A Study on Interior-point Methods to Solve Multiobjective Optimization Problems

Thesis submitted in partial fulfillment

for the Award of Degree

Doctor of Philosophy

by

Jauny

DEPARTMENT OF MATHEMATICAL SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY

(BANARAS HINDU UNIVERSITY)

VARANASI - 221005

Roll No. 17121022

Year 2022

CERTIFICATE

It is certified that the work contained in the thesis titled ***"A Study on Interior-point Methods to Solve Multiobjective Optimization Problems"*** by ***Jauny*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all requirements of Comprehensive Examination, Candidacy, and SOTA for the award of Ph.D. Degree.

17/11/2022

Dr. Debdas Ghosh
(Supervisor)
Assistant Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

DECLARATION BY THE CANDIDATE

I, ***Jauny***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Dr. Debdas Ghosh*** from ***July, 2017*** to ***May, 2022*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: 17/11/2022

Place: Varanasi

Jauny

(Jauny)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the candidate is correct to the best of our knowledge.

17/11/2022

(Dr. Debdas Ghosh)
Assistant Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

17.11.2022

(Prof. Sanjay Kumar Pandey)
Professor and Head
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: A Study on Interior-point Methods to Solve Multiobjective Optimization Problems

Name of the Student: Jauny

Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology (Banaras Hindu University) Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the *Doctor of Philosophy*.

Date: 17/11/2022

Place: Varanasi

Jauny

(Jauny)

Note: However, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for author's personal use provided that the source and the Institute's copyright notice are indicated.

Dedicated to my parents,

Mrs. Sona Devi

and

Mr. Satya Beer

ACKNOWLEDGEMENT

Foremost, I would like to thank my supervisor *Dr. Debdas Ghosh*, for his constant guidance and support. It is my great honor to be his student and to work with him during my candidacy. I started my doctoral studies with a crude knowledge of optimization, particularly multiobjective optimization. I am eternally grateful to my supervisor for mining, polishing, and making me a confident researcher. He has been and will remain my role model as a successful researcher.

Next, I am indebted to my parents *Mr. Satya Beer* and *Mrs. Sona Devi* for their blessings, support and believing in my ability to pursue my career as a researcher. I feel extraordinary fortunate to have moral support from my elder brothers *Mr. Mukesh Prajapati* and *Mr. Ravindra Prajapati* and sister in laws *Ms. Sweta* and *Ms. Monika* during my Ph.D. and in particular while completing the thesis work. I would also like to say a heartfelt thanks to my cute nephews *Vihan, Kush* and *Dhruv.*

I deeply thank to my fiancee *Sakshi Daksh* for her love and constant support, for all the late nights and early mornings, and for keeping me sane over the past few months. Thank you for being my muse, editor, proofreader, and sounding board. But most of all, thank you for being my best friend. I owe you everything. Also, I would like to thank her family, specially my future mother-in-law *Mrs. Rachna Devi* to support me during thesis writing.

I would also like to thank to the Research Progress Evaluation Committee (RPEC) members *Dr. Anuradha Banerjee* & *Dr. Tanima Dutta*, for their insightful comments and encouragement, but also for the hard questions which incented me to widen my research from various perspectives.

I express my cordial thanks to our H.O.D, *Prof. S. K. Pandey.* I would sincerely like to acknowledge the facilities provided by Department of Mathematical Sciences, Indian Institute of Technology (BHU) Varanasi, India.

It is difficult to describe my gratitude and appreciation for the suggestions and support that I received from my co-author *Ashutosh Upadhayay.* My sincere thanks goes to *Prof. Qamrul Hasan Ansari* & *Prof. Matthias Ehrgott* who make me motivated during the Ph.D., and who gave me access to their immense knowledge.

I am thankful to my fellow lab mates *Mr. Abhishek Singh, Mr. Amit debnath, Dr. Ram Surat Chauhan, Mr. Gourav Kumar, Mr. Ashutosh Upadhayay, Mr. Krishan Kumar, Mrs. Anshika, Ms. Megha Pandey, Ms. Suprova Ghosh, Mr. Nand Kishor, Mr. Ravi Raushan, Ms. Jaya Joshi* for their stimulating discussions. I am also thankful to my friends *Mr. Gulab Patel, Mr. Abhay Rajput, Ms. Diksha Gupta, Ms. Pragya Shukla, Mr. Shobit Shrivastava, Mr. Jay Singh Maurya, Mr. Gagan, Mr. Ajay Patel, Mr. Robin Vikram Singh* in the institution for their immense kindness throughout the period and the friendly atmosphere that made me courageous to complete my work. It is my ought to remember my seniors *Dr. Om, Dr. Manushi, Dr. Anuwedita, Dr. Swati, Dr. Vinita, Dr. Sumit, Dr. Rakesh, Dr. Rahul, Dr. Anup and Dr. Anil* for their continuous inspiration and moral support.

My acknowledgment will be incomplete if I do not mention *Pt. Madan Mohan Malaviya* with whose blessings and support I could be able to achieve my goal successfully.

Last but not the least a very special thanks to *Baba Vishwnath* and *Shri Krishna* (whose blessings can be felt in every space of this place), I thank you for the hidden forces of nature brought within me during this scientific endeavor. I am much obliged to my patients who made me able to complete this thesis work effectively. I may be apologized, if I could not acknowledge somebody who supported directly or indirectly and contributed to my research and preparation of this thesis. Thus I recompense my thanks to all.

Date:

Place: Varanasi **(Jauny)**

List of Figures

List of Tables

List of Symbols

Symbol	Description
$\mathbb{R}^n$	Euclidean space of dimension n
$\mathcal{X}$	Feasible decision region
$\mathcal{Y}$	Objective feasible region
$\mathcal{P}$	Number of objective functions in an MOP
$\mathcal{J}$	Number of inequality constraints in an MOP
$\bar{\mathcal{J}}$	Number of equality constraints in an MOP
e	Vector of ones of compatible dimension
I	Identity matrix of compatible dimension

Abbreviations

Abbreviation	**Description**
AWS	Adaptive Weighted Sum
NBI	Normal Boundary Intersection
NC	Normal Constraint
LP	Linear Programming
SOP	Single-objective Optimization Problem
MOP	Multiobjective Optimization Problem
IPM	Interior Point Method
TR	Trust Region
CM	Cone Method
KKT	Karush-Kuhn-Tucker
IIPM	Infeasible Interior Point Method
HV	Hyper Volume
PD-IPM	Primal Dual Interior Point Method

PREFACE

Optimization is not only important in its own right but also integral to many applied sciences such as operations research, management sciences, economics and finance, and all branches of math-oriented engineering. It provides a unique insight into any situation. Optimization introduces a computational situation where the goal is to obtain the best of all possible circumstances. Also, optimization techniques assist us in finding the best under prespecified circumstances. Essentially, optimization is used to detect, characterize and compute the maxima or minima of a function for a set of acceptable points and certain predefined conditions.

The mode of optimization is not just confined to the mathematical arena. Optimization methods can be applied in many spheres of study to find solutions that maximize or minimize some study parameters, such as in producing a good or service, minimizing the cost of production, and maximizing profits. Such instances often have special structures: convex, nonconvex, linear, nonlinear, quadratic, semidefinite, dynamic, integer, stochastic programming, etc. Optimization is the source of vast theoretical foundations and advanced algorithms. Mathematically, identification of the solution is the essence of optimization, i.e., The minimization or maximization of a function or a set of functions in conjunction with a set of constraints, regardless of the number of decision variables. Accordingly, two kinds of optimization problems are distinguished in the literature: single and multiobjective optimization problems.

Interior point methods are developed for solving single objective optimization problems by following a smooth path (central path) inside the feasible region. These are the most efficient methods for searching for optimal solutions for large problems with sparse structures. The variants of interior-point methods are being extended to solve the various problems such as linear, nonlinear, convex and nonconvex. Also, they are being applied to solve many real-world problems such as engineering optimization problems.

This thesis describes a study of various interior-point methods to solve multiobjective optimization problems. This thesis is organized as follows.

Chapter 1 begins with the introduction of a multiobjective optimization problem. Then, a discussion will be provided on some popular existing algorithms which are preferred to solve multiobjective optimization problems. After that, the motivation of "why are interior-point methods preferred to solve multiobjective optimization problems?" will be explained. Thereafter, some historical background and the current status of interior-point methods will be discussed.

In Chapter 2, a study on an infeasible interior-point technique to solve MOPs (convex and nonconvex) is discussed. The algorithmic implementation and convergence analysis of the method with an estimate of the number of iterations to reach an ϵ-precise solution is provided. A performance comparison between the proposed method and popular existing solvers is provided concerning two performance measures (IGD and HV) and the corresponding relative efficiency measures.

Chapter 3 analyzes a Newton-type globally convergent interior-point method for solving multiobjective optimization problems. The global convergence of the proposed method is also shown under some mild conditions. Numerical results of ZDT and DTLZ test problems are presented. Also, three performance metrics (modified generational distance, HV, and IGD) are used on some test problems to investigate the efficiency of the proposed algorithm.

Chapter 4 presents a primal-dual interior-point algorithm is proposed to solve multiobjective optimization problems. To demonstrate the efficiency of the proposed method, we applied it to some constrained test problems. The proposed algorithm has been applied to an optimal control problem of carbon dioxide emission from the energy sector.

In Chapter 5, a trust-region interior-point technique is derived to generate the Pareto optimal solution for multiobjective optimization problems. The efficiency of the proposed method is tested by applying it on some standard test problems. As an application, we apply the proposed algorithm to solve an optimal control problem for a tuberculosis model.

In Chapter 6, a new trust-region algorithm to obtain the Pareto critical points of unconstrained nonsmooth multiobjective optimization problems is developed.

Finally, in Chapter 7, we conclude the thesis with some suggestions for future work.