Chapter 2

Literature Review

2.1 Reservoir characterization

Reservoir characterization typically specifies the total volume inside the trap which has the capacity to contain hydrocarbons. The key elements in interpretation are the precision of reservoir approximation such as thickness as well as other petrophysical parameters of each reservoir, reservoir properties estimation, such as water saturation, porosity and other parameters from well log and seismic data. Seismic data can estimate reservoir parameters quantitatively throughout the analysis; one such fundamental step is to relate the seismic volume at well position often via a synthetic seismogram (a sonic and density logs derived seismic trace). Efforts throughout the study concentrate on predicting the physical sub-surface properties of rocks that are essential in the discovery and extraction of hydrocarbons. A significant element in quantifying producible hydrocarbons is the understanding of reservoir characterization (Schlumberger, 1989). It is possible to obtain precise reservoir characterization from well logs, mainly utilizing resistivity logs and gamma ray (Asquith and Gibson, 1982). Supplementary studies of the geological structure that can keep hydrocarbons in place must be considered in order to map hydrocarbon reservoirs, as hydrocarbons in geological traps, i.e. a rock formations combination that will prevent vertical or lateral migration of gas and oil. The progress of a diversity of methods that try to spread log properties has been motivated by the need to thoroughly analyze targets in order to identify ideal production approaches and also reduce the threat that may be correlated with exploration of hydrocarbon (Muslime and Moses, 2011).

The characterization of reservoirs has been analyzed over many years using geophysical data. Most approaches are based on a concept developed by Nur (1982), who showed that with temperature rise in heavy-oil saturated sands, P-wave velocity is reduced significantly. The observations of Nur have contributed to several time-lapse seismology experiments in the heavy oil fields of Western Canada. Pullin et al. (1986), Lines et al. (1990), De Buyl (1989) and Matthews and Sheriff (1992) explored the uses of seismic observing for the Athabasca oil sands. Isaac and Lawton (2014), Eastwood (1993) and Eastwood et al. (1994) made further developments in the seismic observing of improved oil recovery at Cold Lake, Alberta.

The acquisition and processing of multicomponent data was addressed by Hoffe et al. (2000) for Pikes Peak field data. Dey et al. (2000) used microphone data to reduce near surface noise on geophone data. Stewart et al. (2002) explored the application of recording 5 multicomponent data on cables positioned near the VSP acquisition site at the bottom of a tiny lake. Brittle et al. (2001) used the data to analyze the deconvolution of Vibroseis. The acquiring and processing of VSP data was recorded by Xu et al. (2001) and Osborne and Stewart (2001). Newrick et al. (2001) used the VSP data to present an analysis of Pikes Peak seismic velocity anisotropy. The impact of seismic attenuation through the steamed reservoir was checked by Hedlin et al. (2001). Downton and Lines (2000) examined the effectiveness of time-lapse analysis for Amplitude Versus Offset (AVO). P-P and P-S (converted-wave) data joint inversion was performed by Zhang (2003). The seismic response of a reservoir simulation was modeled by Zou et al. (2000). In and around the Pikes Peak area, Van Hulten (1984) delivered a detailed geological context for the Waseca Formation. At the UNITAR Conference in 1998. Sheppard et al. (1998) presented a paper primarily offering a reservoir engineering summary of Pikes Peak Husky 's thermal project. The problem of bottom water in the Pikes Peak reservoir was explored by Wong et al. (2003) and how field production may be expanded into these regions where heavy-oil saturated sands were underlying water saturated sands. In other areas of Western Canada, multicomponent approaches have been established and are relevant to the monitoring and study of the Pikes Peak heavy-oil reservoir.

Miller (1996) published an analysis of multi-component seismic data in carbonate (Lousana, Alberta) and clastic oil and gas fields (Blackfoot, Alberta). The interpretation of multicomponent 6 provided a base for discerning shale reservoirs rich in sand at Blackfoot, Alberta which was published by Stewart et al. (1996) and Margrave et al. (1998). In the oil and gas industry, the application of geophysical data for characterization of reservoir, monitoring and surveillance has attained wide recognition. The petrophysical and geophysical basis for reservoir detection using seismic technology was explored by Justice (1992) and Sheriff (1992). Wang (1992) summarized their previous study in rock physics for applications for reservoir characterization.

Porosity estimation is very important for reservoir characterization. There are many ways in which porosity can be measured, microscopic and macroscopic analysis, including well core analysis, and well log data analysis (Maity and Aminzadeh, 2015; Bahmaei and Hosseini, 2019). Porosity estimation utilizing seismic data has been performed by different investigators and in different ways. For example, to predict P-wave velocity and porosity in different areas, Holt et al. (2018), Duffaut et al. (2018), Duffaut and Landrø (2007) and Landrø et al. (2019) and applied seismic tests. The influence of a unique attribute on the estimate of P-wave velocity and porosity was studied by Yan et al. (2019). The method of evaluation is what differentiates the present project from related studies. A key problem that is not addressed in any of the supplementary researches is to take into description the attributes which have an important relationship with P-wave velocity and porosity in addition to having a logical-mathematical connection, so that they can be utilized in the estimation process. The accuracy of the estimation may be inaccurate if this important relationship is absent, even if relatively decent results are obtained.

On the one hand, the significance of porosity as an important reservoir petrophyscial parameter, and on the other hand, the type of seismic attributes with significant information to classify petrophysical parameters and lithology, has led to the development of several types of porosity analysis software, provided the solid backing of the oil industry. An example of these improvements is the concurrent assessment of 3D seismic and well-log data in modeling, analysis, inversion and estimation. In view of this, extensive research on porosity and its measurement techniques were carried out, where attributes were discovered to be affected by these parameters, neural networks and computational regression methods were used. A proper analysis of seismic attributes and well log trace started with 3D seismic studies to determine porosity in southern Iran oil field. Forward modeling of acoustic impedance was conducted applying quality management with well-log data and seismic attributes after the interpretation of particular horizons. Regression techniques namely single attribute analysis, multi attribute analysis, and neural networks were employed to predict the porosity in some portions of this area using this model and inversion. Porosity seemed necessary to study, among the attributes centered on numerical equations as the use of some of them decreases the reliability of the estimates. Thus, with concern to a reasonable correlation with porosity that was ignored in alike research, an analysis of current attributes was carefully carried out. The results obtained clearly showed that the estimate improved dramatically after eliminating nonphysical attributes and reanalysis while the porosity coherence prediction also improved.

2.2 Seismic, well log data and its significance

2.2.1 Seismic data

The "time picture" of the subsurface structure is given by seismic data. An attempt to convert the time data to depth should be made for precise structural analysis. Prestack and post-stack seismic data are routinely used for reservoir characterization as these methods estimate various petrophysical properties. Seismic attributes derived from seismic data are a significant aspect of an analytical interpretative method that supports structural and stratigraphic interpretation (channels, pinches, meanders, etc.)

as well as providing guidance on the lithology variety and approximation of fluid content with possible gain of comprehensive reservoir characterization (Strecker et al., 2004). It is essential to know that attribute analysis enhances the traditional structural understanding, and its significance to a specific trouble of an overlook can be critically tested for the discriminating characteristics of the attributes set. For stratigraphic, structural and lithological interpretation, seismic attributes such as cosine of instantaneous phase, amplitude envelope and instantaneous phase were addressed. The attribute is especially useful in the identification of bright spots, sequence boundaries, accumulation of gas, major variations or depositional conditions, irregularities, significant lithology changes, effects of thin-bed tuning, local variations suggesting faulting and porosity spatial relationship (Fomel, 2007). The seismic data was served as an analytical trace (Taner, 2001) containing original input trace and a imaginary component typically produced from the Hilbert transforms from which it is possible to deduce different phase, amplitudes and frequency attributes (Nissen, 2002). This complex trace makes it possible to quantify the attributes of phase, amplitude, reflector polarity of seismic data and frequency in a severe numerical sense (Stuart et al., 2005).

Some qualitative knowledge on the structure and physical parameters of the subsurface is provided to us by the analysis and interpretation of seismic attributes (Taner, 2001). It was noted that for the evaluation of physical characteristics such as reflection coefficients, acoustic impedance, absorption, velocities etc., the seismic data amplitude value is the key element. The phase component is the key element in description of the reflectors forms, their geometrical measurements etc. In terms of time or space, the seismic attributes obtained from seismic data could be amplitude, velocity, frequency, and the rate of change of each of these etc. The chief goal of the attributes is to offer the interpreter with reliable and comprehensive knowledge on the stratigraphic, structural and lithological parameters.

The classification of attributes

Attributes may be computed before or after time migration from pre-stack or from post-stack seismic data. In both of these situations, the process is the same. It is possible to identify attributes in several different ways. Several authors have provided a classification of their own. The classification of the attributes based on the domain characteristics are as follows:

(i) Pre-stack attributes

CDP or image gather traces are used as input data. They can have offset and directional (azimuth) associated details. These calculations produce enormous numbers of data; therefore, they are not realistic for preliminary investigations. Though, they include significant quantities of knowledge that will be specifically linked to orientation of fracture and fluid contents. Pre-stack attributes contain velocities, AVO and azimuthal variation of all attributes.

(ii)Post-stack attribute

Stacking is a combining procedure that excludes azimuth and offset associated details. Migrated or CDP stacked data can be used as input data. It must be remembered that time relationships will be retained in time migrated data, so temporal variables can also hold their physical measurements, such as frequency. For the depth migrated segments, frequency is replaced by wave number which is a function of frequency and propagation velocity. In initial exploration investigations, post-stack attributes are much extra controlled method for examining huge data volumes. Together the post- and pre-stack attributes can be implemented for comprehensive studies.

Few basic characteristics of attributes

The basic characteristics of attributes are described as follows:

- Primarily signifies the contrast of acoustic impedance, hence reflectivity.
- Sequence boundaries.

- Promising gas accumulation and bright spots.
- Effects of thin-bed tuning.
- Key modifications in depositional atmosphere.

Spatial association with porosity and other lithological changes. Suggests the class rather than the phase component of the propagation of seismic waves.
Several researchers (Das et al., 2017; Adler, 1998; Robertson and Fisher, 1988; Lewis, 1997; Alam et al., 1995) have reported the interpretation and use of seismic data for characterization of reservoir.

2.2.2 Well log data

Well log study offers information about the petrophysical parameters and fluid content of the reservoir rock. Well logging (borehole logging) is the way of constructing a thorough record of the geological formations penetrated by the borehole. The log may be focused either on a visual examination of samples were taken to the surface (geological logs) or on physical measurement taken by instruments dropped into the hole (geophysical logs). Throughout any phase of the well, particular kinds of well logs can be recorded: drilling, producing, or abandoning. Well logging is conducted in gas and oil drilled boreholes, groundwater, geothermal exploration and mineral, etc.
Well log analysis is one of best essential procedures for characterizing reservoirs, in the gas and oil industry, it is necessary for geoscientists to explore extra information about the condition beneath the surface by applying the rocks physical characteristics. This tool is very significant for the detection of the hydrocarbon bearing formation, the measurement of the hydrocarbon volume, etc. By exhausting well log data, some methodologies are required for the detection of reservoir, the geoscientists could be able to measure the following:

1. Permeability
2. Shale volume

3. Reflectivity coefficient
4. Elasticity
5. Water saturation
6. Porosity
7. Other required data for the geophysicists

Formation assessment is a practice of data interpretation within a borehole for the identification and quantification of hydrocarbon bearing zones, lithology, physical and chemical characteristics of rock, and fluid varieties. Petrophysical analysis particularly address the porous media characteristics like: permeability, porosity, water saturation, shaliness and fluid identification especially in hydrocarbon bearing rock (Mukerji et al., 2001; Sarasty and Stewart, 2003). Petrophysical assessment of reservoir rock is typically based on log-based approximation of permeability, water saturation, mineral content and porosity (Popielski et al., 2012). Such explanations include calibration and validation of fundamental data. Rock physics design is an alternative process to identify the fluid element of reservoirs from well log data as well as seismic data (Gupta et al., 2012).

Another approach for well log study is crossplot method of elastic characteristics of rock which have been applied to establish improved lithology and pore fill indicators. P-impedance and S-impedance as well as Lame's parameters [lambda (λ), mu (μ), rho (ρ)] calculated from density, V_P, and V_S logs support the detection of fluid variety and lithology (Samantaray and Gupta, 2008; Inichinbia et al., 2014). The objective of the well log data analysis is (a) the detection of reservoir rocks from log responses, (b) the measurement of P-impedance (Z_P), S-impedance (Z_S) and Lambda-mu-rho as fluid type and lithology instruments, and (c) the formulation of petrophysical characteristics mainly shale volume, water saturation and effective porosity from existing traditional log data for wells holding limited prospective reservoir rocks.

Several authors (Omudu et al., 2008; Das and Chatterjee, 2018; Inichinbia et al., 2014; Singha and Chatterjee, 2017) have reported use of well log data for reservoir characterization.

2.3 An overview on seismic inversion methods

Seismic inversion implies the integration of seismic and well log data for estimation of petrophysical properties of the formations. In other words, seismic inversion is a method which uses seismic and well-log data on the physical characteristics of rocks and fluids to extract underlying models. The properties can also be inferred from seismic data inversion alone (Krebs et al., 2009), in the absence of well data. The seismic inversion method has been commonly used in the oil and gas industries as a method for locating hydrocarbon-bearing layers on the subsurface (Lindseth, 1979; Morozov and Ma, 2009). The physical parameters of interest for a model performing inversion are: velocity and density, impedance (Z), P-wave (V_P), and S-wave (V_S). Lamé parameters that are sensitive to fluid and rock saturation can be obtained from inverted impedance models (Clochard et al., 2009). Inverted volumes can be used to measure the petrophysical parameters such as porosity, sand / shale ratio and gas saturation (Goodway, 2001).

Seismic inversion is the method performed to obtain impression of a reservoir's quantitative rock property through converting the seismic reflection data. In other words, in order to obtain an earth model, seismic inversion is a method that converts seismic data to actual rock properties. Other measurements of the reservoir are usually needed, such as logs and cores. Inversion requires both seismic data and well data, where well data acts to improve the high frequency (below the seismic band) and induce the inversion. Firstly, the well logs are conditioned and revised to ensure an acceptable relationship exists between the required properties and impedance logs. Then, the logs are transformed to time, estimate the seismic bandwidth by filtering, and corrected according to quality for borehole consequences. Seismic inversion utilizes forward modeling to generate synthetic seismic data which match the seismic data observed.

The application of such inversion methods provides the researcher with rock properties rather than reflective property. It allows specifics of the geological data to be inferred. Geophysicists execute seismic surveys regularly to collect the geological information of a gas or oil field. All such surveys detect sound waves that have gone through earth's fluid and rock layers. Such waves amplitude and frequency can be determined such

that any tuning and side-lobe effects that the wavelet produces can be avoided.
Seismic inversion methods have been commonly used to identify prospective gas and oil reservoirs, and also to provide information about the reservoir rocks physical properties. Changes in the rock's physical properties like seismic acoustic impedance, density cause major impact that could be found in seismic data of high quality.
To understand seismic inversion better, first consider the physical procedures involved in seismic data production. Therefore, focus at the elementary convolution seismic trace model in frequency and time domains, taking into account the three elements of this model: seismic wavelet, reflectivity, and noise.
Seismic inversion methods are divided into two broad categories: (i) Post-stack inversion methods, and (ii) Pre-stack inversion methods. The first method is the most widely used where the wavelet effect is removed from the seismic data and a high-resolution subsurface image is produced (Chen and Sidney, 1997) and estimates P-impedance. The second method is focused on model construction based on well log, seismic and geological data (Downton, 2005). This also produces a high-resolution view of the subsurface from which to determine the reservoir properties. An accurate estimation of the reservoir properties during the development phase is important in the decision making process (Pendrel, 2006). Pre-stack inversion converts offsets data or seismic angle into volumes of P- impedance (V_P) , S- impedance (V_S), and density (ρ) by combining of offset gathers or seismic angles, well data and a required stratigraphic analysis. Other potential combinations of elastic parameters such as P-wave velocity, S-wave velocity, and V_P/V_S ratio are equally important. Usually, depending on the goal and acquisition, two parameters (e.g., Z_P and V_P/V_S ratio) are trustworthy and can be utilized to estimate properties of reservoir away from the borehole. These post-stack and pre-stack methods are discussed below:

2.3.1 Seismic inversion of post-stack data

The methods of post-stack seismic inversion or seismic inversion of post-stack data is categorized as the first type. This type of inversion results acoustic impedance volume

utilizing seismic data through integration of the well data and an elementary stratigraphic interpretation. Such impedance volumes can be used to estimate properties of reservoir away from borehole (Russell and Hampson, 1991; Morozov and Ma, 2009). The seismic inversion methods described here are using post-stack seismic data to measure the sub-surface's physical properties. The effectiveness of the inversion method is dependent on the successful conversion of seismic amplitudes into impedance values. In order to conserve amplitude throughout inversion, sufficient precaution must be taken because it guarantees that the estimated variations in amplitude are connected to the geological formations (Vestergaard and Mosegaard, 1991). Therefore, the multiples must be processed and removed from the seismic data. This results in a high S/N ratio (signal to noise ratio) and zero offset relocated section free of any mathematical artifacts. The seismic data is limited to certain band (band-limited), and thus the absence of lower frequencies protects traces of transformed impedance from providing a significant elementary velocity framework for geological analysis (Clochard et al., 2009). However, the produced volume of impedance has mediocre resolution which cannot identify the thin layers. So these things are important and need to be looked after during seismic inversion.
Some of the advantages of post-stack inversion are mentioned below:

1. The acoustic impedance (AI) is a layer property; hence stratigraphic analysis on impedance data is easier in comparison to seismic data.

2. Reducing side lobes, tuning and wavelet effects increases subsurface layer resolution.

3. The acoustic impedance can be measured directly and compared with well log dimensions which serve as a connection to the reservoir properties.

4. Porosity, density etc. can be related to the acoustic impedance. Using geostatistical techniques the acoustic impedance volumes can be converted to the porosity, density, etc., volumes within the reservoir.

5. Acoustic impedance can be utilized to locate individual reservoir regions.

6. Post-stack inversion takes very less time in comparison to pre-stack inversion.

Acoustic impedance (P-impedance) provides information about the properties of rock constituting the reservoir (Latimer et al., 2000; Maurya et al., 2018; Anderson, 1996; Russell and Hampson, 1991; Helgesen et al., 2000; Wang, 2006; Dossal and Mallat, 2005; Maurya and Sarkar, 2016; Maurya and Singh, 2018). The inversion method simulates a synthetic seismic trace from the easiest possible reflectivity model that matches with the input seismic trace. After that the derived P-impedance from various post-stack inversion methods are used to predict several petrophysical parameters of sub surface. Further, the present study uses geostatistical techniques to predict various petro-physical parameters in inter well region.

There are four post stack inversion methods, namely, (i) Model based inversion (MBI), (ii) Colored inversion (CI), (iii) Maximum likelihood sparse spike inversion (MLSSI) and (iv) Bandlimited inversion (BLI). A brief description of each method follows:

(i) **Model based inversion**

Model based inversion (MBI) is a method of converting seismic amplitude to impedance value, using zero-offset (stacked) seismic data to provide time or depth pictures of acoustic impedance (AI). The acoustic impedance (AI) is now one of rock physics parameters that is determined by form of lithology, fluid content, porosity, depth, temperature, and pressure (Maurya et al., 2020; Maurya and Singh, 2017).

MBI is dependent on the convolutional principle which suggests that the seismic trace with the reflectivity function can be produced from wavelet convolution. This method represents the subsurface in contexts of acoustic impedance and time, as layers or blocks. The impedance model is produced by interpolating the impedance logs as a guide, using seismic horizons (Maurya et al., 2020; Maurya and Sarkar, 2016). The impedance can differ laterally and vertically within each layer. Reforming the number of layers compared to the seismic samples number takes care of the non-uniqueness (Mallick, 1995).

The basic principle of MBI is to presume a P-impedance low-frequency model and interrupt the model until a decent match is obtained between computed synthetic seismic trace and the seismic trace. Extracted wavelet shall be considered as a reasonable approximation of the seismic section. Seismic wavelet is a source signature and is necessary in many other inversion techniques. The extraction of the wavelet may be statistical in which it is derived from seismic data, or it may be derived from well log data.

MBI method is critically dependent on a good fit between the seismic data and the well log. The correlated wave generated using both well and seismic data with seismic horizons is used to produce preliminary impedance model which is an essential input for MBI method (Leite, 2010). Technology-based inversion also offers an excellent structural model of the subsurface (Lindseth, 1979; , 1988). Unfortunately, the outcomes of the inversion are sensitive to non-uniqueness and error. The outcome of the inversion is in form of velocity, density, and rock properties that may be correlated with rock type, porosity, and pore fill (Lindseth, 1979; Smith and Gidlow, 1987).

Until some time ago, seismic exploration has mainly consisted of interpreting the seismic PP data. Previous lithology approximation methods centered on the PP data's zero-offset inversion. An implication in inversion technique that utilizes P-wave for incident and refection is that the Gardner's equation that connects P- wave velocity and density is valid ($\rho = kV_p^m$), where k and m are a constant (Gardner et al., 1974). Fatti et al. (1994), updated the Smith-Gidlow technique to estimate $\triangle I/I$ and $\triangle J/J$ instead by $\triangle V_P/V_P$ and $\triangle V_S/V_S$ utilizing an observed relationship (as in Gidlow et al., 1993).

Several authors have utilized MBI method to transform seismic data into impedance. Like Stull (1973) employed this method to obtain subsurface model. Mallick (1995) utilized MBI with genetic algorithm and Bayesian analysis to obtain more detailed image of the subsurface. He specified that provided previous information, the MBI could provide a more detailed subsurface model and can also minimize processing time. The mathematical background of MBI is discussed in following section:

MBI is focused on the convolutionary concept which specifies that the seismic trace can be derived from the wavelet convolution with the Earth's reflectivity and inclusion of noise (Mallick, 1995).

$$S(t) = W(t) * R(t) + N(t) \tag{2.1}$$

Where $S(t)$ represents seismic trace, $W(t)$ denotes seismic wavelet, $R(t)$ represents reflectivity, $*$ convolution operator, and $N(t)$ denotes data's noise component.

Unless the data noise is uncorrelated to seismic signal, the trace for the earth reflectivity function can be solved (Latimer et al., 2000). It is a non-linear problem that can be resolved iteratively as follows.

$$Z = \rho * V \tag{2.2}$$

$$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} \tag{2.3}$$

$$AI_N = AI_1 exp\left(\sum_{i=1}^{n} R_i\right) \tag{2.4}$$

Z denotes the impedance of layer whereas i^{th} layer impedance is $Z_i = \rho_i * V_i$, V_i represents velocity of i^{th} layer, and R_i is the i^{th} layer reflection coefficient. This equation is applicable for almost all of the applied cases, where $R_j \leq |0.3|$ (Berteussen and Ursin, 1983). In exercise, these equations are utilized for recursive inversion in order to transform function of reflectivity into acoustic impedance Z_P (Berteussen and Ursin, 1983). AI_1 denotes 1^{st} (top) layer acoustic impedance and AI_N denotes N^{th} layer acoustic impedance.

The low-frequency acoustic impedance model has produced by measuring these values at the wells using kriging interpolation methods over all the seismic section. Acoustic impedance is usually not recorded during well log data acquisition. These variables can be directly derived from the sonic and density log. The objective function [E(R)] that must be minimized is:

$$E(R) = \alpha \sum_{j=1}^{m} |R_j(t)| + \frac{1}{2} \parallel \frac{1}{\sigma}[S(t) - W(t) * R(t)] \parallel^2 \tag{2.5}$$

The first term in the above equation is l_1 norm of earth's reflectivity and α governs the sparsity of the calculation. The next term reduces the dissimilarity between artificial synthetic seismic trace and real seismic trace. $W(t)$ is wavelet which is produced from the seismic or well log data, $R(t)$ denotes the reflectivity of the earth, σ represents Poisson's ratio and $S(t)$ denotes the original seismic trace.

The workflow of model based inversion method is as follows (Ferguson and Margrave, 1996):

(i) Estimate the acoustic impedance (Z_P) using the well log data at well locations.

(ii) In the seismic segment, pick horizons to command the interpolation and deliver model structural knowledge between the wells in the region.

(iii) To obtain the initial acoustic impedance model, use interpolation between the well locations and interpreted seismic horizons.

(iv) Use any selected block size to block the initial impedance.

(v) From the seismic section, extract statistical wavelet.

(vi) Perform convolution with wavelet and reflectivity of earth to produce synthetic seismic trace. The synthetic trace differs from the recorded seismic trace.

(vii) The Least Squares optimization is conducted to minimize the deviation between the modeled and real section of reflectivity. It is obtained by examining the misfit between the real trace and the synthetic trace, and adjusting the extent of the block and the amplitude to minimize the error.

(viii) Repeat step 7 until the least misfit is achieved between the synthetic trace and actual seismic trace. (Maurya and Sarkar, 2016).

(ii) **Colored inversion**

A fast band-limited seismic data inversion method was developed by Lancaster and Whitcombe (2000) termed as colored inversion (CI) which created extensive attention among interpreters. Recognizing that a single operator can approximate the common sparse-spike inversion method, resulting in relative impedance with reflectivity data through simple convolution, the authors revealed that the above operator could be extracted from well log data. Like some other inversion methods, CI can help eliminate the seismic wavelet's smearing effects and improve features unlike discontinuities and thin beds. However, as CI is precisely connected to seismic data, the comparative impedance it generates can be utilized as a basis for discussion with other inversion methods to understand which type of information is added by quantitative restrictions or the model of low-frequency.

Colored inversion method also converts the seismic data into acoustic impedance (Z_P). Colored inversion performs considerably better than conventional model-based approach of inversion (Lancaster and Whitcombe, 2000). Other types of inversion are time-consuming, costly and are not regularly performed by the interpreter. On the opposite, the colored seismic inversion is fast, simple to use, less costly and robust (Chopra et al., 2006).

Seismic colored inversion enables the interpretation of sub-surface layers and layer boundaries, as well as helps to distinguish soft and hard layers together with fluid variations in the pores. A major advantage of seismic colored inversion over MBI method is that the findings are not biased by using the initial low-frequency guess model. A disadvantage of the colored inversion method is that the inverted impedance measurements are not absolute; they are relative, meaning the findings are not valuable for quantitative interpretation (Neep, 2007).

Several authors introduced seismic colored inversion after Lancaster and Whitcombe (e.g. Swisi, 2009; Maurya and Sarkar, 2016 etc.) and found the technique to be very quick and provide a clear subsurface image.

Colored inversion is a process where the acoustic impedance spectra extracted from well log data are utilized to measure the operator's spectrum. This operator's phase is -90^0 which enables its fusing with series of reflectivity to produce the impedance (Lancaster and Whitcombe, 2000). In the following steps, the operator is extracted:

First, for all wells in the field the acoustic impedance (Z_P) is measured and cross-plot between acoustic impedance and frequency is plotted (Fig. 2.1). Second, a regression line is matched to the acoustic impedance amplitude spectrum to display the spectrum of impedance in the subsurface log-log scale (Fig. 2.2); Third, the seismic spectrum is computed from the seismic traces close to the wells. Such two spectra are utilized to compute the spectrum of the operator which converts the spectrum of seismic into the mean spectrum of impedance. Fourth, the last spectrum is paired with a -90^0 phase modification to build the optimal time domain operator (Fig. 2.3). The operator can be defined in frequency domain as shown in Fig. (2.4). Colored inversion is quick and appropriate the 3D datasets (Neep, 2007).

In this approach the inversion could be approximately interpreted as a convolutional process (filtering process). In the frequency domain, it usually uses an operator (O) to transform seismic traces S(t) precisely into impedance (Z). This operator measures the spectrum of seismic amplitude into the spectrum of earth impedance. This operator's phase is -90^0 (Lancaster and Whitcombe, 2000), hence that it transforms the series of reflectivity into impedance.

Lancaster and Whitcombe (2000) have been described colored inversion method in their extended paper. Performing open-source procedures, they explain whole steps from reflectivity information to inverted cubes:

(i) Match the log spectrum(s) function.

(ii) Get a variety of variations by eliminating the seismic spectrum.

(iii) Transform the spectrum of variance to an operator.

(iv) Link the operator to stacked seismic.

(v) As a QC stage check the residuals by matching the section spectrum of log and acoustic impedance.

Several seismic and well log spectra are tested to describe colored inversion operator, which converts the seismic trace into an average acoustic impedance log (Lancaster and Whitcombe, 2000). The assumption is that input seismic data should have a zero phase. The colored inversion operator is transformed to the time domain using a convolution algorithm, and performed to the seismic volume. When the Colored Inversion operator is determined, it can be used as a 'user-defined filter' for the analysis of seismic data.

In this way, the inversion can be performed within hours, as the volume data need not be transmitted to software and no specific wavelet is needed (Veeken and Da Silva, 2004).

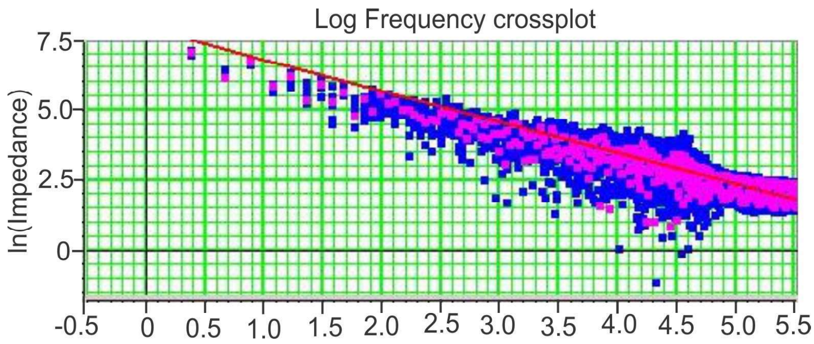

FIGURE 2.1: AI from all wells (blue), one selected well (pink) and frequency on log-log scale (After Maurya and Sarkar, 2016)

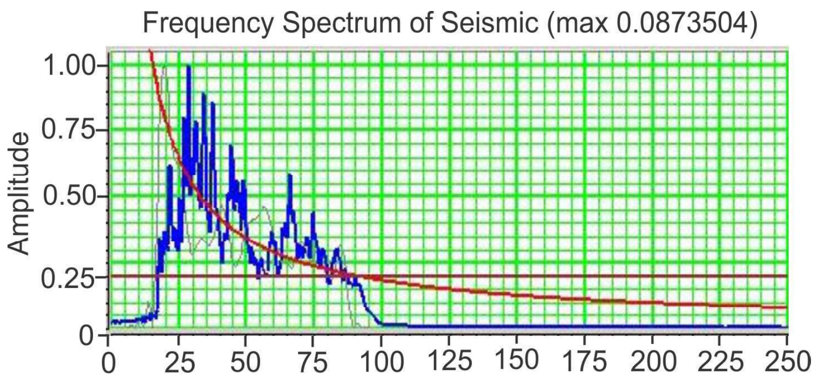

FIGURE 2.2: Seismic spectra near the wells (blue). Red line corresponds to the $f^{-\theta}$ AI spectrum derived in Fig. 2.1. The operator spectrum (black) is the ratio of these two spectra (After Maurya and Sarkar, 2016)

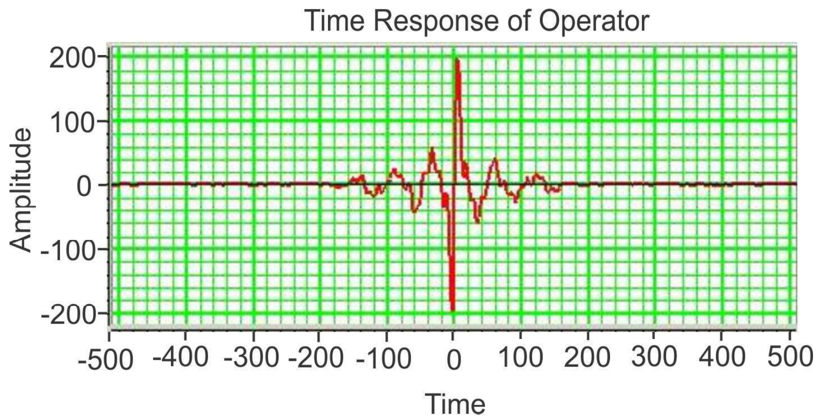

FIGURE 2.3: Time response of operator (After Maurya and Sarkar, 2016)

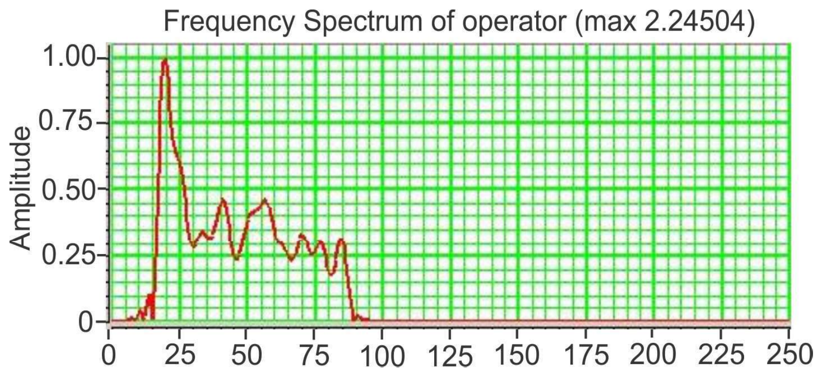

FIGURE 2.4: Frequency spectrum of the operator (After Maurya and Sarkar, 2016)

(iii) **Maximum likelihood sparse spike inversion**

During the 80's maximum likelihood sparse spike inversion (MLSSI) method was introduced (Helgesen et al., 2000). It focuses on the techniques using additional knowledge from well log data (e.g. Russell, 1988). It is generally known as model based inversion (Russell, 1988), when the maximum likelihood sparse spike inversion is forced by a low frequency model (LFM) obtained from acoustic impedance well logs of geologic models.

MLSSI is model-dependent and is based on the principle that the reflectivity consists of a series of major spikes combined with minor spikes on the background (Wang, 2006; Maurya and Singh, 2015, Maurya and Sarkar, 2016). This inversion method imitates a synthetic seismic trace from the easiest possible model of reflectivity that fits with the input seismic trace.

The objective of MLSSI is directed to derive a volume of acoustic impedance from seismic data and well log data. The inverted impedance owns a broad spectrum band, shows a blocky subsurface structure in time domain, and is directly connected to the lithology. Seismic data bandwidth is usually 10-80Hz frequency, so there is deficiency of low and high frequency content. Additional information is required to acquire a broadband spectrum of the inverted impedance volume, which usually attains from well log data.

Although the real seismic data could be used to identify variations in the reservoir, acoustic impedance is a strong indicator of variation because it has better resolution than the seismic data (Pendrel, 2006) and is a dimension of the specific characteristics of the layers, while seismic data only reveals the contrast between layers at their boundaries. This empowers fluid variations in the reservoir to be more simply identified in the seismic data and the observing of fluid migration more effective. One technique of regaining the acoustic impedance from the seismic data is the bandlimited inversion method. Fundamentally, low frequency data is missing in the seismic data which must be provided by other resources for the algorithm to work. Lindseth (1979) proposed that details from a neighboring well log could be utilized to retrieve this missing low-frequency element.

MLSSI is executed in two steps. In the first step, maximum likelihood deconvolution is applied to estimate the seismic reflectivity series. Then in the second step, reflectivity series is transformed into the acoustic impedance which is more important to infer data about the subsurface layer (Hampson and Russell, 1985). MLD is based on the notion that the Earth's sequence of reflectivity consists of significant spikes surrounded in the background by minor spikes. This technique also assumes that only large spikes are essential because they show a deposition gap in the subsurface, hence the goal of the Maximum Likelihood Deconvolution is to identify these large spikes from the seismic data. The seismic trace is expressed as follows.

$$S(t) = R(t) * W(t) + N(t) \tag{2.6}$$

Where $S(t)$ is time dependent seismic trace, $R(t)$ is reflectivity of earth depended on time, $W(t)$ is source wavelet dependent on time and $N(t)$ is time dependent noise component. The Eq. (2.6) can be written as following.

$$S(t) = \sum_{j=1}^{t} R(j)W(t-j) + N(t), t = 1, 2, ...s \tag{2.7}$$

Using the subsurface model assumptions, one can reduce the target function for the best solution i.e. earth's reflectivity series. The objective function $E(R)$ can be written as following.

$$E(R) = \sum_{j=1}^{t} \frac{R^2(j)}{r^2} + \sum_{j=1}^{t} \frac{N^2(j)}{n^2} - 2mln(\psi) - 2(t-3)ln(1-\lambda) \tag{2.8}$$

Where $R(j)$ is reflection coefficient at j^{th} sample, $r$is square root of reflectivity variance, m is number of reflections, t is total number of samples, n is square root of noise variance, $N(j)$ is noise at j^{th} sample, and ψ is the likelihood that a given sample has a reflection (Zhang and Castagna, 2011; Russell, 1988). By minimizing error as given by Eq. (2.8), one can obtain reflectivity series of the earth's seismic data. Thereafter, this reflectivity series is converted into acoustic impedance. If the reflectivity is given by $R(i)$ then the resulting impedance $Z(i)$ can be given as follows.

$$Z(i) = Z(i-1)\left[\frac{1+R(i)}{1-R(i)}\right] \tag{2.9}$$

Inappropriately, the use of Eq. (2.9) to estimate the reflectivity from MLD produces unacceptable results, specifically in cases of more noise. Although the MLD algorithm extrapolates outside the wavelet band width to provide a broadband reflectivity estimate, yet at the bottom of the spectrum the reliability is degraded by noise. Therefore, the impedance's brief wavelength characteristics can be reassembled properly, but the general trend is poorly resolved. This is similar to the fact that the spike's times on the reflectivity approximation are well determined than their amplitudes.

The independent understanding of the impedance trend can be used as a constraint to stabilize the reflectivity estimate. Since $R(i) < 1$, a convolution type equation between acoustic impedance and reflectivity can be derived as follows.

$$lnZ(i) = 2H(i) * R(i) + e(i) \tag{2.10}$$

where $Z(i)$ is the recognized trend in impedance, $e(i)$ is "errors" an input trend and $H(i)$ can be written as follows.

$$H(i) = \begin{cases} 1, & \text{if } i < 1 \\ 0, & \text{if } i > 1 \end{cases}$$

MLSSI is a method that extracts broad-band seismic reflectivity approximation and enables us to invert into an acoustic impedance segment by introducing linear limitations that maintains the main geological characteristics of borehole log information.

In other words, the maximum likelihood sparse spike inversion method use simple subsurface reflectivity model derived from the well log data, and convolution theory to generate synthetic traces. The error between synthetic trace and seismic trace is minimized by introducing more and more spikes in the reflectivity series (Debeye and Van Riel, 1990). Depending on the minimization of error, the sparse spike inversion methods are divided into two groups. The first method is called Linear Programming sparse spike inversion (LPSSI) methods that use l1 norm solution for its implementation and the second is called Maximum Likelihood sparse spike inversion (MLSSI) methods that use l_2 norm solution for its implementation (Russell, 1988; Sacchi and Ulrych, 1996; Zhang and Castagna, 2011). The inverted impedance reveals a time-domain blocky subsurface structure which is directly related to the subsurface lithology (Goutsias and Mendel, 1986; Wang, 2006).

These methods are best for areas with fewer reflectors. They do not require a geological model, so they are better for areas where we have less knowledge of the geology (Xing, 2005).

The positioning of spikes in the reflectivity sequence is determined by using the single most- likely-addition algorithm to add the optimum spike to the current model. This algorithm iterates until it finds a spike with amplitude less than this threshold value multiplied by the average amplitude of all previous spikes (Grana and Della Rossa, 2010). In other words, as a spike is added, its amplitude is compared to the average amplitude of all added spikes detected so far. When the amplitude for a new spike is less than the specified fraction of this average, the algorithm stops adding spikes. That specified fraction is the Spike Detection Threshold.

(iv) **Bandlimited inversion**

Bandlimited inversion (BLI) is used to measure impedance from seismic data. Necessarily, high frequency and low frequency data are missing in seismic data which have to be provided by some other sources such as well log, additional geological information. Lindseth (1979) suggested that inclusion of a nearby well log data could be performed to provide this missing component of low-frequency. Several authors have performed band-limited impedance inversion (Lindseth, 1979) to generate subsurface lithology from seismic data. Ferguson and Margrave (1996) performed this algorithm and observed that band-limited impedance inversion provides accurate impedance measurements for both traces of 10 Hz and 2 Hz. The traces of 2 Hz have been found to carry reliable information at low-frequency.

Llyod and Margrave (2013) also performed this algorithm to acquire information about sand channel in the seismic section. Heather describes that the low frequency cut-off needs to be selected correctly for the frequency to be smooth. Very high values could generate low frequency smearing all over the inversion section and very low value decreases the amount of accurate low frequencies information.

Band limited inversion (BLI) converts post-stack seismic data into impedance. The band limited inversion method starts with identifying the connection between the seismic impedance and seismic trace (Ferguson and Margrave, 1996). Therefore, express the coefficient of normal incidence reflection as:

$$r_j = \frac{Z_{j+1} - Z_j}{Z_{j+1} + Z_j} \tag{2.11}$$

where Z_j is seismic impedance of j^{th} layer, and r_j is seismic reflectivity of j^{th} and $(j+1)^{th}$ interface.
Solve above equation for impedance of $(j+1)^{th}$ layer.

$$Z_{j+1} = Z_j(1 + \frac{2r_j}{1 - r_j}) = Z_j(\frac{1 + r_j}{1 - r_j}), \quad (2.12)$$

Impedance of n^{th} layer if we know impedance of 1^{st} layer is:

$$Z_n = Z_1(\frac{1 + r_1}{1 - r_1})(\frac{1 + r_2}{1 - r_2})...(\frac{1 + r_{n-1}}{1 - r_{n-1}}) \quad (2.13)$$

The first layer acoustic impedance needs to be estimated from a continuous layer above the target area (Llyod and Margrave, 2013). Therefore, the impedance for the j^{th} layer can thus be calculated as follows:

$$Z_{j+1} = Z_1 \prod_{k=1}^{j}(\frac{1 + r_k}{1 - r_k}). \quad (2.14)$$

Divide above equation (2.15) by impedance of 1^{st} layer that is Z_1 and take the logarithm on both side,

$$ln(\frac{Z_{j+1}}{Z_1}) = \sum_{k=1}^{j} ln(\frac{1 + r_k}{1 - r_k}) \approx 2\sum_{k=1}^{j} r_k \quad (2.15)$$

The last step follows from an approximation for ln which is valid only for small r. By solving equation (2.15) for Z_{j+1} we have:

$$Z_{j+1} = Z_1 exp(2\sum_{k=1}^{j} r_k) \quad (2.16)$$

Model the seismic trace as scaled reflectivity: $S_k = \frac{2r_k}{\gamma}$, then above equation becomes:

$$Z_{j+1} = Z_1 exp(\gamma \sum_{k=1}^{j} S_k) \quad (2.17)$$

Thus the above equation incorporates the trace of seismic and afterwards exponentiates the outcome to delivering a trace of impedance. Bandlimited impedance inversion method uses this equation to invert the seismic trace in almost the same manner as Waters (1978), with a few connected to pre-conditioning of the needed impedance estimate and seismic trace scaling (Ferguson and Margrave, 1996). BLI method is restricted to the very similar frequency extent as that of the seismic data (Ferguson and Margrave, 1996). For this purpose, an initial low frequency model (LFM) is applied before the inversion.

(i) The preliminary model is extracted by filtering the wells impedance log. Using wells and seismic horizons, interpolation is used to acquire an initial 3-D model.

(ii) Apply iterative equation to traces of seismic to acquire band- limited inversion.

(iii) Last iterative inversion result is a mixture of the preliminary model with the bandlimited acoustic impedance.

A very significant drawback of BLI method is that seismic data has to be in zero phases. By using extracted wavelet, the seismic pre-stack seismic data could be converted to zero phases (Maurya and Singh, 2016).

2.3.2 Seismic inversion of pre-stack data

The pre-stack inversion or seismic inversion of pre-stack data comes into the seismic inversion methods of the second category. Pre-stack inversion may be performed to estimate the elastic properties of the subsurface, such as S-wave velocity of the subsurface layers that are prone to fluid saturation (Moncayo et al., 2012). Pre-stack inversion converts seismic data (angle/offset gathers) into volumes of P-impedance (Z_P), S-impedance (Z_S), and density (ρ) by combining well data and boundary information from seismic data. Depending on the target depth and acquisition configuration, P-impedance (Z_P), and (V_P/V_S) ratio are efficient, and can be used to estimate reservoir properties away from borehole (Carrazzone et al., 1996). Pre-stack seismic inversion delivers numerous benefits:

1. The layer properties are specified by P-impedance (Z_P), S- impedance (Z_S), and density (ρ), while seismic data is an interface property.

2. It boosted sub-surface layer resolution due to lessening of wavelet effects, tuning and lateral lobes.

3. Acoustic impedance (Z_P) can be compared directly with well log dimensions which in turn are related to properties of reservoir.

4. The data provides additional details compared to other inversion methods (e.g. post-stack inversion) to differentiate between fluid effects and lithology.

The pre-stack inversion method is further divided into several methods namely, (i) Simultaneous inversion (SI), (ii) LMR transform and (iii) Elastic impedance inversion (EI).

(i) **Simultaneous inversion**

The simultaneous inversion (SI) method measures P-impedance (Z_P), S-impedance (Z_S), density (ρ), and (V_P/V_S) ratio by simultaneously inverting each restricted angle stack data, utilizing each angle stack extracted wavelet. The inversion findings such as lambda-rho ($\lambda\rho$), mu-rho ($\mu\rho$) and density (ρ), are known to be useful in characterizing the prospective zone as a reservoir.

Previously, only the post-stack seismic inversion method was regularly used to measure and characterize acoustic impedance (P-impedance) of the reservoir. This method does not provide adequate information about the reservoir, however, since P-impedance alone cannot separate the effects between fluid content and lithology. Thankfully, this limitation has been resolved by extracting details from S-impedance to precisely identify the area's fluid content (Maurya and Singh, 2018; Maurya et al., 2019).

SI method measures P-impedance (Z_P), S-impedance (Z_S), density (ρ), and (V_P/V_S) ratio by simultaneously inverting each restricted angle stack data, utilizing each angle stack extracted wavelet. The inversion findings such as lambda-rho ($\lambda\rho$), mu-rho ($\mu\rho$) and density (ρ), are known to be useful in characterizing the prospective zone as a reservoir.

The purpose of SI method is to provide an accurate estimate of density (ρ), P-wave velocity (V_P), and S-wave velocity (V_S) for predicting the fluid and lithological property of Earth is subsurface. In short this is focused on three hypotheses. First, the linearized hypotheses for reflectivity retains (Ankeny et al., 1986). Second, section of PP and PS reflectivity as an angle gather function could be represented by the equations of Aki-Richards (Ma, 2002). Third, a linear relation exists between the P-impedance (Z_P) with S-impedance (Z_S) and density (ρ) (Nolet, 1978). Utilizing the above three hypotheses, the P-impedance (Z_P), S-impedance (Z_S) and density (ρ) may be measured by disturbing an initial model. This problem has been discussed by several authors. Simmons Jr and Backus (1996) inverted the linearized P-reflectivity (R_P), S-reflectivity (R_S) and density reflectivity (R_D), where

$$R_P = \frac{1}{2}\left[\frac{\triangle V_P}{V_P} + \frac{\triangle\rho}{\rho}\right] \tag{2.18}$$

$$R_S = \frac{1}{2}\left[\frac{\triangle V_S}{V_S} + \frac{\triangle\rho}{\rho}\right] \tag{2.19}$$

$$R_D = \frac{\triangle\rho}{\rho} \tag{2.20}$$

The reflectivity, as given in Eqs. 2.18-2.20 can be approximated from the Aki-Richards equation which is angle-dependent $R(\theta)$ (Richards and Frasier, 1976; Maurya and Sarkar, 2016). Density (ρ) and P-wave velocity (V_P) may be associated with Gardner's relationship ($a = 0.31, m = 0.25$; Gardner et al., 1974), as follows:

$$\frac{\triangle\rho}{\rho} = \frac{1}{4}\frac{\triangle V_P}{V_P} \tag{2.21}$$

On the other hand, the P-wave and S-wave velocities are connected to each other by Castagna's equation (Castagna et al., 1985; Maurya and Singh, 2017) in the following way.

$$V_P = 1.16V_S + 1360 \tag{2.22}$$

A linearized inversion method to explain the reflectivity is used. Buland and

Omre (2003) too employed an equivalent method known as Bayesian linearized AVO inversion. Apart from Simmons Jr and Backus (1996), they employ three terms $\frac{\triangle V_P}{V_P}$, $\frac{\triangle V_S}{V_S}$ and $\frac{\triangle \rho}{\rho}$, using the Aki- Richards estimation techniques. The small reflectivity estimation is also employed by few authors to connect these parameter improvements to the initial parameter. For variations in P-wave velocity, the relationship is written as follows

$$\frac{\triangle V_P}{V_P} \approx \triangle ln V_P \tag{2.23}$$

where ln denotes the natural logarithm. Analogous equation for density and S-wave velocity can also be written.

From several decades numerous inversion methods have been used to analyze and interpret seismic reflection data. Some of them are based on the modified Zoeppritz and Knott equation (Shuey, 1985; Maurya and Singh, 2017), and Aki and Richards (1980), which gives the most specific incarnation. Under some assumptions this modification reduced 16 equations with 16 unknown parameters to a single equation with three unknown Knott-Zoeppritz parameters.

Later the work of Aki and Richard was expanded by a huge number of authors. One method was to reinterpret the Aki and Richards equations in terms of Poisson's ratio and P-wave velocity described by Shuey (1985) and Verm and Hilterman (1995). A second method for defining the unknown terms of their equations from Aki and Richards continued to describe the unidentified terms of their equations in terms of V_P, V_S and ρ . Smith and Gidlow (1987), Fatti et al. (1994), Larsen (1999), Gidlow et al. (1993) and Downton (2005), among others, have suggested many calculations to the Aki and Richards or Knott-Zoeppritz equations. PS-data (Converted seismic wave data) is already used in a number of ways in pre-stack inversion. Tatham (1982), Castagna et al. (1985), Freund et al. (1992) and Mavko and Mukerji (1998), among many others, have illustrated the use of V_P/V_S like a measure of the fluid and rock properties.

The developement of Buland et al. (1996) and Simmons and Backus (2003) analysis to establish a technique which directly invert for P-impedance ($Z_P = \rho V_P$), density and S-impedance ($Z_S = \rho V_S$) through an approximation identical to Buland and Omre (2003) and performing limitations similar to Simmons Jr and Backus (1996). This is also our objective to expand the previous methods of post-stack impedance inversion (Russell and Hampson, 1991), so that this method can be used as a simplification to pre-stack inversion methods. First of all, examine the concepts of post-stack inversion method. Then, by combining Eqs. (2.18) and (2.23) , one can demonstrate that the small approximation of the reflectivity of the P-wave is provided by the

$$R_{pi} \approx \frac{1}{2} \triangle lnZ_{pi} = \frac{1}{2} \left[lnZ_{pi+1} - lnZ_{pi} \right] \tag{2.24}$$

Where i denotes i^{th} interface. Now we assume reflectivity of an N sample, the above equation could be presented in the matrix format as

$$\begin{bmatrix} R_{p1} \\ R_{p2} \\ \vdots \\ R_{pN} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 0 & \cdots \\ 0 & -1 & 1 & \cdots \\ 0 & 0 & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} L_{p1} \\ L_{p2} \\ \vdots \\ L_{pN} \end{bmatrix} \tag{2.25}$$

Where $L_{pi} = ln(Z_{pi})$. Next, seismic trace is produced by the convolution of the seismic wavelet with the reflectivity of the earth (Ma, 2002), the result could be presented in matrix format as

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} w_1 & 0 & 0 & \cdots \\ w_2 & w_1 & 0 & \cdots \\ w_3 & w_2 & w_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} R_{p1} \\ R_{p2} \\ \vdots \\ R_{pN} \end{bmatrix} \tag{2.26}$$

Where T_i denotes the i^{th} seismic trace sample and w_j denotes an extracted seismic wavelet j^{th} term. Combining eqs. (2.25) and (2.26) provide us another forward model that links the seismic trace to the P-impedance logarithm:

$$T = \frac{1}{2} W D L_p \tag{2.27}$$

Where W is the matrix of wavelet provided by equation (2.26) and D denotes derivative matrix provided by equation (2.25). There are two problems, when equation (2.27) is inverted utilizing a typical matrix inversion procedure to provide an estimation of L_P from an information of seismic trace (T) and wavelet (W). First problem is that the inversion of matrix is expensive and possibly unstable too (Hampson et al., 2005). More significantly, the low frequency element of the impedance will not be recovered via a matrix inversion. An alternative approach, including the one accepted in our equation execution (2.27), is to construct a previous impedance guess model and afterwards iterate towards such a solution utilizing the method of conjugate gradient.

(a) *Extension of simultaneous pre-stack inversion*

The equation of Aki-Richards was redefining by (Fatti et al., 1994) for small-angle (Niu et al., 2018) as

$$R(\theta) = c_1 R_P + c_2 R_S + c_3 R_D \tag{2.28}$$

Where $c_1 = 1 + tan^2\theta$, $c_2 = -8\gamma sin^2\theta$, $c_3 = -0.5tan^2\theta + 2\gamma sin^2\theta$ and $\gamma = (V_S/V_P)^2$. The three expressions relating to reflectivity are as per Eqs. 2.18, 2.19 and 2.20 (Hampson et al., 2005).

For an angle trace we can write as follows

$$T(\theta) = \frac{1}{2}c_1 W(\theta) DL_P + \frac{1}{2}c_2 W(\theta) DL_S + \frac{1}{2}c_3 W(\theta) DL_D \tag{2.29}$$

Where $L_D = ln(\rho)$ and $L_S = ln(Z_S)$. Eq. (2.29) could be used for inversion, excluding that it disregards the relationship between L_S and L_D with L_P. Since in this approach, we deal with impedance and take logarithms, these relationships are dissimilar from those provided by Simmons Jr and Backus (1996) and are defined as:

$$ln(Z_S) = kln(Z_P) + k_c + \Delta L_S, \tag{2.30}$$

$$ln(Z_D) = mln(Z_P) + m_c + \Delta L_D, \tag{2.31}$$

Where parameters $(k, k_c$ and $m_c)$ are determined utilizing well log data in the region, $\triangle L_D$ and $\triangle L_S$ define the variation from background style due to the presence of hydrocarbons (Fig. 2.5).

Now, simultaneous inversion can be derived. Starting with the Aki-Richards equations developed by Fatti's, these calculations model the amplitude of reflection as an incident angle function. Utilizing these calculations and the earlier relationships between P-impedance (Z_P), S-impedance (Z_S) and density (ρ) differs, the modified Fatti's equation is written as:

$$T(\theta) = c_1' W(\theta) D L_P + c_2' W(\theta) D L_S + c_3' W(\theta) D L_D, \tag{2.32}$$

Where $c_1\prime = \frac{1}{2}c_1 + \frac{1}{2}kc_2 + mc_3$ and $c_2\prime = \frac{1}{2}c_2$ and $c_3\prime = c_3$. $W(\theta)$ is the wavelet (at θ angle), $L_P = ln(Z_P)$, and and D represents the operator of differentiation derivative.

Equation (2.32) can be depicted in matrix format as follows.

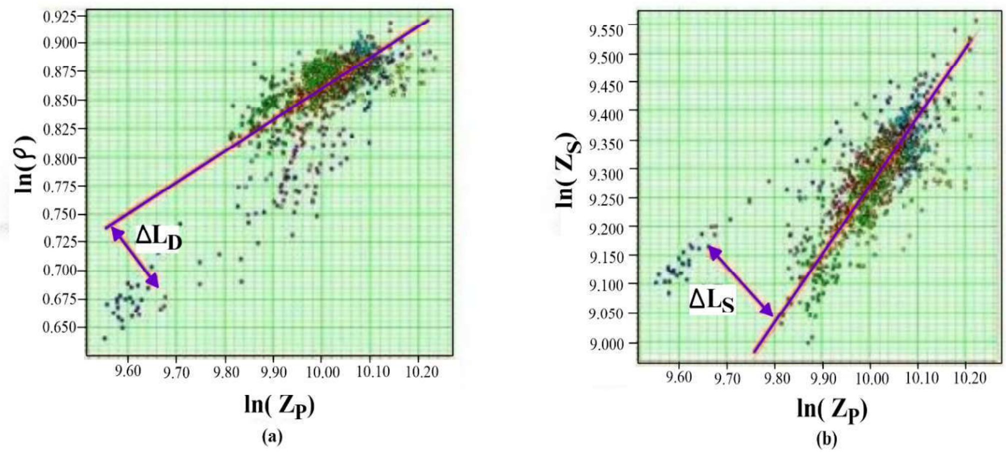

FIGURE 2.5: a) Crossplots of (a) $ln(\rho)$ vs $ln(Z_P)$ and (b) $ln(Z_S)$ vs $ln(Z_P)$

$$\begin{bmatrix} T(\theta_1) \\ T(\theta_2) \\ \vdots \\ T(\theta_N) \end{bmatrix} = \begin{bmatrix} c_1'(\theta_1)W(\theta_1)D & c_2'(\theta_1)W(\theta_1)D & c_3'(\theta_1)W(\theta_1)D \\ c_1'(\theta_2)W(\theta_2)D & c_2'(\theta_2)W(\theta_2)D & c_3'(\theta_2)W(\theta_2)D \\ \vdots & \vdots & \vdots \\ c_1'(\theta_N)W(\theta_N)D & c_2'(\theta_N)W(\theta_N)D & c_3'(\theta_N)W(\theta_N)D \end{bmatrix} \tag{2.33}$$

If Eq. (2.33) is explained by matrix inversion techniques, the difficulty of poor resolution of frequency component is encountered. So, a realistic methodology is to boost the result to $[L_P \triangle L_S \triangle L_D]^T T = [log(Z_P)00]^T$, where Z_P is the original impedance model (Larsen, 1999).

In practice, pre-stack inversion involves the following steps: Remember that while the angle (θ) is zero, thus this equation lowers to zero-offset inversion (model based inversion). We invert for L_P, L_S, and L_D in the above equation. In practice, simultaneous inversion includes the following stages by utilizing the Hampson Russell Software (HRS):

i. We have the following information from the seismic section:
   - N angle traces set;
   - N wavelets set for each angle;
   - Values of the initial model for Z_P .

ii. Using well log data, compute the coefficient values k and m.

iii. Proceed with the preliminary guess model.

$$[L_P \, \Delta L_S \, \Delta L_D]^T = [log(Z_P) \, 0 \, 0]^T \tag{2.34}$$

iv. Perform pre-stack inversion.

v. Calculate Z_P, Z_S and density as:

$$Z_P = exp(L_P) \tag{2.35}$$

$$Z_S = exp(kL_P + k_C + \Delta L_S) \tag{2.36}$$

$$\rho = exp(mL_P + m_C + \Delta L_D) \tag{2.37}$$

The preliminary model guess represents the preliminary P-impedance model, whereas $\triangle L_D$ and $\triangle L_S$ are started in this approximation with zero values (Maurya and Singh, 2015).

(ii) **Lambda-Mu-Rho (LMR) transform**

The LMR (Lambda-mu-rho) transform was firstly introduced by Goodway et al.

(1997). LMR utilizes the following connections between P-wave velocity V_P, V_S, ρ and the Lamé parameters - Lambda (λ) and Mu(μ):

$$V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{2.38}$$

and

$$V_S = \sqrt{\frac{\mu}{\rho}} \tag{2.39}$$

therefore:

$$Z_S^2 = (\rho V_S)^2 = \mu\rho \tag{2.40}$$

and

$$Z_P^2 = (\rho V_P)^2 = (\lambda + 2\mu)\rho \tag{2.41}$$

so,

$$\lambda\rho = Z_P^2 - 2Z_S^2 \tag{2.42}$$

Through seismic inversion, the above equations express P-impedance and S-impedance in rock properties. Many scientists claim that λ and μ help to discriminate fluid effects from lithology effects. Further, the Lamé parameters are obtained by producing reflectivity's of the P-wave and S-wave from pre-stack seismic data, and inverting it to P-impedance and S-impedance (supposing $V_P/V_S = 2$). So $\mu\rho$ is calculated by V_S^2 and $\lambda\rho$ is calculated by $(V_P^2 - 2\mu\rho)$.
A collection of logs that are prone to reservoir lithology and/or fluid, such as P-impedance and S- impedance, LMR attributes, Poisson's ratio, etc., which could be obtained from the seismic data by transformation. The LMR property transforms volumes of S- and P-impedance volumes precisely into volumes of Lambda-Rho and Mu-Rho. This simple but powerful transformation enables you to derive more physically significant details from the results of the inversion. The inverted raw and conditioned P-impedances and S-impedances are used to show the effect of data conditioning.
The process are as follows:

- First, calculate R_P and R_S reflectivity's from pre-stack data.

- Use inversion method to get Z_P and Z_S volumes from the inversion and AVO functions.
- Transform these volumes to calculate lambda-rho and mu-rho.
- A crossplot lambda-rho versus mu-rho is generated to minimize the effects of density. Hydrocarbon sands plot differently than wet sands.

(iii) **Elastic impedance inversion**

Elastic impedance inversion (EI) is a simplification of acoustic impedance inversion that utilizes pre-stack seismic reflection data as input (Connolly, 1999). The inverted elastic impedance is beneficial to detect gas-filled formations (Sakai, 1999; Mallick et al., 2000; Jin et al., 2003; Lu and McMechan, 2002; Lu and McMechan, 2004). There are several methods existing for EI inversion, i.e., full pre-stack inversion, post-stack inversion utilizing amplitude variation with offset (AVO, known as 'AVO inversion'), and elastic impedance post-stack inversion (known as 'EI inversion').

Elastic Impedance is actually a simplification of the acoustic impedance of the offset oriented angle gathers from seismic data. Since P/S mode transformations are noteworthy at oblique incidence angle, EI inversion provides a much more detailed image of the subsurface (Connolly, 1999; Whitcombe, 2002). EI inversion can be achieved from the small-contrast Aki and Richards equation (Eq. (2.43)). In this study, post-stack inversion utilizing elastic impedance is selected for assessment because the method is fast and simple for real-data applications. Though computationally costly, the full pre-stack inversion method is the optimal technique to estimate elastic parameters from seismic reflection data.

Connolly (1999) was the first to invert elastic impedance using range-limited stack sections. The reflection coefficient $R(\theta)$ can be determined performing Zoeppritz equation for P-wave reflectivity for an angle θ as

$$R(\theta) = A + Bsin^2\theta + Csin^2\theta tan^2\theta \tag{2.43}$$

Where

$$A = \frac{1}{2}\left(\frac{\triangle V_P}{\bar{V_P}} + \frac{\triangle\rho}{\bar{\rho}}\right)$$

$$B = \left(\frac{\triangle V_P}{2\bar{V_P}}\right) - 4\frac{V_S^2}{V_P^2}\left(\frac{\triangle V_S}{\bar{V_S}}\right) - 2\frac{V_S^2}{V_P^2}\frac{\triangle\rho}{\bar{\rho}}$$

$$C = \frac{1}{2}\left(\frac{\triangle V_P}{\bar{V_P}}\right)$$

and where

$$\triangle V_P = V_P t_i + V_P t_{i-1}/2$$

$$\triangle V_S = V_S t_i + V_S t_{i-1}/2$$

$$\frac{V_S^2}{V_P^2} = \frac{V_S^2 t_i}{V_P^2 t_i} + \frac{V_S^2 t_{i-1}}{V_P^2 t_{i-1}}/2$$

and likewise for the other variables where ρ the density, V_P and V_S represent the P-wave velocity and S-wave velocity, respectively and t_i is the time at sample i. We should have a function $f(t)$ that has acoustic impedance equivalent properties, so that reflectivity could be calculated from Eq. (2.44) for any incidence as follows

$$R(\theta) = \frac{f(t_i) - f(t_{i-1})}{f(t_i) + f(t_{i-1})} \tag{2.44}$$

Acoustic impedance is estimated using the above equation. This equation may also be developed for small to moderate incidence angles in the following manner

$$R(\theta) \approx \frac{1}{2}\frac{\triangle EI}{EI} \approx \frac{1}{2}\triangle ln(EI) \tag{2.45}$$

This Eq. (2.43) can be modified as

$$\begin{aligned}\frac{1}{2}\triangle ln(EI) = &\frac{1}{2}\left(\frac{\triangle V_P}{\bar{V_P}} + \frac{\triangle\rho}{\bar{\rho}}\right) + \left(\left(\frac{\triangle V_P}{2V_P}\right) - 4\frac{V_S^2}{V_P^2}\left(\frac{\triangle V_S}{\bar{V_S}} - 2\frac{V_S^2}{V_P^2}\frac{\triangle\rho}{\bar{\rho}}\right)sin^2\theta\right)\\ &+ \frac{1}{2}\left(\frac{\triangle V_P}{\bar{V_P}}\right)sin^2\theta tan^2\theta\end{aligned} \tag{2.46}$$

if $K = (\frac{V_S^2}{V_P^2})$, then Eq. (2.46) can be described as

$$\begin{aligned}\frac{1}{2}\triangle ln(EI) = &\frac{\triangle V_P}{\bar{V_P}} + \frac{\triangle\rho}{2\bar{\rho}} + \frac{\triangle V_P}{2\bar{V_P}}sin^2\theta - 4K\frac{\triangle V_S}{\bar{V_S}}sin^2\theta\\ &- 2K\frac{\triangle\rho}{\bar{\rho}}sin^2\theta + \frac{1}{2}\frac{\triangle V_P}{\bar{V_P}}sin^2\theta tan^2\theta\end{aligned} \tag{2.47}$$

After reordering the terms, we get

$$\frac{1}{2}\triangle ln(EI) = \frac{1}{2}\left(\frac{\triangle V_P}{\bar{V_P}}(1+sin^2\theta) + \frac{\triangle\rho}{\bar{\rho}}(1-4Ksin^2\theta) - \frac{\triangle V_S}{\bar{V_S}}8Ksin^2\theta + \frac{\triangle V_P}{\bar{V_P}}sin^2\theta tan^2\theta\right) \tag{2.48}$$

Using $sin^2\theta tan^2\theta = tan^2\theta - sin^2\theta$, Eq. (2.48) becomes

$$\frac{1}{2}\triangle ln(EI) = \frac{1}{2}\left(\frac{\triangle V_P}{\bar{V_P}}(1+tan^2\theta) - \frac{\triangle V_S}{\bar{V_S}}8Ksin^2\theta + \frac{\triangle\rho}{\bar{\rho}}(1-4Ksin^2\theta\right) \tag{2.49}$$

We have utilized only the first two terms of Eq. (2.43). Then the above Eqs. (2.48), (2.49) and following expressions vary only by switching $tan^2\theta$ to $sin^2\theta$. Substitute again $\triangle lnx$ for $\triangle x/x$.

$$\triangle ln(EI) = (1+tan^2\theta)\triangle ln(V_P) - 8Ksin^2\theta\triangle ln(V_S) + (1-4Ksin^2\theta)\triangle ln(\rho) \tag{2.50}$$

Therefore, assume K as a constant and take total expressions within $\triangle s$ as follows

$$\triangle ln(EI) = \triangle ln\left(V_P^{(1+tan^2\theta)}\right) - \triangle ln\left(V_S^{8Ksin^\theta}\right) + \triangle ln\left(\rho^{(1-4Ksin^2\theta)}\right) \tag{2.51}$$

$$\triangle ln(EI) = \triangle ln\left(V_P^{(1+tan^2\theta)}V_S^{-8Ksin^\theta}\rho^{(1-4Ksin^2\theta)}\right) \tag{2.52}$$

And finally, incorporate and exponentiate (i.e. eliminate both the logarithmic and differential factors) by putting the constant of integration to zero:

$$EI = V_P^{(1+tan^2\theta)}V_S^{-8Ksin^\theta}\rho^{1-4Ksin^2\theta} \tag{2.53}$$

Eq. (2.53) is the final outcome which is utilized to measure elastic impedance provided P-wave velocity (V_P), S-wave velocity (V_S), density (ρ) and angle θ are known (Whitcombe, 2002; Connolly, 1999).

2.4 Data conditioning

Presently with the advent of new technologies, the prime aim is to increase the resolution of seismic inversion by enhancing the S/N (signal to noise) ratio of seismic gathers. Therefore, some researchers tried to use pre-conditioning to gather for enhancing the signal to noise (S/N) ratio. However, some other researchers (Chopra and Sharma, 2016; Singleton, 2009) have shown that the pre-conditioning technique degrades inversion results significantly. Data conditioning is generally used before simultaneous inversion to improve the quality of inverted results. The inverted results are also influenced by the random noises present in the pre-stack gathers. The results inverted from the conditioned pre-stack gathers have higher resolution and better correlation coefficients with well logs compared to inverted results from raw gathers. Mainly, data conditioning has five major steps, namely: Band pass filtering, Muting, Super gather, parabolic radon transforms, and Trim statics. Several authors (Alkhalifah, 1997; Fomel and Stovas, 2010; Taner and Koehler, 1969; de Bazelaire, 1988) have already show effect of data conditioning in their work. Zhang et al. (2015) discusses a three-step workflow to execute pre-stack seismic data conditioning prior to pre-stack inversion. First, mitigate the hockey sticks by appling an automatic non-hyperbolic algorithm. Then reduce the stretch at large offset applying an anti-stretch procedure. Finally, enhance the signal to noise ratio (S/N) by using pre-stack-oriented filtering. The workflow is validated over the Fort Worth Basin (FWB), Texas, USA seismic data.

2.5 Additive white Gaussian noise

Gaussian noise is used to evaluate the effect of noise in seismic inversion methods. Gaussian noise is added to the pre-stack seismic data before the inversion to show the influence of noise in the inversion findings. Seismic signal always contains unwanted signals, which mislead seismic data interpretation and hence their effects need to be understood. Seismic noise plays a very important role in the interpretation of seismic inversion results. Sometimes, the existence of noise in the data misleads seismic data interpretation which costs very high in terms of money and time. Hence, it is recommended that before moving towards the interpretation of seismic data, it is compulsary to understand the effect of Gaussian noise on seismic inversion methods. Seismic noises are classified into two broad categories, random noise and coherent noise (Russell, 1988). The random noise is uncorrelated with the seismic signal and generated due to environmental factors such as electric wire, sound nearby, air blow, etc. The random noises can be removed from the seismic data by the stacking process. On the other hand, the coherent noises are predictable on the seismic traces and are generated by multiples which are relatively difficult to remove from the seismic data. There are two popular methods available to remove these coherent noises; the FK filtering and inverse velocity stacking. Maurya and Singh (2020) shows the effect of Gaussian noise on seismic inversion results.

2.6 Scope of geostatistical techniques for reservoir characterization

Geo-statistical methods use sampling points taken at various places and interpolate in the seismic section where no well log data existing. Such sampling points are measurements of petrophysical parameter in the boreholes (Haas and Dubrule, 1994). Geostatistics derive a surface using the values from the estimated locations to measure data points in between the data points for each location. Geostatistics offers two categories of interpolation methods: deterministic and geostatistics (Russell et al., 1997). Mathematical functions are utilizing by the Deterministic methods for interpolation while geostatistics uses both statistical and mathematical methods (Hampson et al., 2001).

Geo-statistical methodology, with its origins in hard-rock mining industry is very valuable for the characterization of reservoirs in exploration geophysics (Bosch et al., 2010). The early use of Geo-statistical methods was reduced due to the dependence on need for well log density. However, modern techniques implement an effective integration

of well logs (high frequency) and seismic information (low frequency).
Geo-statistical techniques play very important role to estimate numerous petrophysical parameters from seismic and well log data. The steps implemented for the Geo-statistical investigation of data are:

1. The structural consistency of well log data is computed via variograms.
2. Using cross-validation plots, a mathematical relationship is established between the seismic and data well log at every well sites. The Multivariate Regression method is linear, while the Probabilistic Neural Network (MLFN) and Multilayer feed forward neural network (MLFN) methods are nonlinear.
3. Then, these linear and non-linear relationships are utilized to measure the volume of petrophysical parameters at all seismic volume locations.
4. The estimated petrophysical parameters are evaluated for its reliability.

The types of geo-statistical methods are: Single attribute, Multi attribute Analysis, Probabilistic Neural Network (PNN), Multilayer Feed Forward Neural Network (MLFN).
There are four geostatistical methods, namely (i) Single attribute analysis, (ii) Multi attribute analysis, (iii) probabilistic neural network (PNN), and (iv) Multilayer feed forward neural network (MLFN).

2.6.1 Single attribute analysis

In the first procedure towards estimation of petrophysical parameters, seismic attributes were extracted for each seismic trace close to well locations. This is really beneficial to examine which attribute will better predict required petrophysical properties. The Single Attribute List examines internal and external attributes to determine the most important attributes for estimating the petrophysical properties (Russell et al., 1997).

2.6.2 Multi attribute analysis

Multi attributes analysis requires a linear regression technique where the matrix of co-variance is utilized to estimate petrophysical parameters performing a linearly weighted sum of the input seismic attributes. The Least Square method calculates weights, which is used to compute the operator of filter for the convolution. The operator is utilized to convert the seismic data to generate the volume of the reservoir characteristic (Hampson et al., 2001). The predicted volume of impedance is then used for study of multi attribute analysis. In porosity, density, P-wave and gamma ray estimation from 3D seismic data, the generated effective porosity, density, P-wave and gamma ray logs are utilized as the target log (Leiphart and Hart, 2001). Some research suggests that the these petrophysical parameters estimated through the technique of linear regression by the multi attribute analysis can be enhanced by employing nonlinear regression methods such as PNN and MLFN (Russell et al., 1997), which is discussed briefly in the fore coming sections.

Mathematical formulation of multi attribute analysis

The easiest method for predicting relationship between seismic attribute and target data is to crossplot the two (Russell et al., 1997). Fig. 2.6 shows crossplot for the property of target log, in this crossplot between density-porosity and seismic attribute is plotted. There is an assumption; the target log has been incorporated to travel time at the matching sample rate as that of the seismic attribute (Hampson et al., 2001).

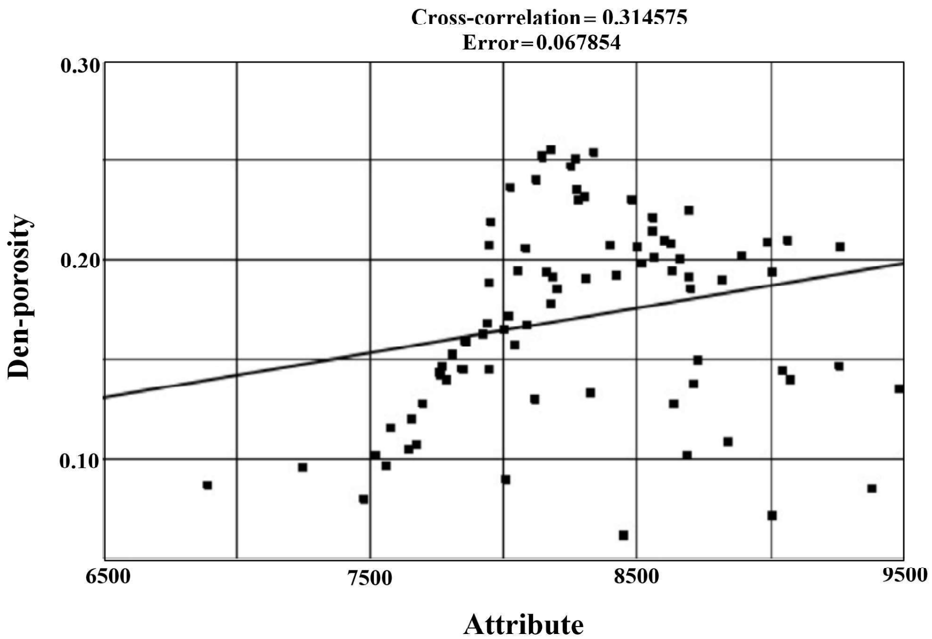

FIGURE 2.6: Traditional crossplot between the target log (density-porosity) and the seismic attribute (After Hampson et al., 2001).

The multiattribute linear regression is used to estimate the well log property in the inter-well area, using seismic and well log information. The process measures and uses a variety of seismic attributes from the seismic information to determine the relation between the attributes and well log characteristics. Then, in the inter-well region, this connection is used to assess petrophysical parameters. It is well known that the cross plot between the two is the easiest way to derive the required connection between target information and seismic attribute for a particular attribute of the seismic data. Assuming that the attribute has a linear relationship between the target log and the attribute, a straight line may be fit by regression which can be defined as:

$$y = a + bx \tag{2.54}$$

In this equation, the coefficients a and b are obtained by reducing the error of mean square prediction as follows.

$$E^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i - a - bx_i)^2 \tag{2.55}$$

The calculated prediction error (E) is a metric of fitness for the regression line defined by Eq. (2.54). Further, the normalized correlation coefficient can be defined as follows.

$$\kappa = \frac{\rho_{xy}}{\rho_x \rho_y} \tag{2.56}$$

Where, ρ_{xy} is correlation coefficient (κ) between x and y, where as ρ_x and ρ_y are variance of x and y respectively. These parameters further can be defind as:

$$\sigma_{xy} = \frac{1}{N}\sum_{i=1}^{N}(x_i - m_x)(y_i - m_y) \tag{2.57}$$

$$\sigma_x = \frac{1}{N}\sum_{i=1}^{N}(x_i - m_x)^2 \tag{2.58}$$

$$\sigma_y = \frac{1}{N}\sum_{i=1}^{N}(y_i - m_y)^2 \tag{2.59}$$

$$m_x = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{2.60}$$

$$m_y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{2.61}$$

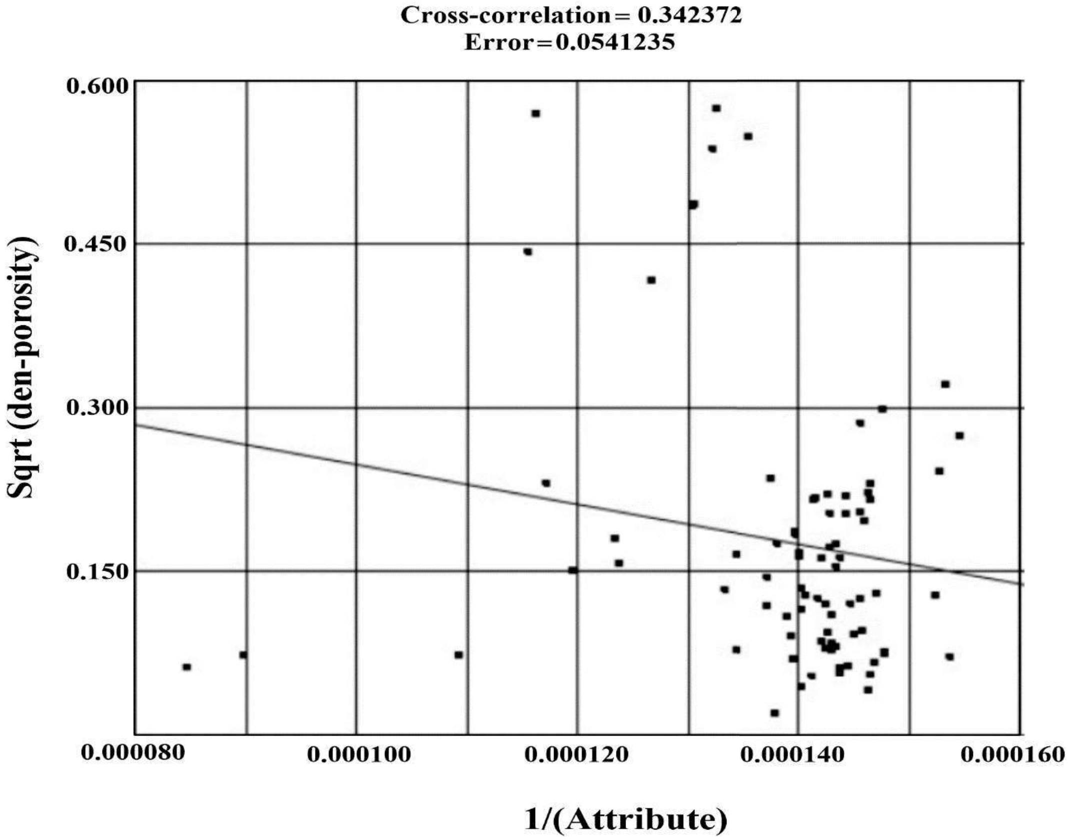

FIGURE 2.7: Performing nonlinear transformation to both attribute and target data enhances the relationship between the tw (After Taner et al., 1994).

It should also be observed that the linear requirement can be simplified to some extent by applying a nonlinear transformation into either target data or attribute data or both (Hampson et al., 2001).

2.6.3 Probabilistic neural network (PNN)

The probabilistic neural network (PNN) is a feed forward neural network that was initially obtained from the statistical algorithm known as Fisher discriminant evaluation and Bayesian network. Neural Networks is being employed for several decades in geophysics (McCormack, 1991; Schultz, et al., 1994). It is a possible benefit, as we can generally better understand its behavior by researching the mathematical formulation (Pramanik et al., 2004).

It was implemented by the Specht in 1990. PNN is a technique of numerical interpolation that uses architecture of the neural network for its execution (Tonn, 2002; Masters, 1995; Specht, 1990; Specht et al., 1991). The weights are determined in the PNN method utilizing the attribute space principle of "distance" from an identified point to an unidentified point. The fundamental concept that powers PNNs is to utilize collection of one or several estimated values (unrelated variables) to predict the single related variable value (Singh et al., 2007).
A sample set extracted using the well logs is divided into training, validation and test subsets like an input to design a PNN model. The discontinuing criterion of training procedure is:(i) a lowest values of mean square error is attained; (ii) non-improvements of mean square error for successive three iterations (iii) to interrupt training procedure, set a maximum number of iterations (Leiphart and Hart, 2001).

Formulation of PNN

Since, PNN is a feed forward neural network with a complex structure. It consists of an input layer, a pattern layer, a layer of summation, and an output layer (Specht, 1990, 1991). The architecture of the PNN method is shown in Fig. 2.8.

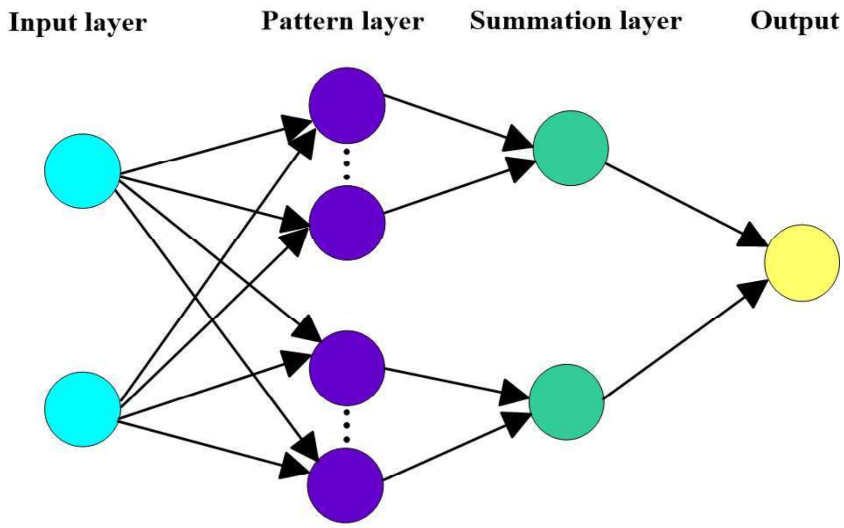

FIGURE 2.8: The architecture of probabilistic neural network.

The probabilistic neural network contains a set of training samples, one for every seismic sample in the examination windows from all of the boreholes:

$$\{A_{11}, A_{21}, A_{31}, L_1\}$$
$$\{A_{12}, A_{22}, A_{32}, L_2\}$$
$$\{A_{13}, A_{23}, A_{33}, L_3\}$$
$$\vdots$$
$$\{A_{1n}, A_{2n}, A_{3n}, L_n\},$$

There are three attributes and n training examples and three attributes. For each example, the L_i values are the calculated target log values. The PNN technique assumes that each new output log value can be written as a linear combination of the log values in the training data. For a new data sample with attribute values (Hampson et al., 2001).

$$x = \{A_{1j}, A_{2j}, A_{3j}\},$$

The innovative measured log value is estimated by

$$L'(x) = \frac{\sum_{i=1}^{n} L_i \, exp(-D(x, x_i))}{\sum_{i=1}^{n} exp(-D(x, x_i))} \tag{2.62}$$

Where,

$$D(x, x_i) = \sum_{j=1}^{3} (\frac{x_j - x_{ij}}{\sigma_j})^2 \tag{2.63}$$

$D(x, x_i)$ reflects the distance between the point of entry and each point of training, x_i in multi-dimensional space covered by the attributes. The value $D(x, x_i)$ is differentiated and scaled by an amount that can vary for each attribute.

Eqs. (2.62) and (2.63) define the use of the PNN network. The network training consists of the ideal set of parameters for smoothing σ_j. The standard for these parameters is that the calculated network should have the lowest validation error.

The authentication outcome for the m^{th} target example is given by

$$L'_m(x_m) = \frac{\sum_{i \neq m}^{n} L_i \, exp(-D(x_m, x_i))}{\sum_{i \neq m}^{n} exp(-D(x_m, x_i))} \tag{2.64}$$

This is the m^{th} target example estimated value when the example is excluded from the training data. As this example's value is known, the prediction error can be calculated for that example. By repeating this process for each of the training examples, you can calculate the complete prediction error for training data.

$$E_v(\sigma_1,\, \sigma_2\, \sigma_3) = \sum_{i=1}^{N}(L_i - L_i')^2 \tag{2.65}$$

The error of prediction is based on selecting the parameters, σ_j. This value is minimized using a Masters, (1994 and 1995) described nonlinear conjugate gradient algorithm. The network calculated is the minimum validation error. The greatest issue with PNN is that it carries all its training information and compares each output sample with each training sample (Specht, 1990 and 1991). The concluding network does have the ability to minimizing the validation error (Quirein et al., 2000; Hampson et al., 2001). The PNN performance is presented in Fig. 2.9 on simple crossplot data. We can see from this figure that the probabilistic neural network does have the desirable property of pursuing the data. The main issue with the PNN is that the application time of geo-statistical methods can be slow as it holds all of its training data and compares every sample output point to every sample training point.

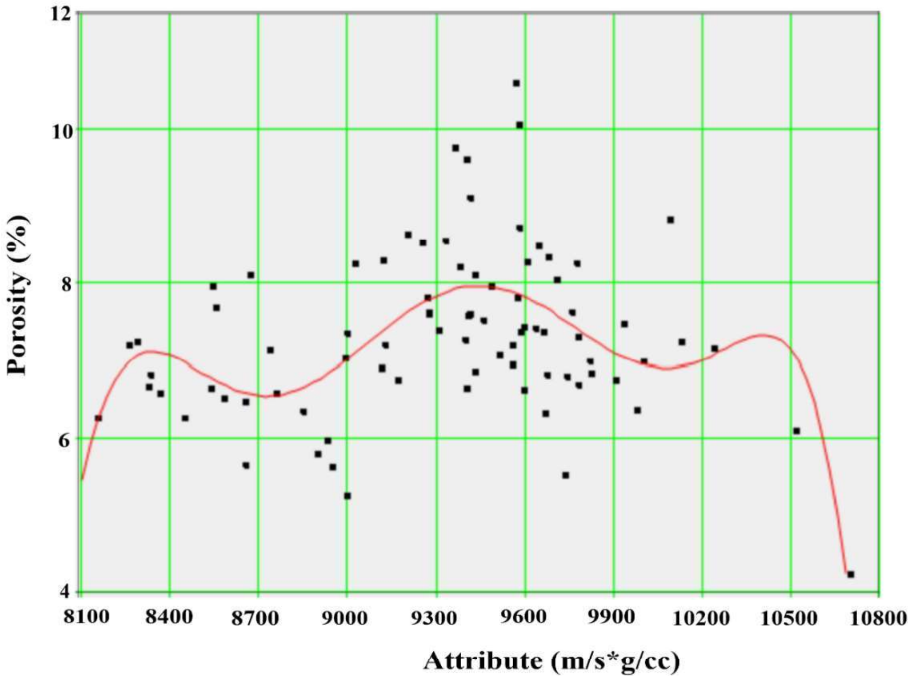

Figure 2.9: Predicted porosity curve derived by the PNN (After Hampson et al., 2001).

2.6.4 Multilayer feed forward neural network

Multilayer feed forward neural network (MLFN) is another non-linear neural network method like probabilistic neural network method. Liu and Liu (1998) explained the benefit of a MLFN technique over the PNN technique to predict the properties of rock straightly from seismic data. The MLFN has been therefore, in this analysis, utilized to estimate petrophysical properties in the inter-well region utilizing well log data and seismic data as input. The MLFN is the conventional network which can be seen in Fig. 2.10. Numerous textbooks have described the characteristics of the multilayer feed forward network (eg, Masters, 1994).

MLFN network architecture comprises of one input layer, one output layer as well as one or more concealed layers. Each network layer comprises nodes that are connected. Their connections are called weights. These weights are the final component of the output decision as they play a key job in the implementation of MLFN. The number of input nodes is determined by the desired amount of attributes analyzed during the

MLFN process. The number of attributes is determined by the convolution in which a specific mixture of attributes has been tested and the mixture with maximum likeness is chosen against the required log property. Mathematically, the MLFN can be expressed as follows.

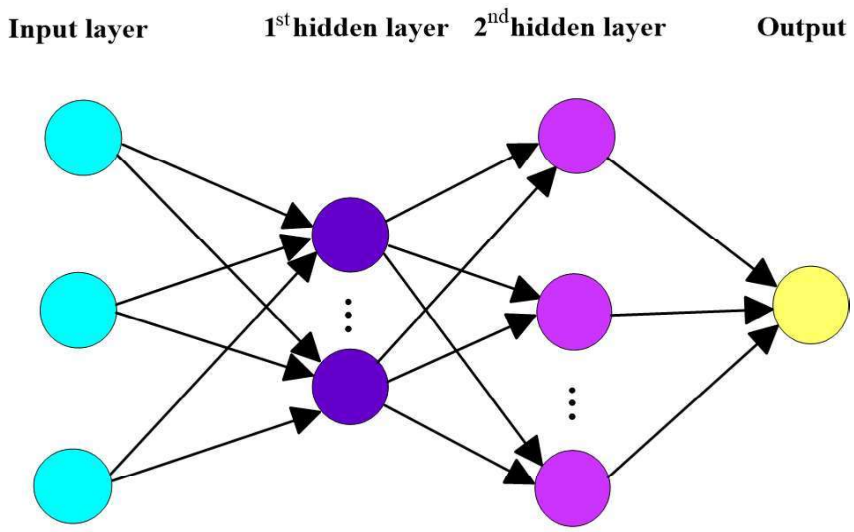

FIGURE 2.10: The architecture of multilayer feed forward neural network (MLFN)

The mapping function Γ could be used for the formal description of the neurons, which assigns every neuron i a subset $\Gamma(i) \subseteq V$, that consists of all the given neuron's ancestors. A subset $\Gamma^{-1} \subseteq V$ which consists of all neuron predecessors. In the next layer, every neuron in a specific layer is linked to all other neurons. The association between the i^{th} and j^{th} neurons are defined through the weight coefficient w_{ij} and the ν_i threshold coefficient of the i^{th} neuron. The weight coefficient represents the degree of significance in the neural network of the specified relationship. Equations 2.66 and 2.67 determine the neuron output value (activity) of x_i.

$$x_i = f(\psi_i) \tag{2.66}$$

$$\psi_i = \nu_i + \sum_{j \epsilon \Gamma_i^{-1}} w_{ij} x_j \tag{2.67}$$

Where ψ_i represents the potential of the i^{th} neuron and function $f(\psi_i)$ is the transfer function (the summation in the Eq. (2.69) is performed over all neurons j carrying the signal to the i^{th} neuron). Transfer function comes from the name transformation and is used for transformation purposes. In machine learning, the sums of each node are weighted as well as the sum is passed through a non-linear function defined as a transfer function to obtain an output. The threshold coefficient of the connection to formally added neuron j can be assumed as a weight coefficient, where $x_j = 1$. For the transfer function, it holds that

$$f(\psi_i) = \frac{1}{1 + exp(-\psi)} \tag{2.68}$$

The monitored adaption method varies the threshold coefficients v_i and weight coefficients w_{ij} to reduce sum of the square alterations between the calculated and observed output values (Pramanik et al., 2004). This is achieved by reducing the objective function e which can also be described as follows.

$$e = \sum_p \frac{1}{2}(x_p - \hat{x_p})^2 \tag{2.69}$$

Where x_p and $\hat{x_p}$ are vectors containing the calculated and required activity of the output neurons, and the summation runs across all output neurons p.

Many researchers (Bhatt and Helle, 2008; Bosch et al., 2010; Chambers and Yarus, 2002; Chen and Sydney, 1997; Doyen, 1988; Eskandari et al., 2004; Hampson et al., 2001; Leiphart and Hart, 2001; Leite and Vidal, 2011; Pramanik et al., 2004; Naeem et al., 2015; Russell et al., 1997; Singh et al., 2007; Maurya et al., 2018) used geostatistical techniques to predict porosity, density, P-wave, etc. for reservoir characterization.