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# CHAPTER 2

## LITERATURE REVIEW

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*This chapter aims to provide an in-depth, comprehensive literature review that summarizes past published work related to the thesis goals. This research focuses primarily on vibration energy harvesting; therefore, the chapter starts with a brief background on energy harvesting applications and necessity, which is followed by a review of the piezoelectric material and mathematical modeling techniques for tapering cross-section PVEHs. After that, a critical literature survey of model validation techniques and performance comparison strategies is conducted.*

*Following a comprehensive review of the literature, the research gaps related to the harvester's shape, mathematical modeling, and experimental analysis are recognized. Finally, the objectives of this thesis are thoroughly explained.*

### **2.1 Outline**

The conversion of vibration energy or deformation energy to electrical power is known as mechanical energy harvesting. The energy harvesters can use a number of energy sources, ranging from the human body to wild animals(Choi et al., 2017; Zhou et al.,

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2020), from industrial machinery to vehicles, from large-scale buildings to bridges, and from water flow to wind(Mohith et al., 2021; Safaei et al., 2019; Yang et al., 2018b). Energy harvesters are now being touted as potential independent power sources for low-power electronic units such as wireless sensors, portable electronics, and medical implants. The ability to provide power to micro-electro-mechanical system (MEMS) or a wireless sensor using sustainable electric power via energy harvesting is appealing because of the environmental issues of chemical batteries. It also eliminates the additional time and cost of replacing, maintaining, and installing complex wired systems(Batra et al., 2016). This is especially important when installing sensor nodes in hostile or difficult-to-reach surroundings. Energy harvesting technology is forecasted to create a new generation of self-powered autonomous sensor networks in wearables(Jung et al., 2015; Xie and Cai, 2014; Zhao and You, 2014), pacemakers(Deterre et al., 2014; Karami and Inman, 2012; Mo et al., 2010; Sohn et al., 2005), smart vehicles(Behroozinia et al., 2018; Matilainen and Tuononen, 2015; Rui et al., 2020), and bridge and civil structure monitoring systems(Erturk, 2011; Lu et al., 2014; McCullagh et al., 2014; Shen et al., 2016), due to the rapid advancements in low-power ICs and high-efficiency energy storage devices.

For the past two decades, energy harvesters have been extensively explored. A variety of research has been carried out in various aspects to improve performance. The main challenges preventing present energy harvesters from being extensively implemented in engineering applications are their limited power output and sensitivity to environmental fluctuations. Researchers have experimented with various transduction methods(Mo and Davidson, 2013; Mohanty et al., 2019), designs(Anton and Sodano, 2007), nonlinear methods(Jia, 2020), optimization techniques(Yang et al., 2018a), and materials(Safaei et al., 2019; Sezer and Koç, 2021) to maximize the power output.

## 2.2 Transduction Methods

The movement of automobiles on bridges and the operation of machinery in production units cause a considerable amount of vibration. These vibration energies dissipated into the environments are wasted mechanical energy. Electrical power can be generated from these wasted mechanical energies using various transduction approaches.

### 2.2.1 Electromagnetic

In this method, the relative motion between the conducting coil and a magnetized body transforms the mechanical energy into electrical energy. The mechanism of electromagnetic energy harvesting (EMEH) is based on Faraday's law of electromagnetics, which states a direct proportionality of voltage and the variation of magnetic flux linkage with respect to time (Zohdi, 2012). Moving magnet, moving conducting coil, and resonant type are the three primary forms of EM harvesting setup. Moving magnet-type harvester is more prone to catastrophic destruction near ferromagnetic components compared to the other two types. The first micro-scaled EMEH was developed around 2000 (Kulah and Najafi, 2004; Williams et al., 2001). Shearwood and Yates (1997) observe an RMS power of 0.3  $\mu\text{W}$  at a resonant frequency of 4.4 kHz. A high resonant frequency is achieved in this design, resulting in low conversion efficiency. Other researchers later dealt with the problem by designing harvesters with a lower resonance frequency. A micro-machined and compact EM harvester with a volume of 1  $\text{cm}^3$  was designed by Ching et al. (2002) to generate an optimum output voltage for powering low-power ICs and sensors. The frequency-

doubling features of some EMEHs generate more energy at lower frequencies(Dai, 2016; Dai et al., 2016). The switching damping, on the other hand, can modify the resonance frequency of the harvester, but it also causes some power loss(Ooi et al., 2015). The EMEHs function best when their size is larger and under periodic excitation but doesn't work well when the vibrations are random(Halim et al., 2016, 2018; Wang et al., 2017).

### *2.2.2 Electrostatic*

The electrostatic charge transfer appears when two surfaces of a charged capacitor move relative to one another, causing changes in the capacitor potential difference, which results in static electricity. An electrostatic energy harvester (ESH) converts vibrational energy to electrical following a two-step conversion. The electrode and circuit impedance are used to bring two dielectrics with different permittivity and thicknesses into direct contact. Then the electrostatic charge starts to flow after this is coupled to a charged electrode. The variable dielectric constant, variable gap and variable area capacitor are the three types of variable capacitors. The improvement of output power in a variable dielectric constant capacitor is related to changes in the dielectric material(Boland et al., 2005; Borno, 2008). The vibration in the variable gap capacitor is primarily caused by changes in the gap-closing arrangement for power harvesting(Guillemet et al., 2010). In a variable area capacitor, plates vibrate in the opposite direction, causing the capacitance to change and power to be generated(Sterken et al., 2003; Tvedt et al., 2008). The requirement of an external power source for ESH reduces the generated power density (PD). The main benefit of

the ESH method is that it produces incredibly high voltage due to its high internal impedance, which allows the power to be transferred across distant locations with minimal loss.

### *2.2.3 Magnetostrictive*

The magnetostrictive (MS) materials generate power from mechanical vibration in two steps. By magneto-mechanical coupling, the mechanical energy is first converted to magnetic energy, which is subsequently transformed to electrical energy via electromagnetic coupling (Deng and Dapino, 2017; Dey, 2017). The orientation of the magnetic field varies when a mechanical deformation is applied to MS material, resulting in a magnetic flux change. The electromotive force, induced as a result of this magnetic flux variation with time, results in electricity generation. The MS sensors have been used to detect time-dependent mechanical stress or strain for decades. Kwun and Bartel (1998) comprehensively presented the theoretical study of the MS sensor mechanism. The MS harvesters do not perform well under high resistive loads due to their inductive properties (Mohammadi and Esfandiari, 2015). The key downsides of this harvester are the pickup coil size, difficulties in integrating with MEMS (Wang and Yuan, 2008), and the fact that it can only produce significant power near resonance frequency.

### 2.2.4 Piezoelectric

The piezoelectric effect is a peculiar material behaviour that generates an electric field across the material with a sensation of mechanical deformation. The piezoelectric material can operate in three modes based on the relationship between stress and polarization direction, namely  $d_{33}$ ,  $d_{31}$ , and  $d_{15}$ . When the piezoelectric materials are exposed to bending stress, the PVEH acts in  $d_{31}$  and  $d_{33}$  modes. However, for  $d_{15}$  mode-operated PVEH, the piezoelectric material is subjected to torsional stress (Kulkarni et al., 2014). Due to its ease of manufacture, the  $d_{31}$  mode is mainly preferred over the  $d_{15}$  and  $d_{33}$  modes. The complexity of interdigital electrode fabrication of  $d_{33}$  mode harvesters, and the involvement of two electrodes for poling and charge collecting separately in  $d_{15}$  mode harvesters, make their use more problematic (Hagood et al., 1993; Wilkie et al., 2000). Significant advancements in energy harvesting technology involving piezoelectric transducers have taken place in recent years. Priya (2007) discussed new developments in the field of low-profile piezoelectric transducers for energy harvesting. According to the study, the PVEH has a three to five times higher PD than other harvesting systems. PVEH, made up of a cantilever configuration, can cause substantial deformation for minor vibration while having a simple design. The key benefit of the PVEH is its compact size, i.e., the thin piezoelectric patch makes the entire system more straightforward and smaller than other harvesters (Lopes and Gallo, 2014). Furthermore, piezoelectric transduction on a small scale is more effective than the electromagnetic counterpart and provides the highest PD than other methods.

## 2.3 Tapering Cross-section PVEH

Scientists have investigated a range of techniques to boost the performance of PVEHs from the standpoint of architectures, materials, and circuitries. Here, only the innovative geometry-modified or tapered cantilever structures for transverse and rotational motion are presented. Rectangular cantilever beams are used in traditional PVEHs, where the bending stress on the piezoelectric patches varies linearly with the length. At the fixed end, the bending stress is highest, while at the free tip, it is lowest(Li et al., 2014). As a result, concerns like overstretch and fatigue may arise at the fixed end, and the piezoelectric material near the free end, on the other hand, does not perform to its full potential(Muthalif and Nordin, 2015). Therefore, researchers have experimented with different geometries to replace the traditional rectangular shape with the objective of achieving a uniform and large strain distribution on the piezoelectric materials in PVEHs(Benasciutti et al., 2010; Mateu and Moll, 2005).

### 2.3.1 Tapered PVEHs for transverse excitation

Roundy et al.(2005) advocated substituting rectangular cantilevers with trapezoidal profiles to boost power production. Experiments have shown that beams having a nonuniform width produce 30% more power than rectangular counterparts. Dietl and Garcia(2010) theoretically and experimentally investigated a number of beams spanning from rectangular to triangular shapes. The research concluded that the beam minorly affects the efficiency, but it substantially impacts the maximum sustainable excitation strength and the highest possible power output. Baker et al.(2005) presented a

trapezoidal shape PVEH by introducing a finite element formulation and examining the geometry's significance on the harvester's PD. The suggested harvester produced 50% more power than a rectangular one because of the uniform distribution of strain over the substrate. Goldschmidtboeing and Woias(2008) used the Rayleigh-Ritz method to formulate a numerical model for varying width cantilever PVEHs and optimized the rectangle, linear, and reverse taper beam shapes. Additionally, Ayed et al.(2014) considered linear and quadratic cantilever profiles. They used the single-mode Galerkin approach to model a unimorph cantilever and reported that the quadratic design could produce roughly 100% higher power than a rectangular one.

Paquin and Amant(2010) proposed a semi-analytical mathematical formulation based on Rayleigh-Ritz approximations for thickness optimization. According to the study, a tapering thickness beam with a slope angle of  $0.94^{\circ}$  increases the power generation by 3.6 times. The improved performance can be attributed to the more uniform strain distribution. They did not give experimental validation; because this procedure is theoretically viable and may not be realistic due to the complications in fabricating thin beams with such a minor taper angle. Wang et al.(2020) recently presented a multi-layered beam with variable thickness for high-efficiency vibration energy harvesting. The PD of their model is not significantly improved because of the linear tapering profile. The design and study of a series of tapering thickness cantilever bimorph PVEHs for broadband operation are given by Keshmiri and Wu(2018). They found that the prototype could produce roughly five times the voltage of the uniform thickness counterpart after conducting optimization experiments. The principal topographies of a few tapering cross-section PVEHs are summarized in Table 2.1. The harvested power of the tapering cross-section harvesters is compared to that of their uniform cross-section counterparts with an equivalent piezoelectric surface area to

determine the performance amplification factor (PAF). By keeping the base width constant, the equivalent surface area of the uniform cross-section harvesters is calculated.

**Table 2.1** A comparison of the performance of some tapering cross-section PVEHs under transverse excitation

References	Tapering profile	Piezoelectric material	Frequency band (Hz)	PD ( $\mu\text{W}/\text{mm}^3$ )	PAF
Salmani and Rahimi(2015)	Exponential	PZT 4	55–75	5.61	1.33
Baker et al.(2005)	Linear	PZT 5H	20–100	0.30	1.30
Dietl and Garcia(2010)	Linear	PZT 5A	10–1000	–	1.01
Paquin and Amant (2010)	Linear	PIC 151	100	23.83	3.60
Keshmiri and Wu(2018)	Exponential	PZT 4	0–795	–	1.89
Kundu and Nemade (2015)	Linear	PZT 5A	40–52	34.32	1.31
Janphuang et al.(2013)	Linear	PZT	19.89	0.1619	2.06
Yang and Zu(2014)	Linear	PZT 5A	10–30	4.6239	1.78

### 2.3.2 Tapered PVEHs for rotational applications

Xie et al.(2017) proposed an analytical technique for the performance evaluation of a tapered rotational piezoelectric vibration energy harvester (RVEH). The group looked into how the taper parameters, patch widths, and other variables affect the harvester's performance. Their model with the optimum design provides an RMS power of 0.1376 W at a rotating frequency of 10 rad/s. However, experimental validation of their prototype is challenging due to the intricate design. Yu-Jen et al.(Yu-Jen et al., 2019; Yu-Jen et al., 2017) proposed a trapezoidal piezoelectric cantilever design for energy harvesting in rotational applications. The numerical analysis indicated that the output power could be efficiently increased by adjusting the design parameters, and the system can reach resonance at any wheel rotation frequency. According to the experimental results, the average generated power was 107.4  $\mu$ W for wheel speeds range of 177 – 796 RPM. However, their design is very complicated to be used in practical applications. Recently, Deng et al. (2020) proposed a FE model of a linearly tapered harvester using ANSYS for rotational motion application. The proposed harvester produced 5-26 mW power at a 10-20 Hz operating frequency range. However, they have not presented any definite mathematical model or experimental method to estimate the harvester's responses. The principal topographies and PAF of a few tapering cross-section PVEHs under rotational motion applications are summarized in Table 2.2.

**Table 2.2** A comparison of the performance of some tapering cross-section PVEHs for rotational motion

References	Harvester shape	Piezoelectric material	Frequency band (Hz)	PD ( $\mu\text{W}/\text{mm}^3$ )	PAF
Xie et al.( 2017)	Trapezoidal	PZT 4	1.6	1.71	1.33
Xie and Wang(2017)	Trapezoidal	PZT 4	0.16	0.51	1.31
Yu-Jen et al.(2017)	Trapezoidal	PVDF	4-20	1.354	1.30
Yu-Jen et al.(2019)	Trapezoidal	PVDF	3.33-11.67	1.06	1.01
Deng et al.(2020)	Trapezoidal	PZT	10-20	2.07	1.60

## 2.4 Mathematical Modeling of PVEHs and Performance Comparison

In the design and development of ideal PVEH systems, mathematical modeling plays a vital role. A brief review of the mathematical models of PVEH systems is presented in this section. The main focus of this section is the mathematical models established to determine the response of the cantilever PVEH, the most popular energy harvesting design. Piezoelectric materials are typically used as thin patches that are bonded or inserted into beam, plate, or shell structures for energy harvesting, vibration isolation, shape sensing, noise control, or condition monitoring applications. Because of the minor ratio of thickness to in-plane dimensions, such thin-walled components can be regarded as 2D surfaces. Again, the normal transverse strain can be assumed to be zero for such thin components due to the negligibly small thickness change.

### *2.4.1 Lumped parameter modeling and improvements*

The majority of the early efforts on mathematical modeling of PVEH systems relied on single degree-of-freedom (SDOF) lumped parameter techniques (DuToit et al., 2005; Roundy and Wright, 2004; Stephen, 2006). While these models were beneficial for understanding the behaviour of piezoelectric energy harvesting, their calculations overlooked features of piezoelectric systems' electromechanical coupling. Researchers began investigating distributed parameter models using the Rayleigh-Ritz discretization to improve lumped-parameter mathematical models, which provided more precise solutions (DuToit et al., 2005; DuToit and Wardle, 2007; Sodano et al., 2004). Furthermore, various researchers used Euler-Bernoulli beam theory to formulate exact analytical models for the cantilever PVEHs (Ajitsaria et al., 2007; Chen et al., 2006; Lu et al., 2004). However, many of the studies mentioned above include flaws in mathematical assumptions that lead to inaccuracy; Erturk and Inman highlighted these flaws in 2008 (Erturk and Inman, 2008b). In addition, the same researchers published an accurate distributed parameter analytical model for a unimorph cantilever PVEH (Erturk and Inman, 2008a), which has gained widespread acceptance in the energy harvesting field. They also came out with accurate solutions for symmetric bimorph cantilever PVEHs in 2009 (Erturk and Inman, 2009). Their method, however, is limited to basic uniform-section cantilevered beams.

Exact analytical solution methods are useful for modeling basic uniform cross-section cantilever PVEHs; however, many practical harvesting setups are sophisticated in design, and exact solutions are impossible to obtain. For example, the exact solutions are impossible to find for tapering beam harvesters, thick cantilevers, and asymmetric

systems. Approximate solution approaches must be used in these situations. Some researchers have proposed such approximate solution methods; DuToit et al.(2005) model a symmetric cantilever bimorph with Rayleigh-Ritz approximation technique, Elvin and Elvin(2009) model a unimorph PVEH using an approximate Rayleigh-Ritz method, and Erturk and Inman(2012) model unimorph, bimorph, and asymmetric cantilever PVEHs using an assumed modes formulation.

### *2.4.2 Nonlinear modeling of PVEHs*

Elastic deformation with large deflections is very common in thin structures. The geometry of the harvesting beam can be changed to optimize the fundamental operating frequency and output performance of any PVEH. This variation in geometry causes nonlinearity in the system. The piezoelectric material can also cause nonlinearity, linked to the higher applied excitation and different constraints, including impact, backlash, and friction(Daqaq et al., 2014; Zhang and Schmidt, 2014a). In such instances, nonlinear modeling is essential.

#### **2.4.2.1 Geometrically nonlinear modeling**

Geometrically linear theories are only applicable to systems that are subjected to small deflections. Alternatively, for structures experiencing high displacements and/or rotations, geometrically nonlinear theories must be incorporated. Including various nonlinear coefficients also leads to a variety of nonlinear approaches. In comparison to works dealing with linear formulations, there are far fewer research articles dealing with

nonlinear formulations. The von Karman nonlinear theory, which takes into account the squares and products of derivatives of the transverse deflection in the strain components, is the most uncomplicated and widely used. Im and Atluri( 1989) were the first to apply this nonlinear theory to the analysis of piezoelectric integrated structures. Zhang and Schmidt(2013, 2014b, 2014c) extended the idea to piezoelectric plates and shells, based on the first-order shear deformation (FOSD) theory for static and dynamic investigation, including additional geometrically nonlinear effects while utilizing simple strain-displacement equations. Moita et al.(2002) developed geometrically nonlinear FE models using nonlinear strain-displacement relation. Masana and Daqaq (2011) presented a model to study axially loaded clamped-clamped tunable PVEHs considering a nonlinear strain-displacement relation. They discovered enhanced steady-state responses, including higher output power and a broader frequency band, when considering the nonlinearity generated by the axial load. Leadenham and Erturk (2015) modelled the nonlinear electric-enthalpy relationship as a polynomial function in strain magnitude and found that the higher-order nonlinear coefficients remain significant in the symmetric bimorph harvesters.

#### **2.4.2.2 Piezoelectric materially nonlinear modeling**

Linear constitutive relations, which incorporate the first-order and higher-order change of electric potential along the thickness, are frequently employed in the analysis of piezoelectric systems. The commonly held belief that first-order electric potential variation occurs through the thickness of piezoelectric materials, resulting in a constant electric field, is only effective for systems with very thin piezoelectric patches(Yang,

1997, 1999). The bending effect of the piezoelectric actuators must be considered when the thickness of the piezoelectric patches is not very thin. Furthermore, the linear models formed by Gopinathan et al.(2000) and Marinković et al. (2007) contained quadratic electric potential variation across the thickness. The linear piezoelectric constitutive relations can only be applicable for systems with low electric potential and small strains. Because piezoceramics are fragile, only minor strains arise in the system before it is damaged. However, a high electric field can arise on the piezoelectric patches, resulting in material nonlinearity. Nelson (1978) and Joshi et al. (1992) first reported the nonlinear electroelastic constitutive relations. Yang and Batra(1995) later extended the nonlinear constitutive equations to transversely isotropic materials such as piezoelectric and PVDF materials. Recently, based on the assumption of a quadratic electric potential distribution across the thickness, Rao et al.(2016) developed a FE model for piezoelectric laminated composites with electroelastic material nonlinearity. The previously presented research papers mainly dealt with geometrically linear models with material nonlinearities.

Furthermore, some researchers developed fully nonlinear models that included both geometrical and electroelastic material nonlinearities (Zhang et al., 2017),(Yao et al., 2004), and Stanton et al.(2010a, 2010b, 2012)). Abdelkefi et al. introduced a multimodal formulation for linear(Abeldelkefi et al., 2012a) and parametric(Abeldelkefi et al., 2012b) base excitation, which considered the geometric, material, and inertia nonlinearities.

### 2.4.3 Performance comparison standards

It's tough to compare the performance of PVEHs with different construction, dimensions, materials, and operating modes. Firstly, the lack of a universal standard is cited for the difficulties in comparing the performances. On the other hand, the amount of data revealed in the publications differs significantly. A review of various standards used for evaluating and comparing the PVEHs is offered in this section. The most widely used standard for PVEHs is the power density, which is the generated power divided by the volume. However, it is not appropriate for comparing different architecture because overall power generation is not always proportionate to the harvester's volume. For example, the power generation from inertial PVEHs is dependent on mass and the maximum displacement(Marin et al., 2011).

Beeby et al.(2007) proposed the normalized power density (NPD) to account for the excitation effect. The NPD is the harvester's reported power output normalized to volume and acceleration amplitude squared. Despite the fact that the NPD isn't a perfect benchmark due to a lack of frequency data, it does provide information about relative performance levels. It's worth noting that in this context, volume might denote either the volume of active piezoelectric materials or the system's whole volume. Both ways of calculation have disadvantages. The computation based solely on active material volume is skewed in favour of PVEHs with smaller piezoelectric materials. This is because: (1) the PVEHs typically have complex shapes; (2) the excitation amplitude range differs widely in the reported models; (3) the accurate dimensions of the majority of harvesters aren't well reported. Again, innovative implementations, such as the use

of rectification and storage circuits as the tip mass, significantly reduce the overall size. For those systems, the whole volume is not recommended for use in calculations.

Another extensively used criterion is efficiency, defined as the ratio of output electric power to input mechanical power. Despite agreement on the criteria, the literature's stated efficiency value varies significantly (Yang et al., 2017). Some (Kim et al., 2015) have reported efficiency of more than 80%, while some (Akaydin et al., 2012) have claimed efficiency lower than 1%, yet others (Shafer and Garcia, 2014) have advocated that efficiency can never surpass 50%. Yang et al. (2017) conducted a comprehensive investigation of efficiency and concluded that the disagreement is primarily due to different methods of estimating the input and output powers. It's important to remember that efficiency isn't the only factor to consider while evaluating harvester performance. It simply finds the energy conversion ratio and ignores the size effect and mechanical energy harvesting ability. High efficiency does not often imply high power generation. A PVEH system with a lower excitation and less damping usually provides high efficiency.

Aside from power density and efficiency, other more complex performance measurement standards have been proposed, which take into account more variables. Roundy (2005) offered a new criterion (effectiveness) to replace the concept of efficiency. The mechanical quality factor, mass density, coupling coefficient, and transmission coefficients are all considered when calculating the effectiveness. The functionality of the "effectiveness" is low because these parameters are dependent on many operating conditions.

In PVEHs, a high-power output isn't always the primary focus. Wide operating bandwidth is also investigated to achieve good environmental compliance and

reliability. Most of the time, the conditions for achieving these two aims are not in sync. PVEHs' mechanical damping effect is typically minimized for enhanced power generation; however, this comes at the cost of limited operating bandwidth. Mitcheson et al.(2008) suggested a bandwidth figure of merit to reflect both bandwidth and power output, which is the multiplication of fractional bandwidth and volume figure of merit (FoM<sub>v</sub>). In addition, Liu et al.(2015) presented a collective standard that considers both bandwidth and generated power. For broadband PVEHs, the suggested SFoM<sub>BW</sub> (figure of merit with bandwidth information) is based on an averaged FoM<sub>v</sub>, mechanical damping, density, and bandwidth.

Overall, numerous attempts have been found to redefine or adopt new standards for evaluating performance. There is no common standard that scholars agree on. The energy harvester performance is significantly influenced by the operating conditions, materials, dimensions, and circuitries used. In the past decades, different energy harvesters have been invented for various operating circumstances. Comparing diverse architectures and approaches with a single standard is neither feasible nor justified. Furthermore, it is sporadic for a publication to provide sufficient data to employ a sophisticated performance comparison standard. So, it is opined that end-user-relevant parameters, such as generated power, output voltage, volume, and mass, are of importance and should be emphasized in discussion. Moreover, the PD and VPM are taken as standards for comparing performance. They aren't universal norms, but they can be determined effortlessly.

## 2.5 Research Gaps

The previous sections presented a comprehensive literature review corresponding to various transduction methods, tapering cross-section PVEHs for transverse and rotational motion application, different linear and nonlinear mathematical formulation techniques, and ultimately, the reported performance comparison standards. From the literature reviews and keeping the motivations for the thesis in mind, the research gaps are identified. The research gaps are identified related to three areas: harvester shape, mathematical modeling of PVEHs, and experimental analysis techniques.

### *2.5.1 Research gaps related to harvester's shape*

The accumulated electric charge in the piezoelectric patch is proportional to the mechanical strain developed in the host beam of a PVEH. Due to inadequate strain distribution in the host structure, the piezoelectric potential remains underutilized, decreasing the effectiveness of most of the reported PVEHs. When the strain is uniformly distributed over the substrate structure, the PVEH's power generating capacity gets maximized.

Tapering the harvesting beam's cross-section improves the strain distribution in the piezoelectric patches and consequently increases the PD and VPM. Only a few researchers have acknowledged the linearly varying cross-section harvesters for rotational motion applications. Innovative designs such as parabolic tapering width are not considered for any PVEH applications, and neither is the exponential taper for rotational motion applications.

### *2.5.2 Research gaps related to mathematical modeling*

Mathematical modeling is essential in the design of optimal PVEH systems. The lumped parameter methodologies were used in most of the reported mathematical modeling of PVEH systems, which neglected piezoelectric systems' electromechanical coupling characteristics. The majority of the reported models include flaws in mathematical assumptions that lead to inaccuracy in performance evaluation and are only applicable to rectangular-shaped harvesters. Moreover, the exact solutions for tapering harvesters, thick cantilevers, and asymmetric systems are impossible to determine. As a result, in those instances, approximate solution approaches must be utilized. Furthermore, the geometry of the harvesting beam can be redesigned to optimize the fundamental operating frequency and output performance which causes nonlinearity in the system. The piezoelectric material also causes nonlinearity at high excitation levels. Therefore, developing a fully nonlinear model using approximate solution methods that includes both geometrical and piezoelectric material nonlinearities is of significant interest.

Designing a PVEH for rotational motion applications requires special modeling techniques, which should consider all the characteristic parameters of rotational motion and piezoelectric energy harvesting. The conventional mathematical model of RVEHs considered the host beam massless and the system's total mass as a single mass. They also implemented the approximate static solution procedures to predict the power response of the harvesters. Moreover, no proper mathematical model was found to study the effects of the variation of the characteristic parameters associated with the rotational motion and energy harvesting.

### 2.5.3 Research gaps related to experimental methods

The experimental analysis of any PVEH is essential to validate the mathematical model and check the proposed harvester's feasibility in practical applications. Prototype fabrication and setup development for the experimental investigation of PVEHs for rotational motion application are significant concerns for researchers. The tapering profile of the piezoelectric coupled beam is an add-on to the problem. Only one research group has proposed an experimental analysis of RVEH using only the linearly tapering beam so far.

## 2.6 Research Aims and Objectives

The aims of this thesis are threefold:

1. *To develop complete electromechanical coupled numerical models for the proposed parabolic and exponentially tapered PVEHs for both transverse and rotational motion applications using an approximate solution approach, with the goal of better understanding the dynamics of piezoelectric vibration energy harvesting devices. This aim has the following objectives:*

– To formulate the mathematical model taking account of the geometric and piezoelectric material nonlinearities (considering the nonlinear strain-displacement, piezoelectric constitutive, and electric displacement relations) for the appropriate evaluation of the harvesters' performance at higher transverse excitation levels.

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- To develop a multimodal mathematical model incorporating all the characteristic parameters associated with the rotational motion and the vibration energy harvesting to conduct a parametric analysis of the frequency responses and power generation performances.
2. *To validate the proposed numerical model of the RVEH using FE simulation and perform a comprehensive experimental validation of the PVEHs under both transverse and rotational motion. This aim has the following objectives:*
- To simulate both the parabolic and exponential tapered harvesters' operation under rotational excitation using ANSYS Mechanical APDL by utilizing the concept of the voltage coupling of the piezoelectric element.
  - To fabricate and assemble both the parabolic and exponentially tapered PVEHs and all other components of the harvesters.
  - To construct setups to test the harvesters' performances under rotational motion and transverse excitation in a wide range of excitation frequencies
  - To identify the critical excitation levels after which the linear formulations fail to provide accurate results
3. *To perform parametric analysis for both parabolic and exponential tapering PVEHs in order to explore and quantify the influence of the characteristic parameters on harvester performance. This aim has the following objectives:*
- To analyze the effects of taper parameter, piezo-patch thickness, and the tip load mass on the harvesters' performance and nonlinearity under transverse excitation.
  - To analyze the effects of taper parameter, piezo-patch to host beam thickness ratio, piezoelectric coupled beam's length, overall radius of rotation, and the tip

load mass on the harvesters' frequency response and voltage generation performance under rotational excitation.