ABSTRACT

In this thesis, the author has developed variants of the interior-point technique to determine the Pareto optimal solution for multiobjective optimization problems. The cone method and modified Pascoletti-Serafini scalarization techniques are utilized to convert a multiobjective optimization problem into a set of single-objective optimization subproblems.

The thesis consists of seven chapters as given below.

Chapter I

This chapter discusses the fundamentals of multiobjective optimization problems and some popular existing algorithms which are preferred to solve multiobjective optimization problems. Furthermore, the motivation of "why are interior-point methods preferred to solve multiobjective optimization problems?" will be explained. Thereafter, some historical background and the current status of interior-point methods will be discussed.

Chapter II

In this chapter, an infeasible interior-point technique is proposed to generate the nondominated set of a nonlinear multiobjective optimization problem with the help of the direction-based cone method. The suggested approach is derived for both convex and nonconvex problems. In order to solve the parametric optimization problems of the cone method, the infeasible interior-point method starts with an initial iterate outside the feasible region, and then gradually reduces the primal and dual infeasibility measures and the objective function value across the iterations with the help of a merit function. Estimates of the reduction of primal and dual infeasibility parameters per iteration are given. The convergence analysis of the method and an estimate of the number of iterations to reach an ϵ-precise solution are also provided. The author provides the performance of the proposed methods on a variety of convex and nonconvex multi-objective test problems. Performance comparison between the proposed method and popular existing solvers is provided with

respect to two performance measures and the corresponding relative efficiency measures. The reduction of a combined infeasibility measure, as the iterations progress, on the test problems is also shown graphically.

Chapter III

This chapter proposes an interior-point technique for detecting the nondominated points of multiobjective optimization problems using the direction-based cone method. Cone method decomposes the multiobjective optimization problems into a set of the single-objective optimization problem. For this set of problems, parametric perturbed KKT conditions are derived. Subsequently, an interior point technique is developed to solve the parametric perturbed KKT conditions. A differentiable merit function is also proposed whose stationary point satisfies the KKT conditions. Under some mild conditions, the proposed algorithm is shown to be globally convergent. Numerical results of unconstrained and constrained multi-objective optimization test problems are presented. Also, three performance metrics (modified generational distance, hypervolume, inverted generational distance) are used on some test problems to investigate the efficiency of the proposed algorithm. The author also compares the results of the proposed algorithm with the results of some other existing popular methods.

Chapter IV

This chapter introduces a primal-dual interior-point technique to determine the Pareto optimal solution for multiobjective optimization problems. A direction-based approach is utilized to transform multiobjective optimization problems into a set of single-objective optimization subproblems. Then, the subproblems are solved by using a primal-dual interior-point method. The primal-dual interior-point method utilizes the Newton method to calculate the primal-dual direction at each step to solve the perturbed Karush-Kuhn-Tucker optimality conditions. A merit function is introduced to take the suitable step lengths along the search directions. To demonstrate the efficiency of the proposed method, we applied it to some constrained test problems. As an application, we use proposed algorithm to an optimal control problem of carbon dioxide emission from energy sector, which aims to derive a mathematical framework to effectively utilize the available mitigation options to curtail CO_2 emission from energy use. We propose a multi-objective approach to find the optimal mitigation strategies to minimize the CO_2 level from energy sector as well as to minimize the cost of implementation of mitigation strategies.

Chapter V

This chapter introduce a trust-region interior-point technique to generate the Pareto optimal solution for multi-objective optimization problems. The modified Pascoletti-Serafini scalarization technique is utilized to convert a multi-objective optimization problem into a set of single-objective optimization subproblems. Then, the subproblems are solved by a trust-region interior-point method. Using the sequential quadratic programming technique, the algorithm proceeds through a sequence of barrier problems. With the help of the stationary points of a merit function, we obtain stationary points of the objective function of the barrier problem. It is shown that the directions that are used to find the sequence of iterates of the proposed method are descent direction of the used merit function. To show the efficiency of the proposed method, we show its performance on some standard test problems. As an application, we apply the proposed algorithm to solve an optimal control problem for a tuberculosis model. The model problem is a minimization problem and it has two objectives: one is the sum of the active infections patient and persistent latent individual, and another is the cost to implement the control strategies.

Chapter VI

This chapter presents a new trust-region algorithm to obtain the Pareto critical points of unconstrained nonsmooth multiobjective optimization problems. The gradients and Hessians of the objective functions that are used when solving trust-region subproblem are approximated by using ϵ-subgradients and quasi-Newton methods, respectively. Using the BFGS updating formula for the Hessian approximation of the model, we show that the proposed algorithm is convergent under some mild and standard conditions on the objective functions. At last the proposed algorithm is implemented in the MATLAB environment and applied on some nonsmooth test problems.

Chapter VII

In this chapter, the author conclude the thesis with some suggestions for future work.