PREFACE

The thesis consists of five chapters.

Chapter 1 provides the introduction to the thesis. Basic definitions that are being used throughout this thesis are collectively provided. It also presents the literature survey and recent works on fractional partial differential equations (FPDEs), variable-order fractional partial differential equation and time-fractional Black-Scholes model. The motivation behind the problem statement of the thesis is explained in this chapter.

Chapter 2 aims to provide a higher-order new numerical approximation for variable order Caputo fractional derivative of order $0 < \alpha(\mathrm{x}, \mathtt{t}) < 1$. Then, by using this approximation, a numerical scheme is presented by using finite difference approach for variable order time fractional reaction-subdiffusion equation (VO-TFRSDE). The unconditional stability of the numerical scheme is examined theoretically. The scheme is implemented on the two test problems. The numerical results are highly accurate with higher order of convergence. The content of this chapter is published in **Chinese Journal of Physics, 85 (2023) 431–444**.

Chapter 3 presents a new numerical approximation for variable order Caputo fractional derivative of order $0 < \alpha(\varkappa, \mathtt{t}) < 1$ by using the idea of interpolation. Later, a numerical scheme is presented by using finite difference approach for variable order time-fractional sub-diffusion equation (VO-TFSDE). The unconditional stability of numerical schemes are examined theoretically. The scheme is implemented on the three test problems and the obtained numerical results are highly accurate with a higher order of convergence. A comparative study of the numerical results with earlier existing scheme is also provided to show the effectiveness and accuracy of our

proposed scheme. The content of this chapter is published in **Iranian Journal of Science**, Vol. 49, (2025) 369-381.
DOI https://doi.org/10.1007/s40995-024-01726-5.

Chapter 4 presents a numerical approach for solving multi-term time-fractional electromagnetic wave model arising in a dielectric medium is provided by using finite difference scheme. We have applied the L3 approximation for approximating the Caputo time-fractional derivative and central difference scheme to approximate the space derivative. The proposed difference scheme is of second order in space and time direction for all $1 < \beta < \alpha < 2$. Some test problems validated the efficiency and accuracy of the difference scheme.

In **chapter 5**, we have designed a semi discrete scheme for the time-fractional Black-Scholes model governing European option. The time-fractional Caputo derivative is approximated using L-123 approximation and spatial derivative is approximated using operational matrix method. The proposed difference scheme is of $(4-\alpha)$ order in time. A test problem of time-fractional Black-Scholes model (TFBSM) is given to validate the efficacy and accuracy of the difference scheme. A comparative study of the existing scheme is also provided to demonstrate the effectiveness of the scheme.

Chapter 1

Introduction

This chapter provides a concise overview of the thesis's objective, methodology, and structure. Sect.1.1 discusses the background of fractional calculus, fractional PDEs and variable order PDEs. In Sect.1.2, basic definitions that are being used throughout this thesis are collectively provided. A brief literature review for some classes of fractional mathematical models is presented in Sect.1.3. Sect.1.4 presents some mathematical preliminaries about polynomial interpolation. In Sect.1.5, some fundamental mathematical results based on proposed numerical methods are discussed that are used in all chapters of the thesis. The challenges and motivations behind the topics are explained in Sect.1.6. Sect.1.7 defines the problem statement and lists the objectives of the thesis.