PREFACE

Mathematical modeling has long been a cornerstone of understanding and solving complex phenomena in diverse fields such as physics, biology, engineering, and finance. Traditional differential equations have provided powerful tools for these purposes, but as the systems under study become increasingly intricate and exhibit nonlocal or memory-dependent behaviors, the limitations of classical approaches become apparent. In this context, fractional calculus has emerged as a compelling extension of traditional calculus, offering the flexibility and depth needed to address these challenges. The growing applications and widespread use of fractional derivatives highlight the significance of having a well-established theory for fractional derivatives that runs in parallel with the theory of ordinary derivatives. Fractional differential equations (FDEs) are mostly used to model complex systems, so in these cases, ensuring properties like existence, uniqueness, regularity, stability, etc. become even more important due to the intricate and sometimes counterintuitive nature of the solutions. In research and practical applications, these properties guide the formulation and analysis of FDEs, leading to reliable and meaningful results. In recent years, significant progress has been made in establishing the existence, uniqueness, stability, and other theoretical results of various categories of differential equations.

In this thesis, we explored several qualitative aspects of fractional differential equations, with a focus on the existence and uniqueness of their solutions. Additionally, we examined the controllability and approximate controllability of fractional control systems, supported by various illustrative examples. This thesis is organized into seven chapters, each contributing to a comprehensive understanding of the subject matter. Chapter 1 focuses on preliminary results, outlining essential notations, definitions, and lemmas. It also introduces the foundational concepts of semigroup theory and other mathematical tools necessary for the subsequent chapters. Chapter 2 provides a brief introduction to the fundamental concepts and context of the study, including a historical background. It offers an overview of key topics such as fractional calculus, nonlinear systems, delay differential equations, nonautonomous differential equations, and evolution equations. Additionally, this chapter presents a concise introduction to the problems addressed in the thesis and includes a detailed literature survey.

Chapter 3 investigates the existence and controllability of solutions for a particular category of nonlinear fractional functional non-autonomous integro-differential equations featuring a nonlocal condition within the framework of Banach space. By applying fractional calculus methods and various fixed-point theorems, we discuss a few outcomes concerning existence, uniqueness, and controllability. This is achieved by converting these problems into fixed-point problems by means of the α-resolvent family. Finally, we provide two examples to demonstrate the application of the proposed results.

Chapter 4 investigates the existence of mild solutions and the approximate controllability of a class of time-fractional, nonlinear, non-autonomous fractional control systems with delay. In this chapter, we used techniques from fractional calculus, semigroup theory, and fixed-point theory to construct a robust framework for addressing the problem. We transform the existence problem into a fixed-point problem using the α-resolvent family, thereby deriving existence results for mild solutions. Furthermore, we explore the approximate controllability of the fractional control system.

In Chapter 5, we look into the existence results for a family of fractional functional differential equations employing the Riesz-Caputo fractional derivative in a Banach space. Fractional calculus techniques, Kuratowski's measure of noncompactness, Carathéodory conditions, and some theorems on fixed points are used to establish existence results. At the end, few examples are showcased to evince the proficiency of the offered results.

Chapter 6, investigates the existence and uniqueness of solutions for a particular category of fractional delay integro-differential equations in the context of Banach space, incorporating the Riesz-Caputo fractional derivative. Employing fractional calculus techniques and multiple fixed-point theorems, we establish a few results regarding both existence and uniqueness. Further, by introducing a partial order in a Banach space of all continuous functions, we look into the existence of extremal solutions. To demonstrate the competence of the suggested outcomes, a few instances are presented at the conclusion.

Finally, Chapter 7 presents a summary of the key findings and draws conclusions based on the research conducted. Apart from this, potential directions for future work are discussed, outlining the areas we intend to explore further.