

**A Study on Generalizations of C_1 , C_2 , C_3 and D_2
Modules**



*THE THESIS SUBMITTED
FOR THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN
MATHEMATICAL SCIENCES*

by

Kaushal Gupta

DEPARTMENT OF MATHEMATICAL SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY

(BANARAS HINDU UNIVERSITY)

VARANASI -221005

Roll No: 18121522

April 2024

CERTIFICATE

This is to certify that the revised thesis entitled " A Study on Generalizations of C1, C2, C3 and D2 Modules" is being submitted by Mr. Kaushal Gupta in partial fulfillment for the award of Ph.D. in the Department of Mathematical Sciences, IIT(BHU), Varanasi is a record of bonafide work carried out by him.

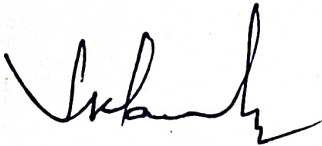
Date of Submission: 24.04.2024



(Supervisor)

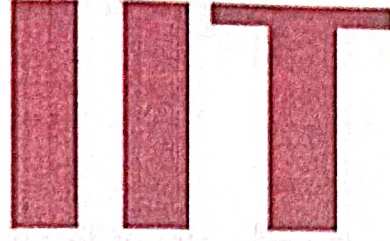
(Co-Supervisor)
(If any)

Forwarded by:



HOD / HEAD
(Signature with Seal)
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
Banaras Hindu University
Varanasi-221005

भारतीय
प्रौद्योगिकी
संस्थान
काशी हिन्दू विश्वविद्यालय



INDIAN
INSTITUTE OF
TECHNOLOGY
BANARAS HINDU UNIVERSITY

CERTIFICATE

It is certified that the work contained in this thesis titled "A Study on Generalizations of C_1 , C_2 , C_3 , and D_2 Modules" by Kaushal Gupta has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy, and SOTA for the award of a Ph.D. degree.

Dr. Ashok Ji Gupta
(Supervisor)

Associate Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

पर्यवेक्षक / Supervisor
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी / Varanasi-221005

DECLARATION BY THE CANDIDATE

I, **Kaushal Gupta**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Ashok Ji Gupta** from **January, 2019 to April, 2024** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for any other degree/diploma award. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, thesis, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: 24.04.2024

Place: Varanasi


(Kaushal Gupta)

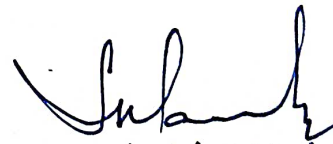
CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my/our knowledge.



(Dr. Ashok Ji Gupta)
Associate Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

पर्यवेक्षक / Supervisor
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी / Varanasi-221005


24.04.2024

(Prof. S. K. Pandey)
Professor and Head
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

विभागाध्यक्ष / HEAD
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी / Varanasi-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: *A Study on Generalizations of C1, C2, C3 and D2 Modules*

Name of the Student: *Kaushal Gupta*

Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology (Banaras Hindu University), Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the Ph.D. degree.

Date: 24.04.2024

Place: Varanasi

Kaushal
(Kaushal Gupta)

Note: However, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for the author's personal use, provided that the source and the Institute copyright notice are indicated.

ACKNOWLEDGEMENTS

It is a matter of great privilege and immense pleasure for me to express my deep sense of gratitude to my thesis supervisor, **Dr. Ashok Ji Gupta**, Associate Professor, Department of Mathematical Sciences, IIT(BHU). His wide experience, learned guidance, stimulating discussion and unstinting moral support helped me immensely in completing this work.

I am very grateful to **Prof.(Retd.) B. M. Pandeya**, Department of Mathematical Sciences, IIT(BHU) Varanasi, for his blessings. I am also thankful to my RPEC members- Prof. A. K. Tripathi and Prof. S. K. Upadhyay, for their constant encouragement and motivation throughout the period of research.

My sincere thanks go to Prof. S. K. Pandey, Head of the Department of Mathematical Sciences, for providing the necessary facilities. I also express my deep sense of gratitude to all faculty members and staff of the department for the necessary help.

I must thank my fellow researchers at IIT(BHU) Mr. Satish Kumar, Mr. Pradeep Rai, and Mr. Mukul Kumar Verma, Mr. Theophilus Gera along with my seniors Sonal Gupta, Dr. Shiv Kumar, Dr. Sanjeev Kumar Maurya, Dr. Varun Kumar Dr. Manoj Kumar Patel. My special thanks to my friends at IIT(BHU) Pradeep, Kashif, Sunny, Rashmi, Anupam, Shiv Shankar, and Ashutosh, for our enjoyable time together and for making my campus life calm and all the research scholars of the department for their moral support.

I gratefully acknowledge the Union Grant Commission (UGC), India, for providing financial support in the form of a Junior Research Fellowship and Senior Research Fellowship for the successful completion of this work.

Nothing can fade my memories of the beautiful time spent with Dr. Himanshu Chauhan, Dr. Shashank Vikram Singh, Dr. Rajkamal Nailwal, Shivam Saxena, Akshay Chauhan, and my elder brother Kailash Mehra in my B.Sc. days. I am always indebted to their unconditional friendship and love since 2011.

I express my sincere and cordial gratitude to my grandfather, Late Mr. Anandi Lal Vaish, my grandmother, Late Mrs. Urmila Devi, my elder father, Mr. Kripanandan Vaish, and my elder mother, Mrs. Manju Lata, my father, Mr. Naresh Varshney, my mother Mrs. Kiran Varshney who always stood my decisions and provided all kinds of supports, moral as well as financial. Also, I would like to thank my elder sisters, brothers-in-law, my younger sister Dr. Himanshi, and my younger brother Er. Rishabh for their constant encouragement. Special love to my nephews and nieces. Their love, care, and patience encouraged me to work and do better.

This acknowledgment would be incomplete if the name of great visionary Pt. Madan Mohan Malaviya is not mentioned as who made this divine center of knowledge. Deepest regards to him.

Finally, I bow down before the "Lord Mahadeva" for his showers of blessings throughout my research work to complete it successfully.

Date: 24.04.2024

Place: Varanasi

Kaushal
(Kaushal Gupta)

ABBREVIATION

\mathbb{N}	The set of natural numbers
\mathbb{Z}	The set of integers
\mathbb{Z}_n	$\mathbb{Z}/n\mathbb{Z}$ for some $n \in \mathbb{N}$
\mathbb{Z}_{p^∞}	The prufer p -group
\mathbb{Q}	The set of rational numbers
\mathbb{R}	The set of real numbers
\mathbb{C}	The set of complex numbers
\subseteq	A subset
\leq	A submodule
\leq^\oplus	A direct summand
\leq^p	A pure submodule
\ll	A small submodule
\trianglelefteq	A fully invariant submodule or An ideal
\leq^e	An essential submodule
$E(N)$	Injective hull of a module N
$PE(N)$	Pure Injective hull of a module N