CHAPTER 7

SUMMARY OF THE THESIS AND FUTURE SCOPE

7.1 Summary

The present thesis deals with the problem of stability of nonlinear dynamical systems under the effect of impulses. The impulsive sequence we have considered is fixed at each impulse point, and there are two categories of impulsive sequences that we have taken into consideration: stabilizing impulses and destabilizing impulses. The stabilizing impulses contribute to strengthening the stability of dynamical systems, whereas destabilizing impulses refer to the disturbed impulses which hinder the stability of dynamical systems and also lead to undesired dynamical behaviours. Our primary focus is on employing mathematical analysis to investigate the finite-time/fixed-time stability of nonlinear dynamical systems under the effects of stabilizing impulses as well as destabilizing impulses. In the context of finite-time stability, the upper bound of settling-time depends on the initial condition of the considered system but for fixed-time stability, we determine a upper bound of settling-time that is bounded and independent of the initial conditions. This thesis work covers the extensive study of the class of fixed-time stability of nonlinear systems for destabilizing impulses, which has not been reported by any existing work. Notably, the derived stability results exhibit lesser conservatism and higher precision compared to the existing results. Furthermore, the theoretical results on the nonlinear systems are extended to synchronization of various types of general neural network models which include inertial Cohen-Grossberg neural networks, memristor neural networks, complex-valued inertial neural networks and coupled neural networks. The this consists of Seven Chpaters, out of which, in **Chapter 1**, we elaborately described the impulsive dynamical systems and the classification of impulsive dynamical systems. The basic definition of delay differential equation and Lyapunov stability theory have been defined along with their properties. The physical interpretations of Cohen-Grossberg, bidirectional associative memory, memristor and complex-valued neural networks are presented. These models are extended to find the fixed-time stability results for time-delay and impulsive sequences. Finally, we conclude this chapter with a relevant literature review.

Chapter 2 focuses on achieving fixed-time stability (FxTS) of nonlinear dynamical systems under the influence of impulses. This is an extension of the FxTS theory proposed by [87]. In their study, [87] only considered the stabilizing impulses to achieve the FxTS but there is no relevant discussion about the destabilizing impulses. We have extended the result for stabilizing and destabilizing impulses based on a novel Lyapunov inequality. Two different problems have been investigated in this chapter. **Subchapter 2.1** and **Subchapter 2.2** developed some new theories on FxTS of nonlinear dynamical systems under the impulsive effects. In Subchapter 2.1, the FxTS theory of nonlinear impulsive systems with stabilizing and destabilizing impulses has been proposed using the Lyapunov functional and average impulsive interval. Contrary to the claim given in [87, 89] that FxTS cannot be possible for destabilizing impulses, we have derived a condition to achieve the FxTS of such nonlinear impulsive systems. Further, we verified numerically that the estimated settling-time functions are more effective than those reported previously in the literature. Subchapter 2.2 attempts to examine FxTS of the Cohen-Grossberg bidirectional associative memory neural network with destabilizing impulses. Firstly, a novel sufficient condition based on a new Lyapunov inequality is established for achieving FxTS in the presence of impulsive perturbations. The comparison method and the concept of average impulsive interval have been applied to derive the results. From the sufficient condition, we conclude that if the impulsive perturbations to the systems activate at the times that are separated by a certain length, then FxTS of impulsive systems is possible to obtain even in presence of destabilizing impulses. Furthermore, two continuous controllers: one containing signum terms and another without it, have been constructed based on the Lyapunov inequality to investigate FxTS of Cohen-Grossberg bidirectional associative memory neural networks. The obtained settling-time functions depend on the parameter of impulsive sequences. This implies that the desired settling-time of FxTS varies for different classes of impulsive sequences. Finally, we provide two numerical examples to demonstrate the efficacy of the proposed theoretical results. The first example involves a cyber-physical model, while the second one encompasses a neural network model.

Chapter 3 illustrates the fixed-time synchronization of neural networks under destabilizing impulses and time-varying delays. The neural network we considered here is the inertial Cohen-Grossberg neural network. In fixed-time synchronization, the state trajectories approach the equilibrium point within a fixed-time. To guarantee the FxTS for destabilizing impulses we design a lemma based on Lyapunov inequality that takes less number of parameters as compared to the results existing in literature. Here, we obtained the results by the utilization of comparison principle and average impulsive interval. To deal with the inertial term, an appropriate variable substitution has been considered to transform the second order neural networks into the system of first order neural networks and constructed a suitable Lyapunov function. Furthermore, based on the proposed lemma, a unified controller is developed and sufficient conditions are established to achieve fixed-time synchronization of the drive and response inertial Cohen-Grossberg neural network in the presence of desynchronizing impulses. To validate the robustness and applicability of the proposed theoretical results, two numerical examples are incorporated.

Chapter 4 deals with the problem of synchronization of neural networks with time delay and impulsive effects. Two different kinds of impulses, synchronizing and desynchronizing impulses have been considered separately to analyze the synchronization of neural networks. Further, the results for fixed-time synchronization have been derived in which the drive and response system can achieve synchronization in a fixed-time for any initial conditions. **Subchapter 4.1** explores the fixed-time synchronization of the drive and response memristor neural networks under the effects of synchronizing and desynchronizing impulses. Due to the discontinuous nature on the right-hand side of the memristor neural network, the solution has not existed in the usual method. Then based on the Lyapunov stability theory, set-valued map, and differential inclusion theory, we effectively dealt with the system discontinuity caused by the memristor's weight coefficient in the drive and response system by using the measurable selection theorem. Moreover, some sufficient conditions for fixed-time synchronization of memristor neural networks are presented utilizing the comparison method and average impulsive interval for synchronizing and desynchronizing impulses. The results for desynchronizing impulses have not been addressed in any of the existing works [95, 96, 111, 139]. The main highlight of the present subchapter is that, it provides lesser conservative results through numerical simulation and theoretical derivation for the case of synchronizing and desynchronizing impulses. **Subchapter 4.2** aims to analyze the fixed-time synchronization of complex-valued inertial neural networks with time delay and impulsive effects. For the complex-valued inertial neural networks in which the neural networks are of second order, we firstly reduce the second order complex-valued neural network into first order complex-valued neural network by using the variable substitution and then separated the complex-valued neural networks into two equivalent real-valued neural networks subsystems equivalently. Then, the sufficient conditions have been proposed for achieving fixed-time synchronization of complex-valued neural networks for synchronizing and desynchronizing impulses by using the comparison principle and the average impulsive interval. For desynchronizing impulses, we show that the condition $\tau_a > \frac{\ln \xi}{\rho}$ is required to achieve fixed-time synchronization of complex-valued neural networks. The settling-time function obtained in this subchapter depends on the impulsive sequence as well as the parameters of the continuous-time subsystems. Finally, two numerical examples have been taken to demonstrate the effectiveness of the proposed theoretical results.

Chapter 5 is concerned with the problem of fixed-time synchronization of coupled neural networks with mixed-delays. A novel pinning impulsive controller has been developed which is based on impulsive control and pinning control strategy. This controller only control the nodes with larger error values and thus, avoids the waste of control resources. Hence, using the concept of the average impulsive interval, comparison principle method and Lyapunov function theory, some unified sufficient condition has been derived for realizing the fixed-time synchronization of coupled neural networks with mixed delay under the influence of stabilizing and destabilizing impulses. Furthermore, the proposed results have been extended to the general class of neural networks. Also, the fixed-time synchronization problem of coupled neural networks with mixed-delay under delayed impulsive effects has been thoroughly examined and discussed. A synchronization criteria have been established to gurantee the fixed-time synchronization. Finally, the time-delayed Chua systems is employed to verify the proposed fixed-time synchronization results.

Chapter 6 aims to provide some results on the finite-time stability of nonlinear systems under the influence of stabilizing and destabilizing delayed impulses. Using the Lyapunov stability theory, we have introduced certain sufficient condition for finite-time stability of impulsive systems, where the impulses are dependent on delay. Delay-dependent impulsive sequences have been designed for which the impulsive system will achieve the desired settling-time. For stabilizing impulses, time delay affects only the construction of impulsive sequences whereas it affects both impulsive sequences as well as settling-time functions for destabilizing impulses. The upper bound of time delay has been estimated up to which the impulsive sequences are defined for the finite-time systems. Two numerical examples have been considered to verify the effectiveness of obtained theoretical results. It is shown with the help of numerical examples that time delay in impulsive control acts as a perturbation for the finite-time stability systems. The obtained results of this chapter are based on certain assumption (i.e., when the inequality $t_{l+1} - t_l \geq \tau_l$ holds for all $l \in \mathbb{Z}_+$). This implies that after hitting impulse moments t_l for $l \in \mathbb{Z}_+$, the solution of the finite-time system will depend only on the solution between the impulsive moments t_{l+1} and t_l.

7.2 Future Scope

The study of dynamical properties of impulsive dynamical systems has been paid much attention because of its rich applications in neural networks [185] and orbital transfer of satellite [186]. Based on different kinds of switching rules, the impulsive systems are divided into impulsive systems with fixed-time impulses and impulsive systems with variable-time impulses. In fixed-time impulsive systems, the impulse time can be set in advance to control the dynamical behaviors, but the same does not hold for variable-time impulsive systems because, in the later case, impulsive points depend on the states of systems. The solution of variable-time impulsive systems may hit the same surface of discontinuity finite or infinite number of times, and this phenomenon is called the pulse or beating phenomenon [187]. Therefore, investigating the dynamical behaviours of variable-time impulsive systems is complicated. It will be very interesting to study the effects of variable time impulses on fixed-time stability problems of nonlinear systems. Another future research direction is to investigate the predefined-time and prescribed-time stability of nonlinear impulsive systems. In fixed-time stability, we can see that the upper bound of settling time is too hazardous and it can not be diminished as per requirements of the users. To deal with it the concept of predefined-time stability has been introduced recently by [188]. Here, the upper bound of the settling time can be determined by the user in advanced by appropriate tuning of the parameters. Further, in case of both fixed-time and predefined-time stability the settling time depends on the initial conditions and the system parameters which may not be always suitable in practical applications. To deal with it recently the concept of prescribed-time stability is introduced by [189]. Here, the state trajectories can be made to converge exactly at the desired time without any dependency on the system parameters and the initial conditions. Moreover, currently very few research articles are reported on predefined and prescribed-time stability. Hence, it will be a very interesting and challenging task to incorporate the aforementioned stability notions for the impulsive systems. The present thesis is concluded with this hope in mind for future work.