55	0.17
55.57	21.63	0.10	0.37	0.53	0.17
55.57	32.94	0.12	0.44	0.55	0.17
55.57	44.26	0.14	0.53	0.55	0.17
55.57	55.57	0.17	0.62	0.57	0.18

4.7 Multivariable Process Transfer Function Matrix

The elements of multivariable process transfer function matrix are evaluated for

$F_{1s} = 44.26$ LPH and $F_{2s} = 55.57$ LPH (using Table 4.10 and 4.11) as:

$$\mathbf{G}(s) = \begin{pmatrix} \frac{0.16}{(54.31s+1)} & \frac{0.61}{(54.31s+1)(73.01s+1)} \\ \frac{0.50}{(41.34s+1)(54.75s+1)} & \frac{0.16}{(41.34s+1)} \end{pmatrix} \quad \text{Eq. 4. 91}$$

4.7.1 Transmission Zeros

The transmission zeros of the process are evaluated as:

$$z_1 = 0.0386 \quad \text{Eq. 4. 92}$$

$$z_2 = -0.0705 \quad \text{Eq. 4. 93}$$

The Two Input Two Output (TITO) Quadruple Tank Process (QTP) has one Right Half Plane (RHP) transmission zero and hence exhibits non-minimum phase behaviour, under the operating condition, $(\gamma_1 + \gamma_2) < 1$. In the present study, $(\gamma_1 + \gamma_2) = 0.47$

4.7.2 Control loop Interactions

The control loop interactions in a Two Input Two Output Process is shown in Figure 3.7. Each Controlled Variable (CV) (liquid levels in tank 1 and tank 2) is affected by change in any of the Manipulated Variable (MV) (volumetric flowrates of pumps P1 and P2). The Relative Gain Array (RGA) analysis is therefore useful in identifying the proper pairing of the Controlled Variable (CV) and Manipulated Variable (MV) for control system design.

4.7.2.1 Relative Gain Array (RGA) Analysis

Each elements of Relative Gain Array (RGA) is defined as the ratio of two steady state gains:

$$\Psi_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j} \right)_{Open-loop}}{\left(\frac{\partial y_i}{\partial u_j} \right)_{closed-loop, \neq u_j loop}} \quad \text{Eq. 4. 94}$$

Where, y_i represents the i^{th} Controlled Variable (CV) and u_j represents the j^{th} Manipulated Variable (MV). The Relative Gain Array (RGA) for a Multiple Input Multiple Output (MIMO) process is represented as:

$$\Psi = \begin{pmatrix} \psi_{11} & \cdots & \psi_{1n} \\ \vdots & \ddots & \vdots \\ \psi_{n1} & \cdots & \psi_{nn} \end{pmatrix} \quad \text{Eq. 4. 95}$$

For the Two Input Two Output (TITO) Quadruple Tank Process (QTP), the elements of Relative Gain Array (RGA) are defined as:

$$\left(\frac{\partial y_1}{\partial u_1} \right)_{open-loop} = K_{11} \quad \text{Eq. 4. 96}$$

$$\left(\frac{\partial y_1}{\partial u_1} \right)_{Closed-loop-2} = K_{11} \left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}} \right) \quad \text{Eq. 4. 97}$$

Defining ,

$$\zeta = \frac{K_{12}K_{21}}{K_{11}K_{22}} \quad \text{Eq. 4. 98}$$

The elements of (2x2) Relative Gain Array (RGA) are calculated as:

$$\psi_{11} = \psi_{22} = \frac{1}{1 - \zeta} \quad \text{Eq. 4. 99}$$

$$\psi_{12} = \psi_{21} = \frac{-\zeta}{1-\zeta} \quad \text{Eq. 4. 100}$$

The Relative Gain Array (RGA) for the Quadruple Tank Process (QTP) is obtained as:

$$\Psi = \begin{pmatrix} -0.092 & 1.092 \\ 1.092 & -0.092 \end{pmatrix} \quad \text{Eq. 4. 101}$$

From the elements of Relative Gain Array (RGA), it is observed that $\psi_{12} \& \psi_{21} > 1$. It is recommended to avoid pairing u_j with y_i if ψ_{ij} has a large value. u_i is therefore paired with y_i .

4.8 Closed loop Response of Quadruple Tank Process (QTP) with interactions

Closed loop response of the Quadruple Tank Process (QTP) to step change in set point of liquid level in tank 1 is studied using the conventional PID controller. The effect of controller gain, K_c on the closed loop performance is shown in Figure 4.14. The liquid level in tank 1 exhibits inverse response (due to the effect of RHP transmission zero). The effect of interaction is clearly observed from the dynamics of tank 2. Moreover, good set-point tracking is not achieved even for different controller gains. This behaviour emphasizes the need of decoupler design for the Quadruple Tank Process (QTP).

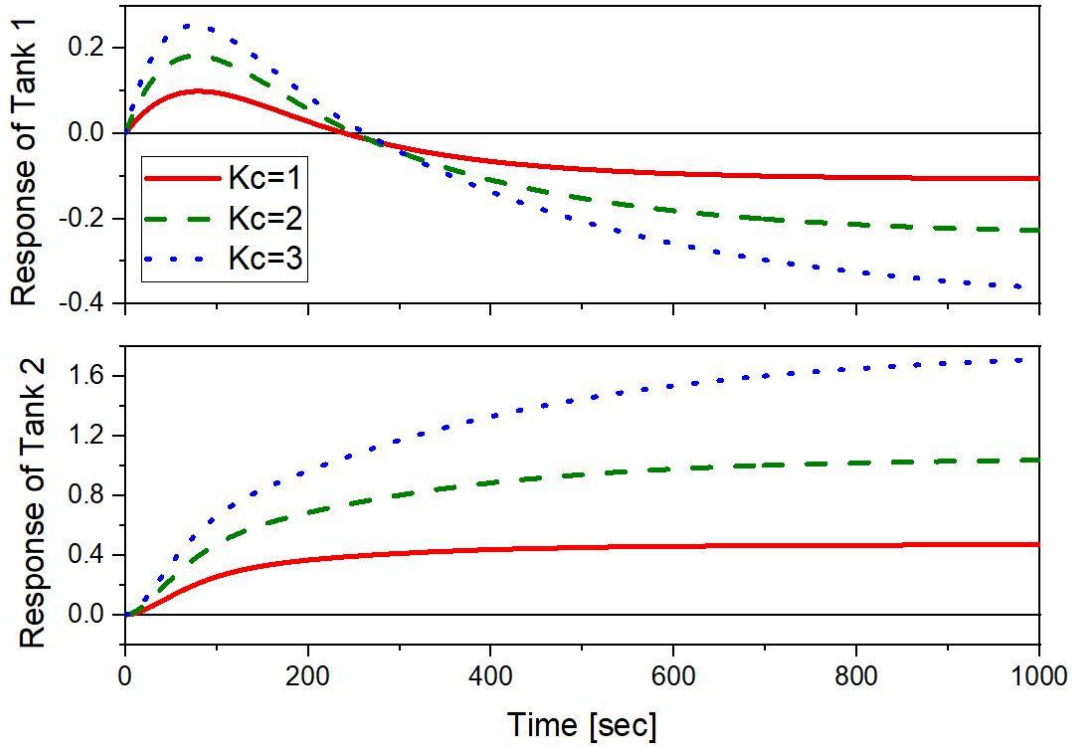


Figure 4. 14 Closed loop Response of Conventional Controller (With Interactions)

4.9 Design of Inverted Decoupling Controller

The decouplers effectively decouple the control loops (by nullifying the effect of interactions) and convert the Two Input Two Output (TITO) process into two Open Loop Equivalent Transfer Functions (OLETFs), $g_{11}^*(s)$ and $g_{22}^*(s)$, for which, independent controllers are designed (Rajapandiyan & Chidambaram, 2012). The closed loop block diagram of inverted decoupling controller for the Two Input Two Output (TITO) Quadruple Tank Process (QTP) is shown in Figure 4.15. The decouplers for the Quadruple Tank Process (QTP) are designed based on the following relations:

$$d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)} \quad \text{Eq. 4. 102}$$

$$d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)} \quad \text{Eq. 4. 103}$$

$$g_{11}^*(s) = g_{11}(s) + d_{21}(s)g_{12}(s) \quad \text{Eq. 4. 104}$$

$$g_{22}^*(s) = g_{22}(s) + d_{12}(s)g_{21}(s) \quad \text{Eq. 4. 105}$$

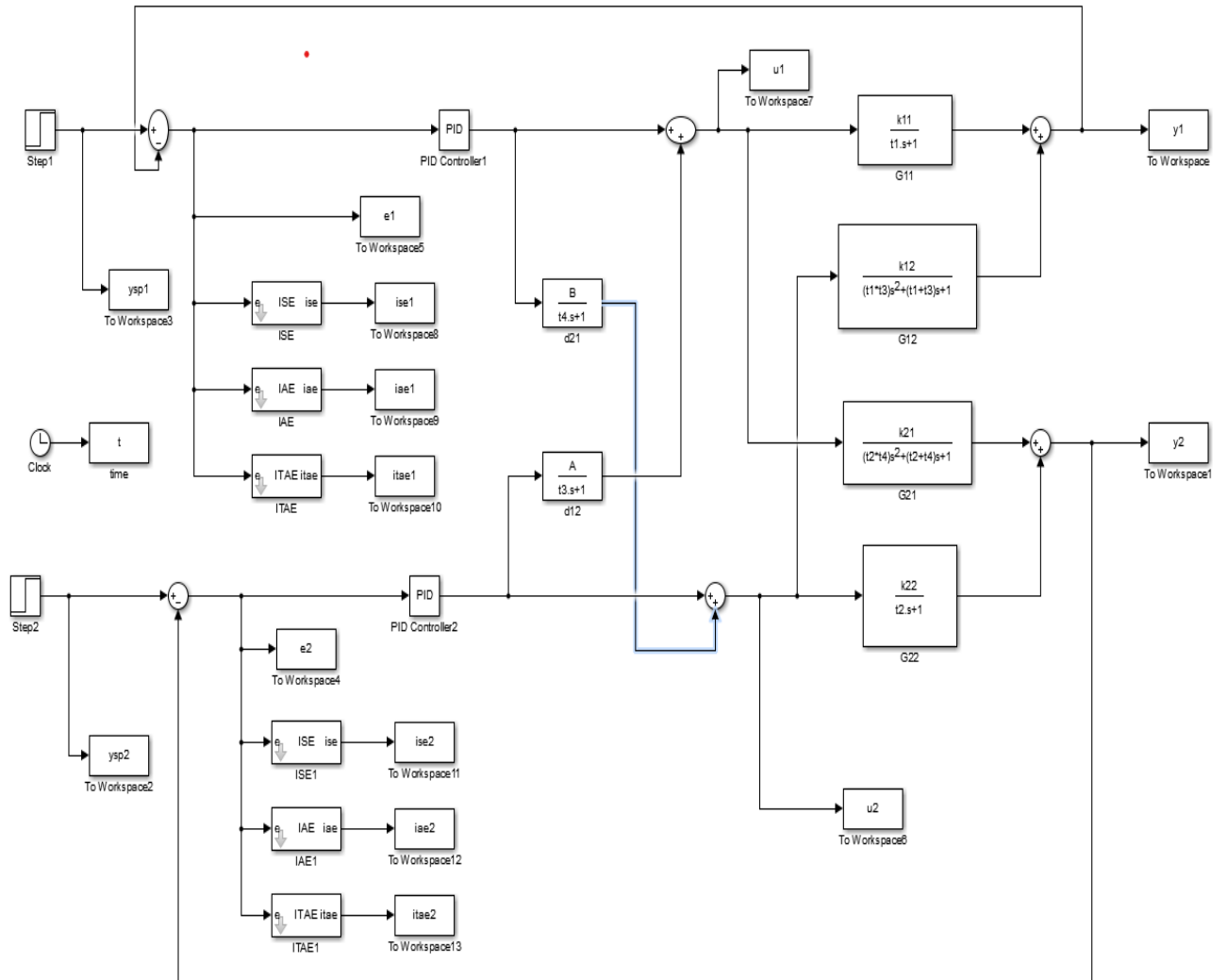


Figure 4. 15 Closed loop Block Diagram with Inverted Decoupling Controllers

4.10 Closed loop Response of Quadruple Tank Process (QTP) with Inverted Decoupling Controllers

Closed loop response of the Quadruple Tank Process (QTP) with Inverted Decoupling controllers is shown in Figure 4.16. Comparison of the two closed loop responses in Figures 4.14 and 4.16 demonstrate the effectiveness of a decoupler.

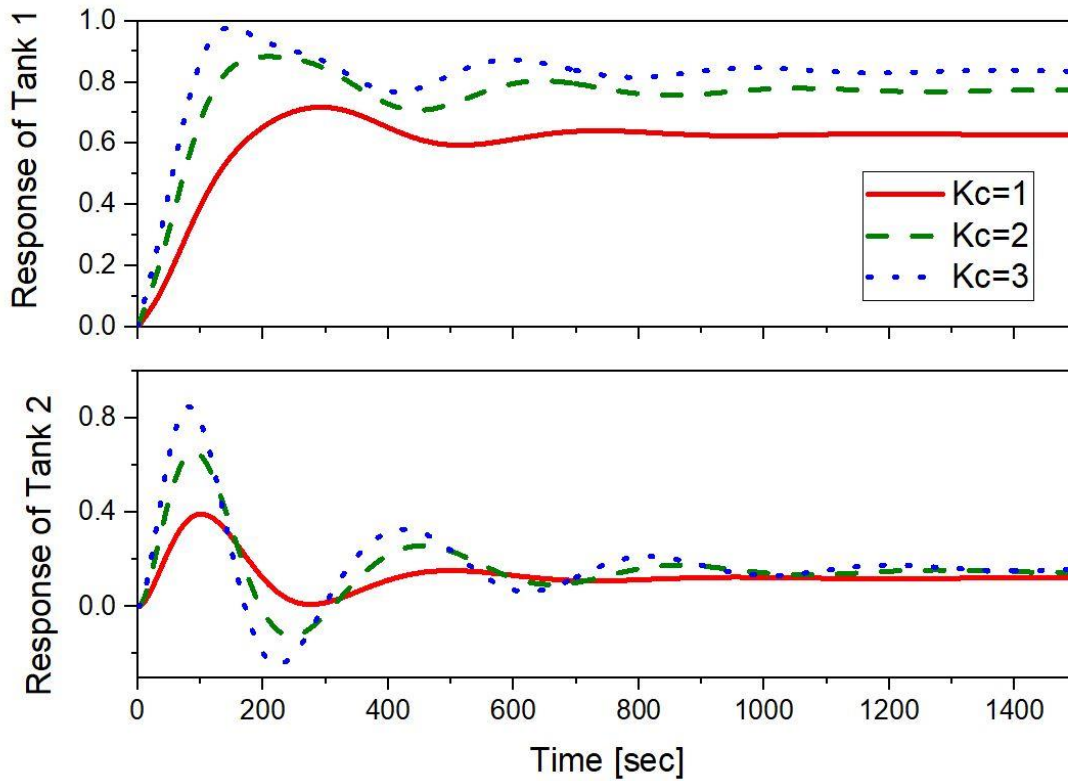


Figure 4. 16 Closed loop Response of Conventional Controller (With Decouplers)

The set-point tracking performance of the controller is evaluated in terms of Quantitative Performance Indices (QPI), as shown in Table 4.12. It is observed that by increasing the controller gain, there is improvement in the closed loop response of tank 1, but the response of tank 2 becomes more oscillatory. The controller performance is improved by using a PID controller. However, the closed loop stability is an important factor to be kept in mind while designing the conventional PID controller for a non-minimum phase Multiple Input Multiple Output (MIMO) process. This study therefore emphasizes the need of an Internal Model Control (IMC) based PID controller design technique (discussed in chapter 3) for the decoupled Quadruple Tank Process (QTP).

Table 4. 12 Quantitative Performance Indices (QPI) for Conventional Controller

Controller tuning parameter	Quantitative Performance Indices (QPI)		
	Integral of Square of Error (ISE)	Integral of Absolute Error (IAE)	Offset
K_c			
1	262.05	599.3	-0.37098
2	118.7955	373.8609	-0.22601
3	73.53796	271.5006	-0.16481

4.11 IMC based PID Controller Design for the Decoupled Quadruple Tank Process

Internal Model Control (IMC) based PID controller is designed based on the input-output pairing suggested by the Relative Gain Array (RGA) analysis of the process. Since the decoupled open loop equivalent transfer functions (OLETFs) as shown in equations 4.101 and 4.102 exhibit higher order underdamped response, the Internal Model Control (IMC) based PID controllers are designed based on second order stable process transfer functions, $g_{12}(s)$ and $g_{21}(s)$ (equation 4.91). The design of IMC based PID controller for second order stable processes is discussed at length in section 3.3.2. The PID controller parameters (K_c , τ_I and τ_D) are expressed in terms of the Internal Model Control (IMC) tuning parameter λ , as shown in equations 3.38 to 3.40. In the present study, λ_1 & λ_2 are the two Internal Model Control (IMC) tuning parameters for the two independent Internal Model Control (IMC) based PID controllers designed for the decoupled Quadruple Tank Process (QTP). As an initial guess, the nominal values of λ_1 & λ_2 are selected based on the process time constants (Table 4.10).

4.12 Closed loop Response of IMC based PID Decoupling Controller

The closed loop response of liquid level in tank 1 (control loop 1) and tank 2 (control loop 2), for different values of the Internal Model Control (IMC) based PID controller tuning parameters, λ_1 & $\lambda_2 = \lambda$, is shown in Figure 4.17 and the quantitative performance indices (QPI) are shown in Table 4.13. It is observed that for lower values of λ , the response is more oscillatory and with larger overshoots. Hence, the controller performance is tested at three different values of $\lambda = 200, 300$ and 400 . For all the values of λ under consideration, the error values are comparable and offset is completely eliminated. However, the overshoot decreases as λ is increased from 200 to 400. Hence the value of $\lambda = 400$ is recommended.

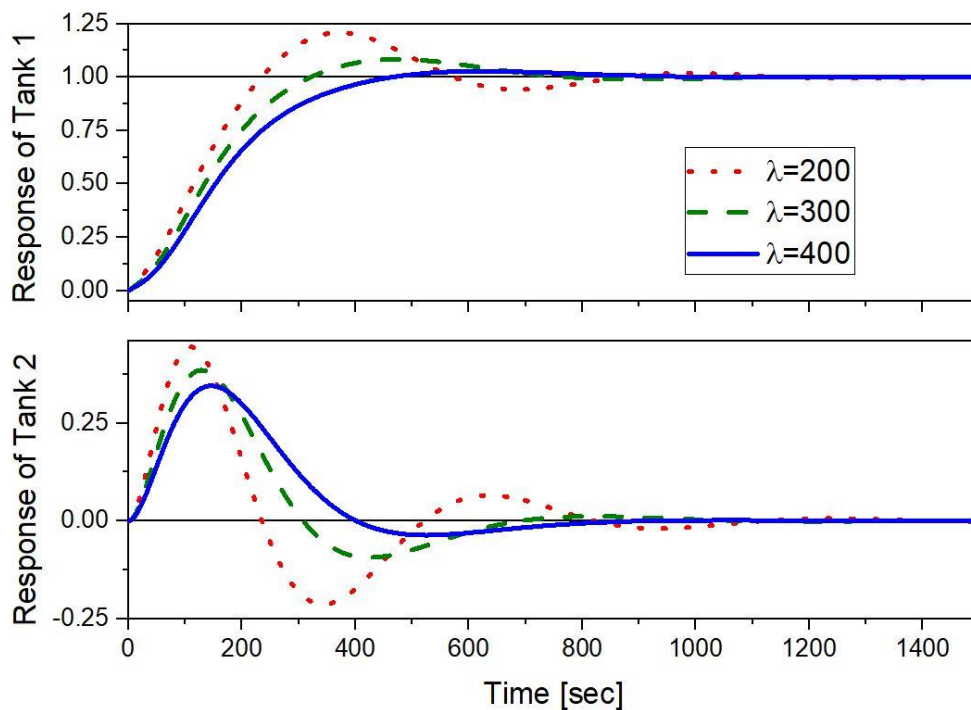


Figure 4. 17 Closed loop Response of Inverted Decoupling IMC-PID Controllers

Table 4. 13 QPI for IMC based PID Decoupling Controller

IMC tuning parameter $\lambda_1 \& \lambda_2 = \lambda$	Closed loop response of tank 1 (Control loop 1)				Closed loop response of tank 2 (Control loop 2)			
	ISE	IAE	Offset	% Overshoot	ISE	IAE	Offset	% Overshoot
200	88.86	175.09	0.00	20.80	27.40	114.44	0.00	44.47
300	98.94	167.01	0.00	8.35	20.86	89.68	0.00	38.52
400	113.48	180.35	0.00	2.76	19.22	83.42	0.00	34.54

4.13 Comparison with Open Literature

(Komathi et al., 2017) studied the Internal Model Control (IMC) based Proportional Integral (PI) control and Fractional Order Proportional Integral (FOPI) control of a Two Input Two Output (TITO) Quadruple Tank (QTP) minimum phase process. The multivariable process transfer function matrix and its transmission zeros, (studied by (Komathi et al., 2017)) are reproduced for comparison:

$$\mathbf{G}_1(s) = \begin{pmatrix} \frac{11.89}{(121.4s+1)} & \frac{6.875}{(121.4s+1)(3.967s+1)} \\ \frac{6.738}{(84.73s+1)(3.109s+1)} & \frac{0.155}{(84.73s+1)} \end{pmatrix} \quad \text{Eq.}$$

4. 106

The multivariable transmission zeros of $\mathbf{G}_1(s)$ are:

$$z_1 = -0.4560 \quad \text{Eq. 4. 107}$$

$$z_2 = -0.1177 \quad \text{Eq. 4. 108}$$

The process studied by (Komathi et al., 2017) exhibits minimum phase behaviour since all its transmission zeros are negative. The Internal Model Control (IMC) based PID

decoupling controller designed in the present work is implemented on the process transfer function of (Komathi et al., 2017), for the sake of comparison. The closed loop simulation results of comparative study are shown in Figure 4.18 and Table 4.14. The proposed Internal Model Control (IMC) based PID decoupling controller is shown to exhibit better performance than the IMC-PI controller of (Komathi et al., 2017).

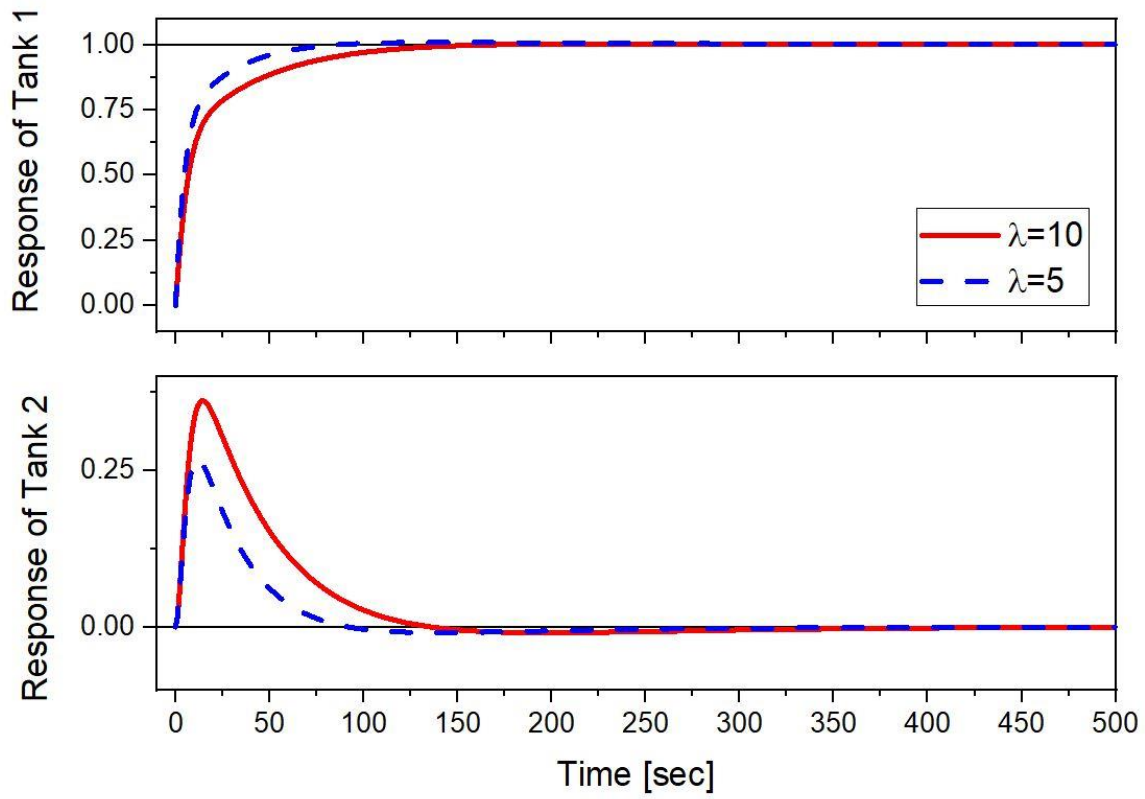


Figure 4. 18 Comparative Performance of IMC-PID Decoupling Controllers

Table 4. 14 Quantitative Performance Indices (QPI) for comparative study

Controller	Closed loop response of tank 1 (Control loop 1)				Closed loop response of tank 2 (Control loop 2)			
	ISE	IAE	Offset	% Overshoot	ISE	IAE	Offset	% Overshoot
IMC based PID, $\lambda = 10$ (Present work)	6.39	19.13	0.00	0.36	3.85	18.22	0.00	36.14
IMC based PID, $\lambda = 5$ (Present work)	3.98	12.14	0.00	0.89	1.55	10.17	0.00	26.75
IMC based PI (Komathi et al., 2017)	44.27	92.03	-	-	30.46	64.2	-	-

4.14 Conclusion

In the present work, an Internal Model Control (IMC) based PID decoupling controller is designed and tuned for a Two Input Two Output (TITO) experimental Quadruple Tank Process operated at non-minimum phase. Identification of the Quadruple Tank Process is carried out based on open loop experimental data. Transmission zeros of the multivariable process transfer function matrix are evaluated to confirm the non-minimum phase behaviour. Relative Gain Array analysis is carried out to study the interaction among the two control loops and selection of appropriate input-output pairing of the controlled and manipulated variables. Decouplers are designed to obtain two independent open loop equivalent transfer functions, for which, two independent Internal Model Control (IMC) based PID controllers are designed. The Internal Model Control (IMC) based PID controller parameter, λ is tuned to provide good set point tracking properties. The controller performance is evaluated from the various Quantitative Performance Indices. The developed control scheme is tested on a similar Two Input Two Output Quadruple Tank Process and the results are compared.