Abstract

Rapidly revolving air masses which are observed during summer in open fields or maybe by the roadsides are called a dust devil, as they are identified by the dust they carry with them. They are born all of a sudden and generally die out soon. A cyclone is similar but rather huge in size, and is generated by prolonged, intense heating of the sea/ocean surface. Another similar phenomenon that is observed coming out of the clouds is a tornado. Despite their huge differences in size, dust devils, tornadoes, and tropical cyclones have the same fundamental vortex properties that can be examined using mathematical models. Several mathematical models have been developed to explore velocity, pressure field, and temporal evolution of atmospheric vortices across different physical scenarios. To investigate the impact of physical parameters affecting the flow, both inviscid and viscous formulations, as well as steady and unsteady states, are taken into consideration. The focus is on developing analytical models that can capture important elements of vortices in order to provide a better understanding of the mechanisms behind vortex formation, growth, and decay.

Chapter 1 is introductory. This gives an overview of atmospheric vortices, which are witnessed as revolving air masses of various forms. It further discusses their distinctive patterns that shape their size and structure. It follows a detailed description of nomenclature, anatomy, areas of occurrence, etc. Descriptive photographs are included to help with comprehension. In order to support the discussions in subsequent chapters, a concise summary of related background is also provided.

An critical part of the thesis, Chapter 2, establishes the framework for the current study. It highlights the scientific path that has resulted in the current understanding by charting the chronological evolution of research in this field. The literature is reviewed in detail, covering analytical, lab-based, and simulation studies. Key elements of vortex dynamics and a variety of modelling techniques are covered, providing insight into both traditional and modern viewpoints.

Chapter 3 constructs an analytical inviscid model for dust devils which obtains azimuthal velocity depending on radial and axial coordinates both. This generalised model ensures that all the velocity components are bounded. Axial velocity profiles display realistic central updrafts; radial velocity, directed toward the centre of the vortex, initially

increases in magnitude and then finally decreases to zero; and pressure exhibits a central low-pressure consistent with atmospheric vortex dynamics. This models dust devils' circulation which correlates with electrical charging mechanisms and particle lifting. Results indicate that regions of maximum tangential and vertical velocities align with zones of strongest particle acceleration and probable electrification.

Chapter 4 focuses on modelling the structure of tropical cyclones, specifically the eye, eyewall, and rain-band regions. It explores variations in eye size on the intensity of tropical cyclones by modelling the eye and eyewall regions while including viscous flow effects and adding axial and radial dependencies to the velocity components. The analysis reveals that larger radial velocity leads to a thicker eye-wall, and if the eye size is larger, the updraft has the tendency to rise faster. Azimuthal velocity increases slightly with higher Reynolds numbers in the inner eyewall surrounding the eye. This trend remains consistent within the inner eye-wall, but undergoes a sharp reversal in the outer eye-wall. It attains a pronounced maximum in the outer eye-wall, beyond which the variation begins to decline, at a gradual pace, and persists through the rain bands.

The scope includes the formulation and validation of analytical models describing steady, axisymmetric, viscous, and incompressible flows in cylindrical coordinates, applied to the study of whirlwinds. Chapter 5 presents a generalised whirlwind model, where the velocity components vary with both radial and axial positions. For the derivation of the velocities, Rankine and Burgers-type profiles are used as boundary conditions to impose cyclostrophic balance. All of the velocity components of this model are bounded. This model incorporates exponential localisation not only in the radius but also in height, yielding a novel vortex structure with distinct regions in the radial direction: an inner region facilitating upward flow and an outer region directing flow downward.

In order to improve the model's representation of vortex dynamics, Chapter 6 extends on the work given in Chapter 5 by adding a central low-pressure zone to the stream function. Pressure is derived from the momentum equations in this generalised analytical model, which incorporates all three velocity components. The no-slip condition at the ground is enforced by a stream-function that has a central low-pressure core and a viscous correction. The model can replicate a variety of vortex intensities due to parametric control via a shape factor and Reynolds number. At lower altitudes, the radial velocity is directed toward the centre, indicating an inflow from the outer regions. It increases in magnitude radially inward, reaching a maximum for all curves. Beyond this point, the velocity decreases in magnitude and approaches zero at the periphery of the low-pressure zone. For heights above $z = 1$, an outflow is observed. Vertical velocity attains its maximum near the periphery of a low-pressure zone. As radius increases, vertical velocity decreases and eventually becomes negative, indicating regions beyond the vortex influence. Rotational velocity rises from zero near the core, reaches a peak around the periphery of the low-pressure zone, and then decays rapidly to zero as radius increases. The magnitude of maximum azimuthal velocity is higher for the larger low-pressure zone.

The objective of Chapter 7 is to develop a time-dependent viscous model that captures the full life cycle of a dust devil—genesis, intensification, and decay—using a modified stream function. This aims to provide a unified framework for analysing the temporal and spatial evolution of velocity components and pressure, revealing key dynamic features such as intensification and weakening phases. The radial velocity is minimal at the vortex centre, peaks outward, and then declines. It increases during intensification ($t \in [0, 1]$), indicating outward propagation, and decreases during decay ($t > 1$), reflecting dissipation. The axial velocity peaks at the centre and decreases radially. It strengthens during intensification due to surface heating and weakens during decay, indicating reduced vertical motion. The azimuthal velocity is zero at the centre, peaks at an intermediate radius, and declines outward. It grows during intensification, showing stronger rotation, and diminishes during decay, indicating a loss of rotational energy.

The thesis concludes with overall conclusions and a further scope of investigation appended to it.