Chapter 7

Application with Simulation and Experimental Validation

7.1 Application to a Coupled Tank System

In this segment, MATLAB/Simulink simulations and experiments are included to authenticate the effectiveness and precision of the formulated reaching law for discrete-time sliding mode control. Complementing the simulations, the practical experiments are essential to validate the reaching law's applicability in real-world situations. The experimental aspect becomes particularly crucial in assessing how well the proposed approach translates from theory to application, especially in scenarios where real-world dynamics and uncertainties come into play. The emphasis on experimentation is paramount, particularly to assess the proper elimination of chattering and the accuracy of reference tracking facilitated by the proposed reaching law.

The coupled tank system serves as our test bench, with the schematic diagram illustrated in Figure 7.1 depicting the model. The objective is to regulate the water level in tank 1 to meet the desired specifications. In its current configuration (Configuration 1), the control input is directly applied at the inlet of tank 1.

The mathematical model for this specific configuration is expressed as follows:

$$\frac{d}{dt}L_1 = \frac{1}{A_{t1}}\left[K_pV_p - A_{o1}\sqrt{2gL_1}\right] \tag{7.1}$$

In the equation, L_1 corresponds to the water level in tank one, V_p signifies the pump input voltage, K_p is the constant of proportionality, A_{t1} and A_{o1} denote the cross-sectional area

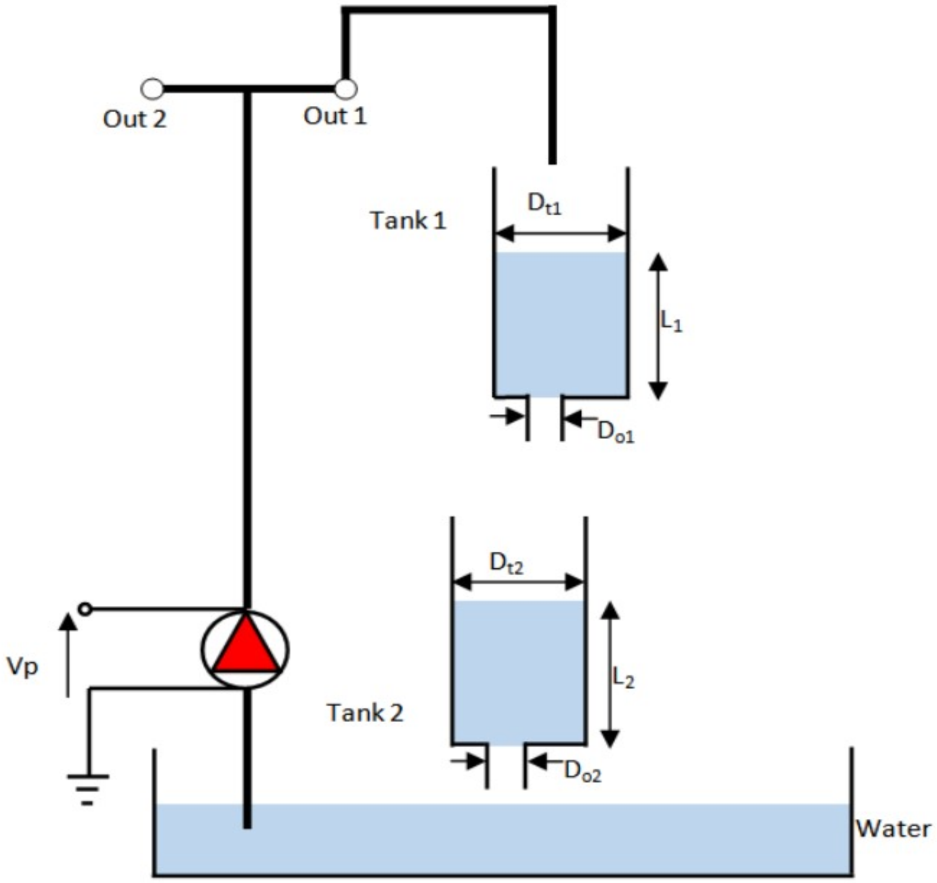


Figure 7.1: Block diagram of a coupled water tank system

of tank one and its outlet hole, respectively, and g represents the acceleration due to gravity. Let's designate the state variable as $z(t) = L_1$, and the control input variable as $u(t) = V_p$.

Then we change the system variable and write the system dynamics further as

$$\dot{z}(t) = \frac{1}{A_{t1}}\left[K_p u(t) - A_{o1}\sqrt{2gz(t)}\right] \tag{7.2}$$

The tank system parameters are: $A_{o1} = 0.1781\, cm^2$, $A_{t1} = 15.517\, cm^2$, $K_p = 3.3\, cm^3/s/V$ and $g = 981\, cm/s^2$. The maximum voltage that can be applied to the tank system is 22 V. We linearize the above system about the operating point z_{1o} and obtain the following:

$$\dot{z}(t) = -\frac{A_{o1}}{A_{t1}}\sqrt{\frac{g}{2z_{1o}}}z(t) + \frac{K_p}{A_{t1}}u(t) \tag{7.3}$$

Additionally, we discretize the above system with sampling time T_s in order to construct the proposed reaching law based discrete sliding mode control. The discrete-time model for the tank system is obtained via zero-order hold on the input with sampling time

$T_s = 0.1\ sec$ and $z_{1o} = 15\ cm$ as

$$z(k+1) = 0.9935z(k) + 0.0212u(k) \tag{7.4}$$

Let us consider the desired water level trajectory for tank 1 as $z_d(k)$. The tracking error is defined as $e(k) = z_d(k) - z(k)$. The dynamics in the error coordinates play a crucial role in evaluating the control system's performance. In the following section, we will explore these dynamics to evaluate the control signal to maintain the desired water level trajectory. The dynamics in the error coordinates are given as

$$\begin{aligned} e(k+1) =& z_d(k+1) - z(k+1) \\ =& z_d(k+1) - \{0.9935z(k) + 0.0212u(k)\} \\ =& z_d(k+1) - \{0.9935(z_d(k) - e(k)) + 0.0212u(k)\} \end{aligned} \tag{7.5}$$

Further, computing the control law according to the proposed reaching law as

$$\begin{aligned} u(k) =& -\frac{1}{0.0212}\bigg\{ - z_d(k+1) + 0.9935(z_d(k) - e(k)) + e(k) \\ & - T_s \operatorname{sign}(e(k)) \min\left\{\frac{|(e(k))|}{T_s}, \gamma\right\}\bigg\} \end{aligned} \tag{7.6}$$

7.2 Results and Discussions

In this section, we intend to show the effectiveness of the proposed reaching law via simulation results as well as experimental validation. A Quanser setup for coupled tank system is shown in Figure 7.2. For this tank system, we choose the value of the reaching law design parameter as, $\gamma = 2$.

7.2.1 Unperturbed Case

The simulation results for unperturbed case are shown in Figure 7.3. The reference tracking of the water level is shown in Figure 7.3 (a), where we can see that system state perfectly tracks the reference trajectory without oscillations. Also, from the driving input which is shown in Figure 7.3 (b), we can notice that there is no chattering present in the control signal.

The corresponding experimental outcomes for the undisturbed case are depicted in Figure 7.4. Water level tracking is illustrated in Figure 7.4 (a), and the control input is

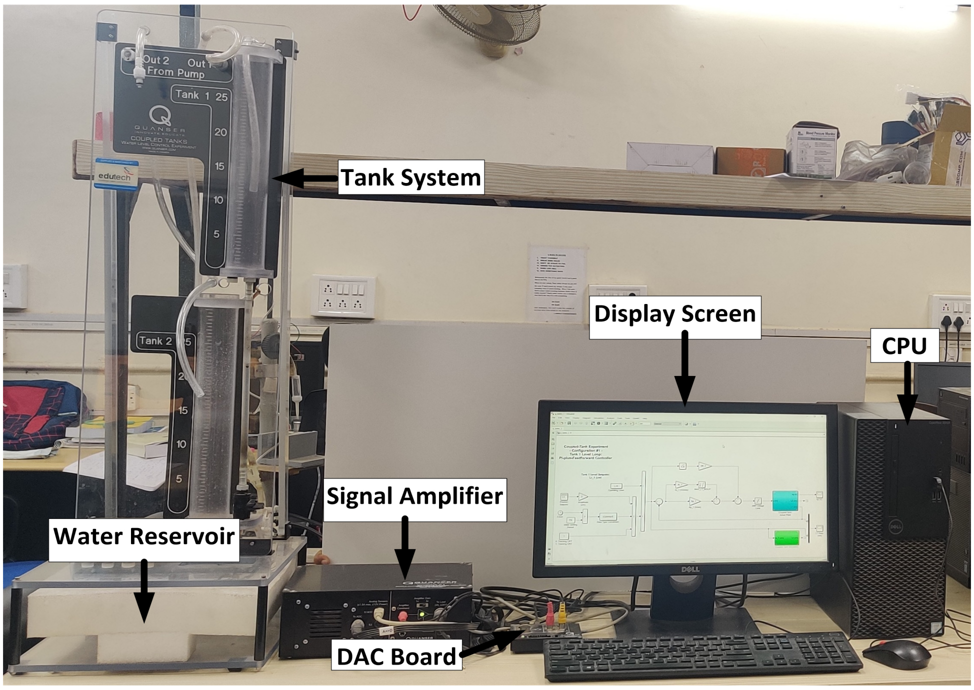


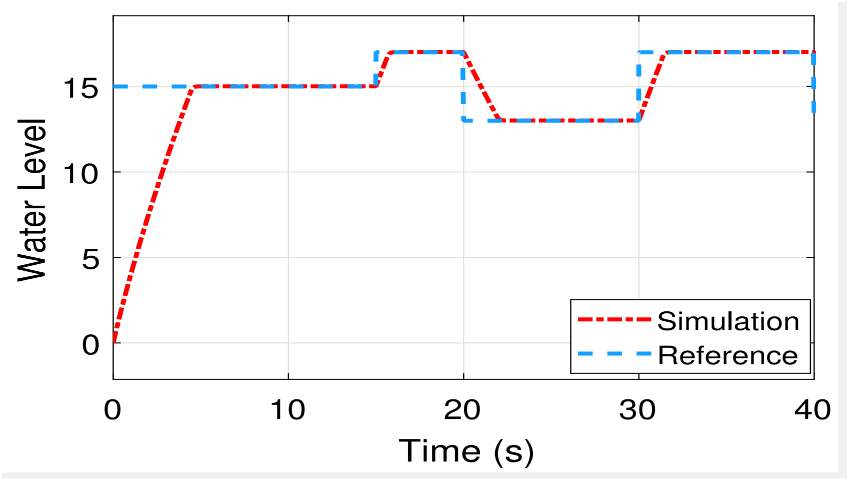
Figure 7.2: Experimental setup of a coupled water tank system

displayed in Figure 7.4 (b). The water level tracking aligns closely with the simulation result. However, it is noteworthy that the control input undergoes rapid changes when the reference value is held constant, potentially attributed to inherent uncertainties arising from sensor noise.

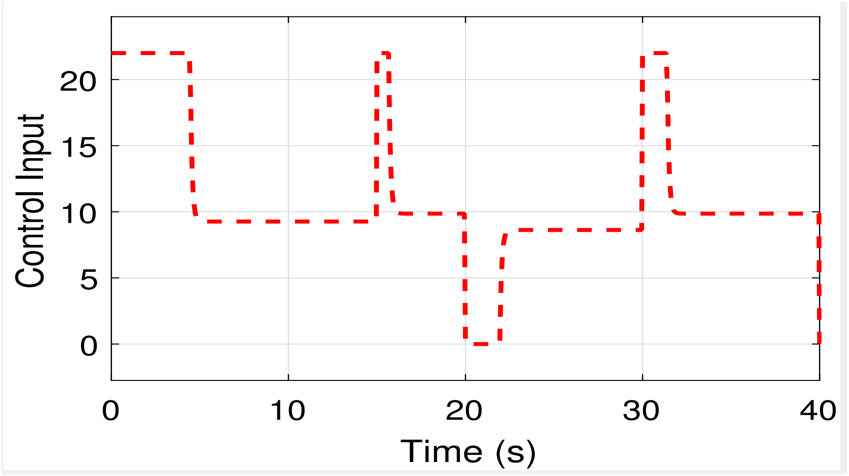
7.2.2 Perturbed Case

The simulation results for the perturbed case are displayed in Figure 7.5. In the simulation, the perturbation is introduced by adding a sinusoidal signal to the tank dynamics. In the presence of such perturbations, the water level's reference trajectory tracking is impeccable without any oscillations, as illustrated in Figure 7.5 (a). Additionally, the corresponding control input signal, shown in Figure 7.5 (b), exhibits an absence of chattering, although there are small amplitude ripples due to the introduced perturbation.

The corresponding experimental results for the perturbed case are presented in Figure 7.6. In this scenario, perturbations are introduced by intermittently opening and closing a thin outlet at the bottom of the tank. Water level tracking is depicted in Figure 7.6 (a), while the corresponding control input is shown in Figure 7.6 (b). The water level tracking closely aligns with the simulation result. However, it is important to note that the control input undergoes rapid changes when the reference value is held constant,



(a)



(b)

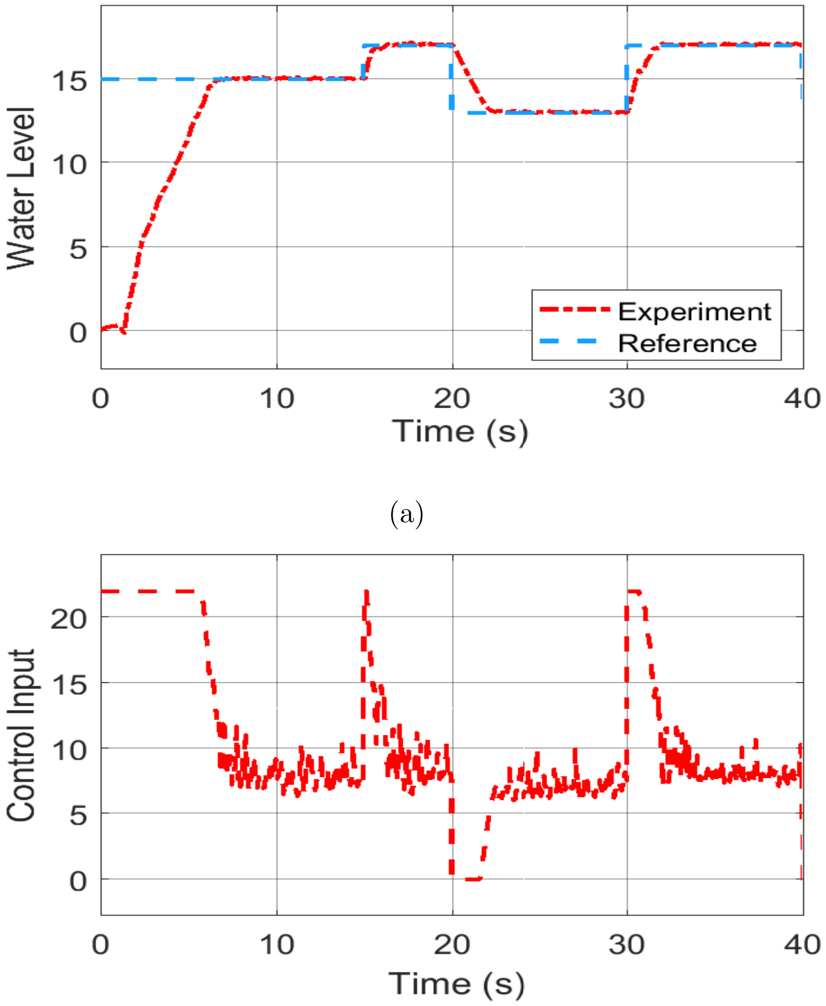
Figure 7.3: Simulation results for unperturbed system: (a) Water level tracking response, (b) Applied control input

potentially attributed to the external perturbation and inherent uncertainties arising from sensor noise.

7.3 Effect of Sampling Time

In this section, we consider a simple first-order discrete-time system to show the dependency on the sampling time. The lesser the sampling time, more is the control effort and vice-versa. To visualize this, let us consider the following system:

$$z(k+1) = z(k) + T_s u(k) \tag{7.7}$$



(a)

(b)

Figure 7.4: Experiment results for unperturbed system: (a) Water level tracking response, (b) Applied control input

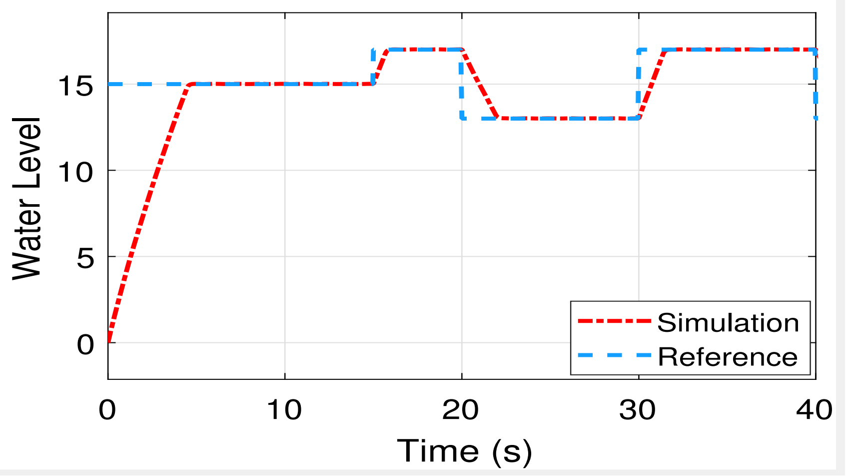
where $z \in D \subset \mathbb{R}$ is the system state, $u \in \mathbb{R}$ is the control input, and $T_s \in \mathbb{R}_+$ is the sampling time. We design the control law as

$$u(k) = \frac{1}{T_s}\{-\text{sign}(z(k)) \min\{|z(k)|, \gamma\}\} \tag{7.8}$$

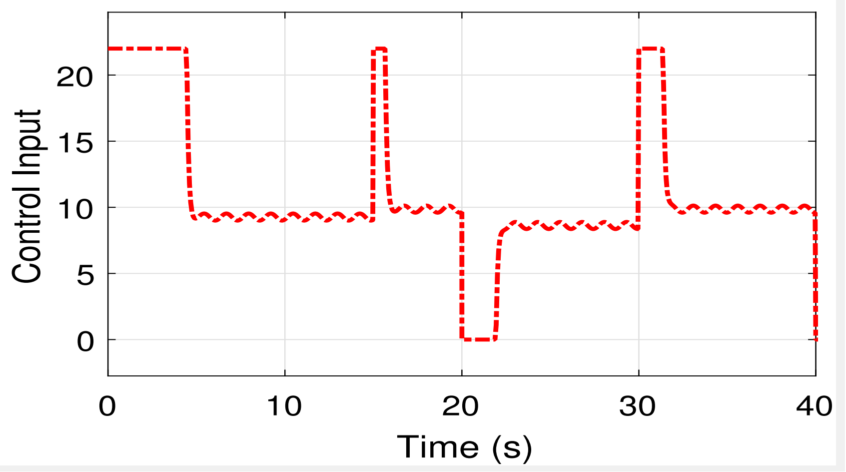
where $\gamma \in \mathbb{R}_+$.

To visualize the effect of sampling time, we simulate the above system with three different values of sampling time keeping initial condition and design constant fixed. We considered $z(0) = 50$, $\gamma = 3$, and $T_s = 0.9, 0.5, 0.3$.

From Figure 7.7, we can see that as the sampling time is reduced, the convergence rate of system state increases, however chattering phenomenon also becomes prominent.



(a)



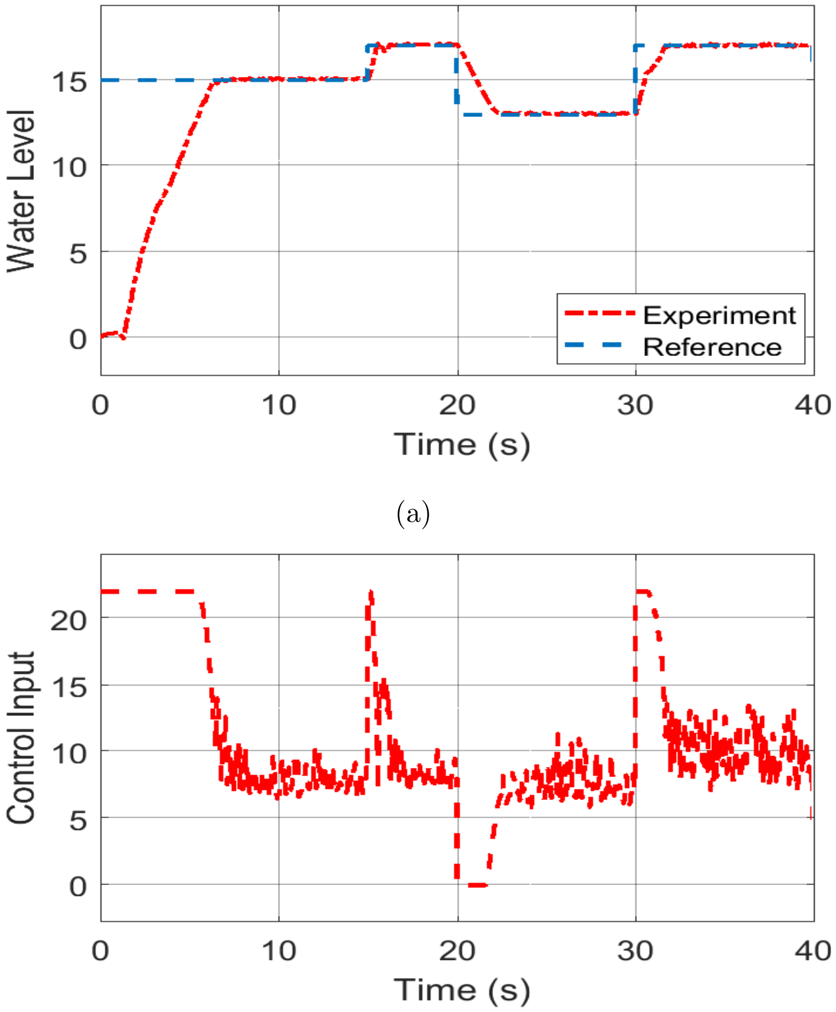
(b)

Figure 7.5: Simulation results for perturbed system: (a) Water level tracking response, (b) Applied control input

Similarly, in Figure 7.8, one can notice the increase in the amplitude of the control signal as the sampling time is reduced. From the design perspective, one should arrive at a suitable trade-off between speed of convergence and the amount of control effort required.

7.4 Conclusion

In this chapter, we explored the practical application of the proposed reaching law based on a difference equation with minima for designing discrete-time sliding mode control in a tank system. Specifically, we implemented this approach in a coupled tank system provided by Quanser, operating it in configuration 1. Two scenarios were considered: the



(a)

(b)

Figure 7.6: Experiment results for perturbed system: (a) Water level tracking response, (b) Applied control input

first without any external perturbation and the second with an external perturbation. For both cases, we presented both simulation and experimental results. The experimental water level tracking for both cases proved to be highly satisfactory, aligning well with the expectations derived from the simulation results. Regarding control effort, in the case of the unperturbed system, experimental results indicated rapid changes in the control signal attributed to sensor noise. Conversely, in the perturbed system, rapid changes in the control signal were observed due to the combined effects of external perturbation and sensor noise. In conclusion, the proposed controller successfully achieved precise water level tracking within a finite time. The experimental results were consistent with the simulation outcomes, affirming the effectiveness of the proposed approach in practical

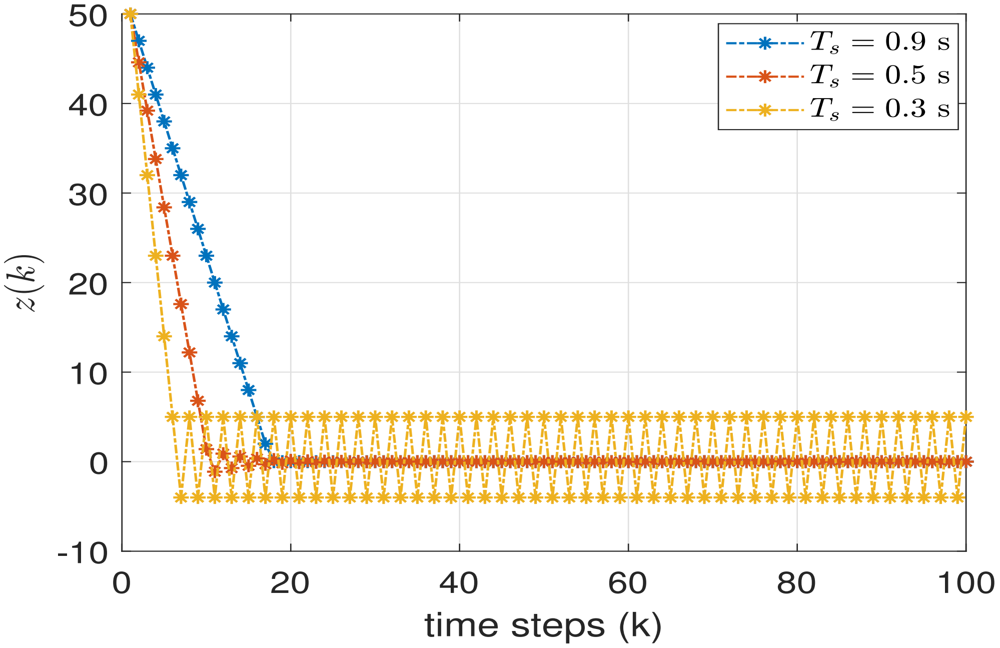


Figure 7.7: Evolution of system state for different sampling time

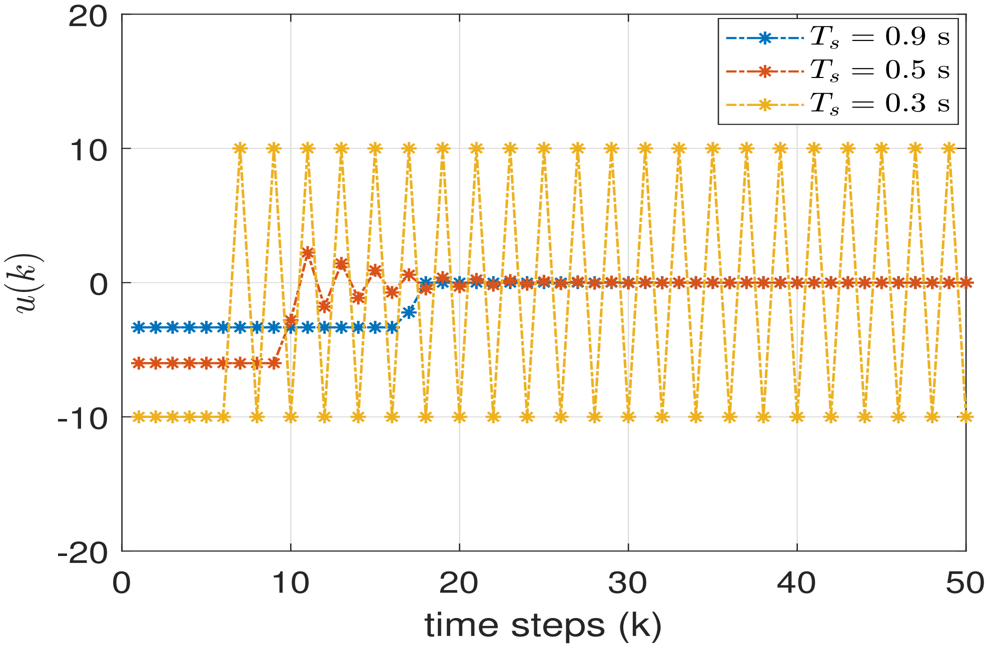


Figure 7.8: Control effort for different sampling time

implementations.