

A Study on Interior-point Methods to Solve Multiobjective Optimization Problems



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by

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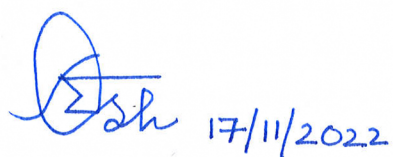
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Dedicated to my parents,

Mrs. Sona Devi

and

Mr. Satya Beer

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List of Symbols

Symbol	Description
\mathbb{R}^n	Euclidean space of dimension n
\mathcal{X}	Feasible decision region
\mathcal{Y}	Objective feasible region
\mathcal{P}	Number of objective functions in an MOP
\mathcal{J}	Number of inequality constraints in an MOP
$\bar{\mathcal{J}}$	Number of equality constraints in an MOP
e	Vector of ones of compatible dimension
I	Identity matrix of compatible dimension

Abbreviations

Abbreviation	Description
AWS	Adaptive Weighted Sum
NBI	Normal Boundary Intersection
NC	Normal Constraint
LP	Linear Programming
SOP	Single-objective Optimization Problem
MOP	Multiobjective Optimization Problem
IPM	Interior Point Method
TR	Trust Region
CM	Cone Method
KKT	Karush-Kuhn-Tucker
IIPM	Infeasible Interior Point Method
HV	Hyper Volume
PD-IPM	Primal Dual Interior Point Method

PREFACE

Optimization is not only important in its own right but also integral to many applied sciences such as operations research, management sciences, economics and finance, and all branches of math-oriented engineering. It provides a unique insight into any situation. Optimization introduces a computational situation where the goal is to obtain the best of all possible circumstances. Also, optimization techniques assist us in finding the best under prespecified circumstances. Essentially, optimization is used to detect, characterize and compute the maxima or minima of a function for a set of acceptable points and certain predefined conditions.

The mode of optimization is not just confined to the mathematical arena. Optimization methods can be applied in many spheres of study to find solutions that maximize or minimize some study parameters, such as in producing a good or service, minimizing the cost of production, and maximizing profits. Such instances often have special structures: convex, nonconvex, linear, nonlinear, quadratic, semidefinite, dynamic, integer, stochastic programming, etc. Optimization is the source of vast theoretical foundations and advanced algorithms. Mathematically, identification of the solution is the essence of optimization, i.e., The minimization or maximization of a function or a set of functions in conjunction with a set of constraints, regardless of the number of decision variables. Accordingly, two kinds of optimization problems are distinguished in the literature: single and multiobjective optimization problems.

Interior point methods are developed for solving single objective optimization prob-

lems by following a smooth path (central path) inside the feasible region. These are the most efficient methods for searching for optimal solutions for large problems with sparse structures. The variants of interior-point methods are being extended to solve the various problems such as linear, nonlinear, convex and nonconvex. Also, they are being applied to solve many real-world problems such as engineering optimization problems.

This thesis describes a study of various interior-point methods to solve multiobjective optimization problems. This thesis is organized as follows.

Chapter 1 begins with the introduction of a multiobjective optimization problem. Then, a discussion will be provided on some popular existing algorithms which are preferred to solve multiobjective optimization problems. After that, the motivation of “why are interior-point methods preferred to solve multiobjective optimization problems?” will be explained. Thereafter, some historical background and the current status of interior-point methods will be discussed.

In Chapter 2, a study on an infeasible interior-point technique to solve MOPs (convex and nonconvex) is discussed. The algorithmic implementation and convergence analysis of the method with an estimate of the number of iterations to reach an ϵ -precise solution is provided. A performance comparison between the proposed method and popular existing solvers is provided concerning two performance measures (IGD and HV) and the corresponding relative efficiency measures.

Chapter 3 analyzes a Newton-type globally convergent interior-point method for solving multiobjective optimization problems. The global convergence of the proposed method is also shown under some mild conditions. Numerical results of ZDT and DTLZ test problems are presented. Also, three performance metrics (modified generational distance, HV, and IGD) are used on some test problems to investigate the efficiency of the proposed algorithm.

Chapter 4 presents a primal-dual interior-point algorithm is proposed to solve multiobjective optimization problems. To demonstrate the efficiency of the proposed method,

we applied it to some constrained test problems. The proposed algorithm has been applied to an optimal control problem of carbon dioxide emission from the energy sector.

In Chapter 5, a trust-region interior-point technique is derived to generate the Pareto optimal solution for multiobjective optimization problems. The efficiency of the proposed method is tested by applying it on some standard test problems. As an application, we apply the proposed algorithm to solve an optimal control problem for a tuberculosis model.

In Chapter 6, a new trust-region algorithm to obtain the Pareto critical points of unconstrained nonsmooth multiobjective optimization problems is developed.

Finally, in Chapter 7, we conclude the thesis with some suggestions for future work.