

Chapter 5

A Lyapunov Characterization of Difference Equation with Minima based Discrete Sliding Mode Control

5.1 Introduction

In this dedicated chapter, a comprehensive exploration of the Lyapunov characterization for the discrete-time sliding mode control is undertaken. The focus of this analysis is on the difference equation with minima (DEM) based law, aiming to establish the Lyapunov stability conditions and demonstrate the finite-time input-to-state stability for perturbed discrete-time systems. Through these derived conditions, we unveil the distinctive features of the proposed method, strategically positioned between Gao's reaching law and Utkin's reaching law, effectively mitigating the inherent limitations of both, namely chattering in the former and excessive control action in the latter. The applicability of the difference equation with minima-based discrete-time sliding mode control is extended to address first-order and second-order perturbed systems. To substantiate the effectiveness of this approach, a series of simulations are conducted, presenting a comparative study against existing DSMC design methods and showcasing the superior performance of the proposed methodology.

The structure of this chapter is outlined as follows. Section 5.2 presents the preliminaries that lay the foundation for obtaining the main results. In Section 5.3, the main results of the study are presented. Section 5.4 focuses on validating the effectiveness of

the proposed scheme through simulations and experiments. Finally, in Section 4.5, the chapter concludes the work.

5.2 Preliminaries

In this section, we revisit some the background results which will be helpful for comprehensive understanding of this chapter.

The following inequality will be useful while deriving the key results.

If $|a| \geq |b|$, with either $a, b \in \mathbb{R}_+$, or $a, b \in \mathbb{R}_-$, then

$$|a - b| \leq |a| - |b| \quad (5.1)$$

5.2.1 Discrete-Time First-Order Unperturbed System

Here, the finite-time stabilization of first-order unperturbed discrete-time systems [37] is revisited. Let us consider the following discrete-time system

$$z(k+1) = u(k), \quad z(0) = z_0 \quad (5.2)$$

where $z \in D \subset \mathbb{R}$ denotes the state vector of the system, $k \in \mathbb{Z}_{\geq 0}$ and $u \in \mathbb{R}$ denotes the control signal. Designing the control law based on difference equation with minima as

$$u(k) = z(k) - \text{sign}(z(k)) \min\{|z(k)|, l\} \quad (5.3)$$

where $\text{sign}(z) = \frac{z}{|z|}$, for $z \neq 0$ and $\text{sign}(0) = 0$ with $l > 0$. Substituting (5.3) in (5.2), we get closed-loop system as $z(k+1) = z(k) - \text{sign}(z(k)) \min\{|z(k)|, l\}$. This system exhibits finite-time stability as discussed in [37] and the settling time function is expressed as $\mathcal{K}(z_0) = \lceil \frac{|z_0|}{l} \rceil$, where $\mathcal{K}(z_0)$ exhibits lower-semicontinuity property at each point $z \in D$. The incorporation of $\min\{|z(k)|, l\}$ gives the system, a reaching phase for $|z(k)| > l$ and dead-beat response for $|z(k)| \leq l$.

In the next section, we discuss these results for the perturbed case in the framework of DSMC.

5.3 Lyapunov Characterization of DEM based DSMC

Here, a new reaching law, which is based on difference equation with minima is presented. The stability conditions for such a law are developed by using Lyapunov methods. Let

us go through the development process in the upcoming subsections.

5.3.1 Discrete-Time First-Order Perturbed System

Let us consider the following discrete-time system with perturbation as

$$z(k+1) = u(k) + d(k), \quad z(0) = z_0 \quad (5.4)$$

where $z \in D \subset \mathbb{R}$, $k \in \mathbb{Z}_{\geq 0}$, $u \in \mathbb{R}$ and d is the matched type of bounded perturbation satisfying $|d| \leq d_0$. We design the control law as in (5.3), which yields the closed-loop system as $z(k+1) = z(k) - \text{sign}(z(k)) \min\{|z(k)|, l\} + d(k)$, where $l > d_0$. The closed-loop dynamics are FTISS in presence of the disturbance $d(k)$ and the solution $z(k)$ is ultimately bounded by d_0 . We propose the following theorem to set forth these claims.

Theorem 5.1 *Consider the dynamical system (5.4) with control law (5.3). Let there exist a continuous function $V : D \rightarrow \mathbb{R}_+$, which satisfies the following:*

$$\alpha_1(z(k)) \leq V(z) \leq \alpha_2(z(k)), \quad \forall z \in D \setminus \{0\}, \quad (5.5)$$

$$V(z=0) = 0, \quad (5.6)$$

$$\Delta V(z) \leq -\min\{V(z), l\} + \alpha_3(|d(k)|) \quad (5.7)$$

where $\alpha_1, \alpha_2, \alpha_3 \in$ class \mathcal{K} function. Then the system (5.4) is FTISS and $z(k)$ is ultimately bounded by d_0 . Further, the expression for settling time is given as $\mathcal{K}(d_0, z_0) \leq \lceil \frac{V(z_0) - d_0}{l - d_0} \rceil$, $z_0 \in D \setminus \mathcal{B}_{d_0}(0)$, $l > d_0$, where $\mathcal{K}(d_0, z_0)$ exhibits lower-semicontinuity property on D .

Proof. We consider the case where $|z_0| > l$, since otherwise for $|z_0| < l$, the solution converges to $z(k) = d(k)$ in finite time with $\mathcal{K}(d_0, z_0) = 1$. To carry out the analysis, a Lyapunov candidate functional selected as $V(z(k)) = |z(k)|$. Then,

$$\begin{aligned} \Delta V(z) &= V(z(k+1)) - V(z(k)) \\ &= |z(k) - \text{sign}(z(k)) \min\{|z(k)|, l\} + d(k)| - |z(k)|. \end{aligned}$$

Since, $|z(k)| \geq \min\{|z(k)|, l\}$, then using (5.1),

$$\begin{aligned} \Delta V(z) &\leq |z(k)| - |\min\{|z(k)|, l\}| + d_0 - |z(k)| \\ &= -|\min\{|z(k)|, l\}| + d_0 \\ &= -\min\{V(z), l\} + d_0. \end{aligned} \quad (5.8)$$

This implies that the system (5.4) with reaching law (5.3) exhibits FTISS with $|z(k)| \leq d_0$, $\forall k \geq \lceil \frac{V(z_0) - d_0}{l - d_0} \rceil$ and the expression for settling time is given as $\mathcal{K}(d_0, z_0) \leq \lceil \frac{V(z_0) - d_0}{l - d_0} \rceil$, $z_0 \in D \setminus \mathcal{B}_{d_0}(0)$, $l > d_0$. This settling time expression can be obtained from (5.8) as shown in the following. Rewriting (5.8) for $V(z(k)) > l$ as: $V(k+1) \leq V(k) - l + d_0$. Further, we can write

$$\begin{aligned} V(k) &\leq V(k-1) - (l - d_0) \\ &\leq V(k-2) - 2(l - d_0) \\ &\vdots \\ &\leq V(0) - k(l - d_0). \end{aligned} \tag{5.9}$$

If at k^{th} time step, $V(0) - k(l - d_0) \leq l$, then from (5.9), $V(z(k)) \leq l \implies V(k+1) \leq d_0$, $\forall k \geq \lceil \frac{V(z_0) - d_0}{l - d_0} \rceil$. This completes the proof. \square

Remark 5.2 Note that, in system (5.4), if there is no perturbation, i.e. $d(k) = 0$, then the closed-loop system exhibits finite-time stability with $\Delta V(z) \leq -\min\{V(z), l\}$. Further, the expression for settling time is reduced to $\mathcal{K}(z_0) = \lceil \frac{V(z_0)}{l} \rceil$.

5.3.2 Discrete-Time Second-Order Perturbed System

Consider the following discrete-time dynamical system

$$\begin{aligned} z_1(k+1) &= z_2(k) \\ z_2(k+1) &= u(k) + d(k) \end{aligned} \tag{5.10}$$

where $z = [z_1, z_2]^\top \in D \subset \mathbb{R}^2$ represents the state vector of the system, $k \in \mathbb{Z}_{\geq 0}$, $u \in \mathbb{R}$ denotes the control signal and d is the disturbance satisfying $|d| \leq d_0$.

We construct a switching function as, $s(k) = c z_1(k) + z_2(k)$, where $-1 < c < 1$ is a positive scalar. The dynamics of the switching variable are $s(k+1) = c z_1(k+1) + z_2(k+1)$. Using (5.10), we can further write, $s(k+1) = c z_2(k) + u(k) + d(k)$. The difference equation with minima based reaching control law $u(k)$ is designed as

$$u(k) = -c z_2(k) + \mu_1 s(k) - \text{sign}(s(k)) \min\{\mu_1 |s(k)|, l_1\} \tag{5.11}$$

where $l_1 > d_0$, $\mu_1 \in (0, 1)$. Substituting (5.11) in the sliding variable dynamics, we further

get closed-loop system dynamics as

$$z_1(k+1) = s(k) - c z_1(k), \quad (5.12)$$

$$s(k+1) = \mu_1 s(k) - \text{sign}(s(k)) \min\{\mu_1 |s(k)|, l_1\} + d(k). \quad (5.13)$$

The dynamics of the switching variable exhibits FTISS, due to which $s(k)$ is ultimately bounded by d_0 . Moreover, the system states converge asymptotically to an invariant set $\omega = \{z \in D \mid |s(k)| \leq d_0\}$ and remain there for all future time. We propose the following theorem to put forward these claims.

Theorem 5.3 *Consider the dynamical system (5.12)-(5.13). Let there exist functions $V : D \rightarrow \mathbb{R}_+$ and $V_1 : D \rightarrow \mathbb{R}_+$, which are continuous and satisfies the following:*

$$\alpha_1(s(k)) \leq V_1(s) \leq \alpha_2(s(k)), \forall s \in D \setminus \{0\}, \quad (5.14)$$

$$V_1(s = 0) = 0, \quad (5.15)$$

$$\Delta V_1(s) \leq -\min\{V_1(s), l_1\} + \alpha_3(|d(k)|) \quad (5.16)$$

where $\alpha_1, \alpha_2, \alpha_3 \in$ class \mathcal{K} function. Then the sliding variable dynamics (5.13) is FTISS and $s(k)$ is ultimately bounded by d_0 . Further, the expression for settling time is given as, $\mathcal{K}(d_0, s_0) \leq \lceil \frac{V(s_0) - d_0}{l_1 - d_0} \rceil$, $s_0 \in D \setminus \mathcal{B}_{d_0}(0)$, $l_1 > d_0$, where $\mathcal{K}(d_0, s_0)$ is lower semicontinuous on D . Moreover, if $V(z_1, s) > 0$, $\forall (z_1, s) \in D \setminus \{0\}$, $V(0, 0) = 0$ and $\Delta V(z_1, s) \leq 0$, $\forall |z_1| \geq \frac{\mu_1 \gamma - l_1 + d_0}{1 - |c|}$ and $|s| \leq \gamma$, then the closed-loop system (5.12)-(5.13) is Lyapunov stable.

Proof. By carrying out the similar analysis as done for the first-order systems, we can show that dynamics (5.13) is FTISS and $s(k)$ is ultimately bounded by d_0 . For this, considering Lyapunov function $V_1(s) = |s(k)|$, similar results can be obtained (similar to the proof of Theorem 1) with settling time function given as $\mathcal{K}(d_0, s_0) \leq \lceil \frac{V(s_0) - d_0}{l_1 - d_0} \rceil$, $s_0 \in D \setminus \mathcal{B}_{d_0}(0)$, $l_1 > d_0$. Moreover, the condition $\Delta V_1(s) \leq 0$ within the set $|s| \leq \gamma$ ensures its positive invariance. Now, we will show that closed-loop system (5.12)-(5.13) is Lyapunov stable. Consider the Lyapunov function as $V(k) = |z_1(k)| + |s(k)|$. Then,

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= |z_1(k+1)| + |s(k+1)| - |z_1(k)| - |s(k)| \\ &= |s(k) - cz_1(k)| + |\mu_1 s(k) - \text{sign}(s(k)) \min\{\mu_1 |s(k)|, l_1\} \\ &\quad + d(k)| - |z_1(k)| - |s(k)|. \end{aligned}$$

Since, $\mu_1|s(k)| \geq \min\{\mu_1|s(k)|, l_1\}$, then using inequality (5.1),

$$\begin{aligned} \Delta V(k) &\leq |s(k)| + |c||z_1(k)| + \mu_1|s(k)| - |\min\{\mu_1|s(k)|, l_1\}| + d_0 \\ &\quad - |z_1(k)| - |s(k)|. \end{aligned}$$

Since, $l_1 < |s(k)| \leq \gamma$, it further follows

$$\begin{aligned} \Delta V(k) &\leq \mu_1\gamma + |c||z_1(k)| - l_1 + d_0 - |z_1(k)| \\ &\leq 0, \quad \forall |z_1(k)| \geq \frac{\mu_1\gamma - l_1 + d_0}{1 - |c|}. \end{aligned} \quad (5.17)$$

Therefore, if $|z_1(0)| \leq \frac{\mu_1\gamma - l_1 + d_0}{1 - |c|} \implies |z_1(k)| \leq \frac{\mu_1\gamma - l_1 + d_0}{1 - |c|}, \forall k \geq 0$, and the set $\Omega = \{|z_1| \leq \frac{\mu_1\gamma - l_1 + d_0}{1 - |c|}, |s| \leq \gamma\}$ is positively invariant if $l_1 \geq d_0$. Moreover, every solution which starts in Ω reaches the set $\omega = \{z \in \Omega \subset D \mid |s(k)| \leq d_0\}$ within settling time $\mathcal{K}(d_0, s_0)$. Hence, the closed-loop system (5.12)-(5.13) is Lyapunov stable. This completes the proof. \square

5.3.3 Backstepping-based SMC Design for Discrete-Time Second-Order System

In this subsection, we discuss the backstepping approach to design a difference equation with minima based control law such that all the states converge to an invariant set in finite time. Let us consider the following dynamical system

$$\begin{aligned} z_1(k+1) &= z_2(k) \\ z_2(k+1) &= u(k) + d(k) \end{aligned} \quad (5.18)$$

where $z = [z_1, z_2]^\top \in D \subset \mathbb{R}^2$ is the system state vector, $k \in \mathbb{Z}_{\geq 0}$, $u \in \mathbb{R}$ is the control input and d is the disturbance satisfying $|d| \leq d_0$.

We consider a nonlinear sliding surface as, $s(k) = z_2(k) - \phi(z_1(k))$, where $\phi(z_1(k)) := \mu_1 z_1(k) - \text{sign}(z_1(k)) \min\{\mu_1|z_1(k)|, l_1\}$, $\mu_1 \in (0, 1)$, $l_1 \in \mathbb{R}_+$.

Further, designing control as, $u(k) = \phi(z_1(k+1)) + \mu_2 s(k) - \text{sign}(s(k)) \min\{\mu_2|s(k)|, l_2\}$, $\mu_2 \in (0, 1)$, $l_2 \in \mathbb{R}_+$, then closed-loop system becomes

$$z_1(k+1) = s(k) + \phi(z_1(k)), \quad (5.19)$$

$$s(k+1) = \mu_2 s(k) - \text{sign}(s(k)) \min\{\mu_2|s(k)|, l_2\} + d(k). \quad (5.20)$$

The closed-loop system (5.19)-(5.20) is FTISS with system states converging to an invariant set in some finite time. We propose the next theorem to justify these claims.

Theorem 5.4 Consider the dynamical system (5.19)-(5.20). Let there exist a continuous function $V : D \rightarrow \mathbb{R}_+$, which satisfies the following:

$$\alpha_1(z(k)) \leq V(z) \leq \alpha_2(z(k)), \forall z \in D \setminus \{0\}, \quad (5.21)$$

$$V(z = 0) = 0, \quad (5.22)$$

$$\Delta V(z) \leq -\min\{V(z), l\} + \alpha_3(|d(k)|) \quad (5.23)$$

where $\alpha_1, \alpha_2, \alpha_3 \in$ class \mathcal{K} function. Then the system (5.19)-(5.20) is FTISS and sliding variable is ultimately bounded by d_0 . Moreover, the settling time function is given as, $\mathcal{K}(d_0, z_0) \leq \lceil \frac{V(z_0) - 2d_0}{l - 2d_0} \rceil$, $z_0 \in D \setminus \mathcal{B}_{2d_0}(0)$, $l = \min\{l_1, l_2\} > 2d_0$ and $\mathcal{K}(d_0, z_0)$ is lower semicontinuous on D .

Proof. To carry out the analysis, we choose a Lyapunov candidate functional as $V(z(k)) = |z_1(k)| + |s(k)|$. Then,

$$\begin{aligned} \Delta V(z) &= V(z(k+1)) - V(z(k)) \\ &= |s(k) + \phi(z_1(k))| + |\mu_2 s(k) - \text{sign}(s(k)) \min\{\mu_2 |s(k)|, l_2\} \\ &\quad + d(k)| - |z_1(k)| - |s(k)|. \end{aligned}$$

Since, $\mu_1 |z_1(k)| \geq \min\{\mu_1 |z_1(k)|, l_1\}$ and $\mu_2 |s(k)| \geq \min\{\mu_2 |s(k)|, l_2\}$, then using inequality (5.1),

$$\begin{aligned} \Delta V(z) &\leq |s(k)| + \mu_1 |z_1(k)| - \min\{\mu_1 |z_1(k)|, l_1\} + \mu_2 |s(k)| \\ &\quad - \min\{\mu_2 |s(k)|, l_2\} + d_0 - |z_1(k)| - |s(k)| \\ &= (\mu_1 - 1) |z_1(k)| + \mu_2 |s(k)| - \min\{\mu_1 |z_1(k)|, l_1\} \\ &\quad - \min\{\mu_2 |s(k)|, l_2\} + d_0. \end{aligned} \quad (5.24)$$

Based on the outcome of $\min\{\cdot, \cdot\}$, inequality (5.24) is deduced further. This leads to four scenarios which cover the complete admissible state space.

Scenario I: $\mu_1 |z_1(k)| \leq l_1$ and $\mu_2 |s(k)| > l_2$

With this, inequality (5.24) can be written as $\Delta V(z) \leq (\mu_1 - 1) |z_1(k)| + \mu_2 |s(k)| - \mu_1 |z_1(k)| - l_2 + d_0$. Since, $\mu_1 |z_1(k)| \leq l_1 \implies \phi(z_1(k)) = 0$, $z_1(k) = s(k)$ follows immediately (from (5.19)). Thus, the inequality is further written as

$$\begin{aligned} \Delta V(z) &\leq - (1 - \mu_2) |s(k)| - l_2 + d_0 \\ &\leq - l_2 + d_0. \end{aligned} \quad (5.25)$$

Scenario II: $\mu_1|z_1(k)| > l_1$ and $\mu_2|s(k)| \leq l_2$

Inequality (5.24) is deduced to $\Delta V(z) \leq (\mu_1 - 1)|z_1(k)| + \mu_2|s(k)| - l_1 - \mu_2|s(k)| + d_0$.

Further, it simplifies as

$$\Delta V(z) \leq -l_1 + d_0. \quad (5.26)$$

Scenario III: $\mu_1|z_1(k)| > l_1$ and $\mu_2|s(k)| > l_2$

This scenario deduces the inequality (5.24) as, $\Delta V(z) \leq (\mu_1 - 1)|z_1(k)| + \mu_2|s(k)| - l_1 - l_2 + d_0$. Further, since at steady state $|s(k)| \leq d_0$, then we can further, write

$$\Delta V(z) \leq -(l_1 + l_2) + 2d_0. \quad (5.27)$$

Scenario IV: $\mu_1|z_1(k)| \leq l_1$ and $\mu_2|s(k)| \leq l_2$

Here, inequality (5.24) reduces to, $\Delta V(z) \leq (\mu_1 - 1)|z_1(k)| + \mu_2|s(k)| - \mu_1|z_1(k)| - \mu_2|s(k)| + d_0$, which solves to $\Delta V(z) \leq -(V(z(k)) - |s(k)|) + d_0$. Further, with this scenario, governing dynamics (5.20) results in, $|s(k)| \leq d_0$, thus, it can be written that

$$\Delta V(z) \leq -V(z(k)) + 2d_0. \quad (5.28)$$

Moreover, if $s(0) \neq 0$, then the system dynamics (5.19)-(5.20) will further evolve and *Scenario IV* will transform to either of the other three scenarios. Hence, combining (5.25), (5.26), (5.27) and (5.28), it can be written that

$$\Delta V(z) \leq -\min\{V(z(k)), l\} + 2d_0. \quad (5.29)$$

Here, the system states are made to converge to an invariant set in some finite time.

Further, the calculation of the settling time involves similar steps to that of the Theorem

1. Thus the settling time function is obtained as, $\mathcal{K}(d_0, z_0) \leq \lceil \frac{V(z_0) - 2d_0}{l - 2d_0} \rceil$, $z_0 \in D \setminus \mathcal{B}_{2d_0}(0)$, $l > 2d_0$. This completes the proof. \square

5.4 Simulation Examples

In this section, we discuss perturbed discrete-time systems of first-order and second-order. We design a difference equation with minima based control law for these systems verify the effectiveness of the proposed law.

5.4.1 First-Order System

Consider the perturbed system as in (5.4). We consider a sinusoidal disturbance which is given by $d(k) = \sin(0.6k)$. We design the difference equation with minima based reaching control law as in (5.3). The initial condition for the system is taken as $z(0) = 100$ and the value of the control gain is selected as $l = 6$. Figure 5.1 shows the evolution of system state in presence of a bounded perturbation. Note that after some finite-time steps, the system state has reached to an invariant set, thereby remaining there for all future time. The evolution of the control input is shown in the Figure 5.2, which shows that there is no chattering involved.

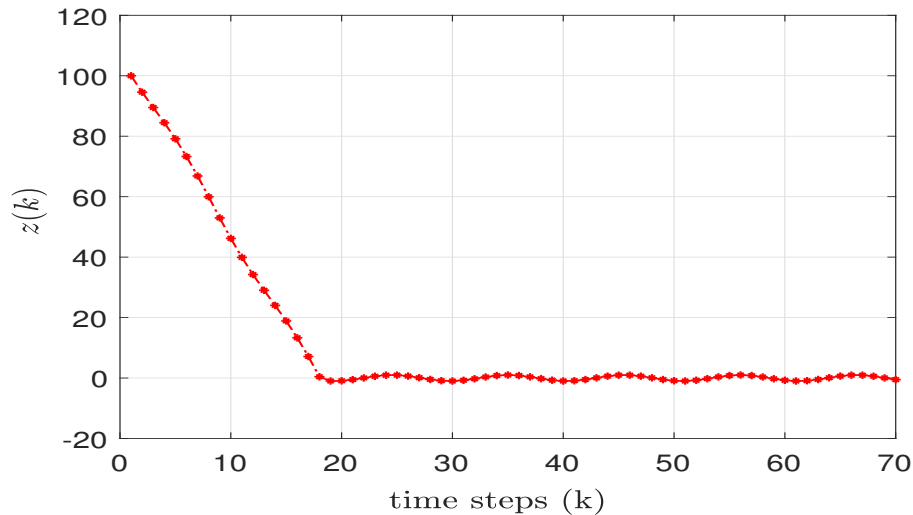


Figure 5.1: State $z(k)$ for the perturbed system

5.4.2 Second-Order System: A Comparative Analysis

Here, proposed method is compared with reaching laws proposed by [57] (Chakrabarty et al., 2015) and [60] (Bartoszewicz et al., 2020). Consider the following unstable perturbed dynamical system

$$\begin{aligned} z_1(k+1) &= z_2(k), \\ z_2(k+1) &= 2z_1(k) - 0.3z_2(k) + u(k) + d(k). \end{aligned} \quad (5.30)$$

We consider a sinusoidal disturbance which is given by $d(k) = \sin(0.6k)$, with $d_m = 1$. The initial conditions for system were chosen as, $z_1(0) = 100$ and $z_2(0) = 50$. The

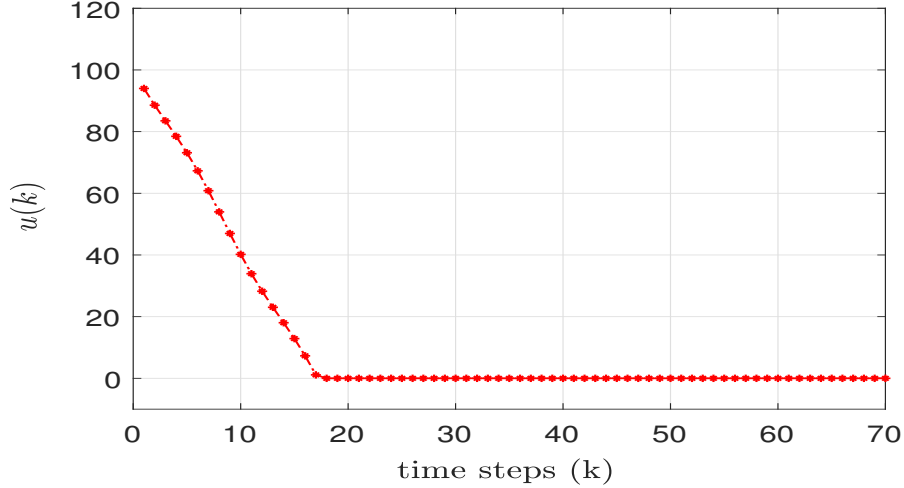


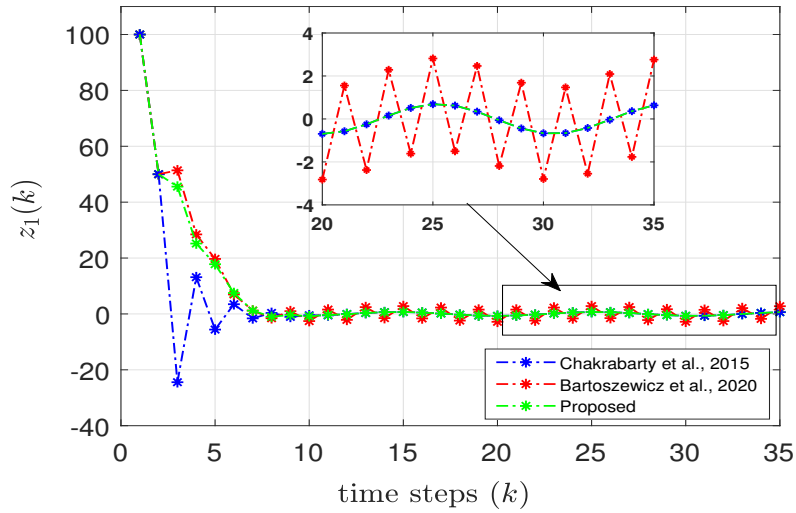
Figure 5.2: Control $u(k)$ for the perturbed system

sliding surface parameter for all the methods is selected as $c = 0.5$. The notations denoting different parameters are taken from the corresponding references. As per the band approach analysis in [57] ($d_m \leq \delta \leq 2d_m$), the control law designed by choosing the minimum possible band which is $\delta = d_m$. For this, the design parameters were computed as $\mu = 1$ and $\epsilon = -d_m$, which results in Utkin's reaching law. Further, the design parameters for reference trajectory based approach in [60] are chosen as $\sigma_0 = 30$ and $\epsilon = 1.1$. For the proposed method, parameters were chosen as $l_1 = 8$ and $\mu_1 = 0.8$.

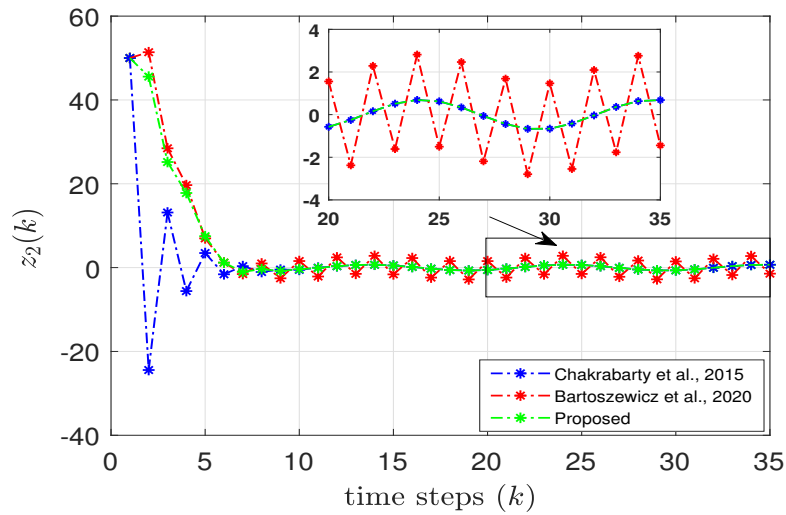
The evolution of system states is shown in the Figure 5.3. The states driven by [57] show oscillatory behavior along with high rate of change of states in the initial phase. On the contrary, the rate of change of states for [60] and proposed method is not very high, however, the occurrence of oscillatory behavior in sliding phase is prevalent in [60] unlike [57] and proposed method. Further, the evolution of switching function and control signal is depicted in Figure 5.4. Since, [57] follows Utkin's law from the beginning, it hits the manifold in just one instant leading to very high rate of change of the sliding variable and highest control signal magnitude at the initial phase unlike [60] and proposed method which have less rate of change of the sliding variable and comparatively less control magnitude. Moreover, we can see the chattering in [60], which is absent in [57] and proposed method. Also, the ultimate band size in [57] and proposed method is d_m , however, in [60], it is $d_m + \epsilon$.

Remark 5.5 *The minimum ultimate band size assured by [57] is d_m , but is achieved at*

the cost of overly-large control action. On the other hand, [60] ensures the less control effort at the initial phase, but results in chattering along with larger band width, $d_m + \epsilon$. However, the proposed method guarantees the less band width (d_m) without requiring large control at the initial phase and eliminates chattering.

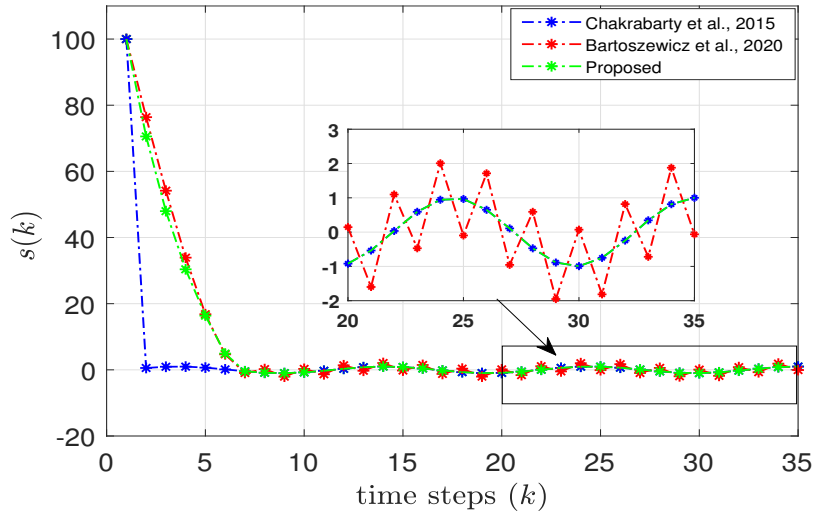


(a)

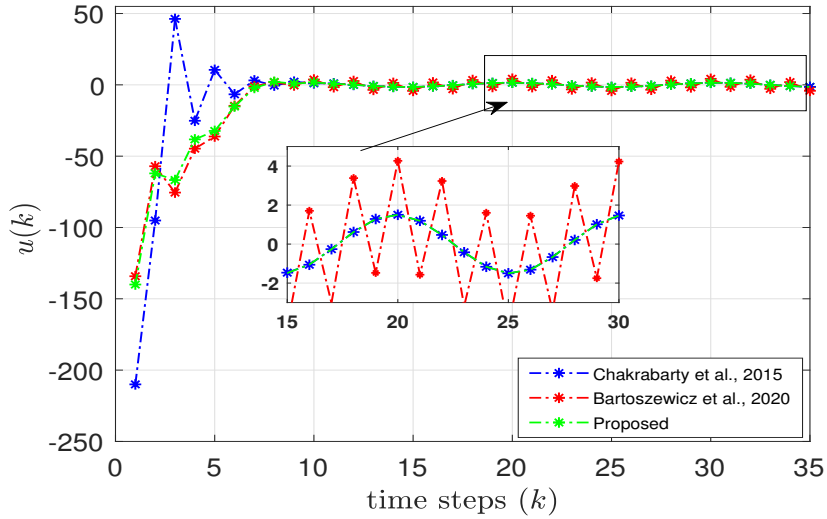


(b)

Figure 5.3: Evolution of system states of the perturbed DTS



(a)



(b)

Figure 5.4: Evolution of switching function and control signal of the perturbed DTS

5.5 Conclusion

In conclusion, this chapter systematically conducted a Lyapunov characterization for difference equation with minima based discrete-time sliding mode control. The established Lyapunov stability conditions serve as a testament to the stability of the systems and, crucially, highlight the successful alleviation of limitations inherent in both Gao's reaching law approach and Utkin's equivalent control-based approach. The proposed methodology's efficacy is explored in the context of both first-order and second-order perturbed

discrete-time systems. To provide empirical support for its effectiveness, a comparative study, facilitated by simulations, was conducted, underscoring the robust performance of the DEM based discrete-time sliding mode control.

In the upcoming chapter, we will introduce discrete-time twisting-like and super-twisting-like algorithms, providing a comprehensive examination of their stability through the application of Lyapunov stability methods, building upon the analytical techniques utilized in this current chapter.