A Study on Generalizations of Direct Projective Modules and Direct Projective Covers of Modules

THE THESIS SUBMITTED

FOR THE AWARD OF THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MATHEMATICAL SCIENCES

by

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May 2023

Conclusion and Future Scope

The work of this thesis has close connections with *direct projective* modules, as we studied some of its generalizations such as Semi-Simple Direct Projective modules, Finite Direct Projective Modules, Finite Direct Projective Cover and Envelopes, and Pure Direct Projective Modules.

This thesis is divided into five chapters that provide a comprehensive understanding of different types of generalizations of direct projective modules and their characterizations over several rings. Chapter 1 is the collection of notations, definitions, and basic results. Chapter 2 focuses on the study of direct projective modules with respect to their endomorphism rings. Also, we introduce the concept of semi-simple direct projective modules which is a generalization of direct projective modules, and characterize semi-simple Artinian rings. Chapter 3 deals with the new notion of finite direct projective modules, and investigates the relationships among Rickart modules, D_3 modules, endoregular modules, and finite direct projective modules. Also, it provides the characterization of semi-hereditary rings, and semi-simple Artinian rings in terms of finite direct projective modules. Chapter 4 explores the Finite Direct Projective Covers and Envelopes and characterizes semi-perfect rings, semi-regular rings, and S-rings. Chapter 5 delves into the new notion of pure direct projective modules, offering an in-depth analysis of their properties and characteristics.

Overall, these chapters offer a thorough understanding of strict generalizations of direct projective modules and their characterizations over several rings.

Future Scope

During the study, we found some research problems that are unsolved and unavailable in the literature on which work can be done in the future. We would like to enlist a few of them as follows:

1. We can generalize the class of direct projective modules in terms of Copure modules, which can be another generalization and dual of pure direct projective modules.

2. We can study rings over which every cyclic module is a finite-direct projective module.

3. We can also extend the theory of cover and envelopes for Pure Direct Projective Modules as "Pure Direct Projective Cover and Envelopes".

4. We can also extend the theory of semi-simple direct projective modules using the concept of polysimple modules.