PREFACE

The theory of error-correcting codes plays a vital role in modern communication systems, ensuring the reliability and efficiency of data transmission across noisy channels. Cyclic codes have emerged as one of the most important classes of error-correcting codes due to their ease of implementation, wide range of applications, and their connection to elegant algebraic structures. In the past few decades, the study of codes over rings has gained significant popularity due to their applications in several modern technologies, such as 5G networks and flash memory systems. This thesis is devoted to the study of certain algebraic codes like cyclic, constacyclic, skew cyclic, and skew constacyclic codes over specific finite non-chain rings. Additionally, it explores their applications in constructing quantum error-correcting codes, which have become increasingly important in the emerging fields of quantum information and computation. Moreover, it examines another significant class of error-correcting codes called linear complementary dual (LCD) codes over these rings.

Throughout this research journey, I have been deeply fascinated by the interplay between algebra, coding theory, and quantum computing. The development of quantum error-correcting codes is critical for realizing practical quantum computers, and this work aims to contribute to this exciting and rapidly evolving field.

This thesis consists of an **Introduction** and **seven chapters**, including **Preliminaries** as the first chapter and **Conclusion and Future Scope** as the last chapter. Chapter 1 provides a comprehensive overview of preliminary concepts, definitions, and results that will be useful for the subsequent chapters.

Introduction gives a general introduction. It also includes an extensive literature review on cyclic codes, their generalizations, and their applications in constructing quantum and LCD codes.

Chapter 1 serves as the foundation of the thesis, introducing essential concepts and definitions from ring theory, classical coding theory, and the theory of quantum error correction.

Chapter 2 studies cyclic codes over a non-chain ring $\mathbb{F}_q[\mathsf{u}, \mathsf{v}, \mathsf{w}]/\langle \mathsf{u}^3 - \mathsf{u}, \mathsf{v}^2 - \mathsf{v}, \mathsf{w}^2 - \mathsf{w}, \mathsf{uv}, \mathsf{vu}, \mathsf{uw}, \mathsf{wu}, \mathsf{vw} - \mathsf{wv}\rangle$ denoted as $\mathcal{S}$. This chapter discusses the structural properties of cyclic codes over $\mathcal{S}$ and their duals. It establishes some important results on the generator matrix of the Gray image of linear codes over $\mathcal{S}$ and the Gray image of cyclic codes over $\mathcal{S}$. Additionally, it presents the construction of quantum and LCD codes from cyclic codes over $\mathcal{S}$.

Chapter 3 investigates skew cyclic codes over a non-chain ring $\mathbb{F}_q[u_1, u_2, \ldots, u_r]/\langle u_i^3 - u_i, u_iu_j - u_ju_i\rangle_{i,j=1}^r$ denoted as $\mathcal{R}$. This chapter examines the structural properties of skew cyclic codes over $\mathcal{R}$ and their duals. Furthermore, it provides methods for constructing quantum and LCD codes from skew cyclic codes over $\mathcal{R}$, leading to the construction of many new codes with improved parameters.

Chapter 4 delves into skew constacyclic codes over a general non-chain ring $\mathbb{F}_q[u_1, u_2, \ldots, u_r]/\langle f_i(u_i), u_iu_j - u_ju_i\rangle_{i,j=1}^r$ denoted as $\mathcal{T}$. It presents several key results on the structural properties of skew constacyclic codes over $\mathcal{T}$. It is proved that the Gray image of a skew constacyclic codes over $\mathcal{T}$ is a skew quasi-$(\alpha_{11\ldots1}, \alpha_{11\ldots2}, \ldots, \alpha_{l_1l_2\ldots l_r})$-twisted code.

The findings of Chapter 4 are utilized in Chapter 5 to construct quantum and LCD codes. This chapter investigates the Euclidean and Hermitian duals of skew constacyclic codes over $\mathcal{T}$ develop methods for constructing quantum codes from Euclidean and Hermitian dual-containing skew constacyclic codes over $\mathcal{T}$. Moreover, quantum codes over nine different rings (belonging to the class of non-chain rings) are also constructed. Consequently, many new and improved quantum codes are obtained. Moreover, this chapter studies Euclidean and Hermitian skew constacyclic LCD codes over $\mathcal{T}$, leading to the construction of several MDS, AMDS, and BKLCs as Gray images of these codes.

The construction of quantum codes which are discussed in Chapters 2, 3, and 5 requires dual-containing codes. In 2006, Brun [21] proposed the construction of quantum codes using shared entanglement which does not require dual-containing codes. Chapter 6 focuses on the construction of entanglement-assisted quantum error-correcting codes (EAQECCs) from constacyclic codes over $\mathcal{T}$. It is proven that under a polynomial Gray map, the image of constacyclic codes over $\mathcal{T}$ is a cyclic code. Furthermore, a method to construct EAQECCs from these codes is established, resulting in many new EAQECCs.

Chapter 7 provides a chapter-wise summary of this thesis. It also outlines the future scope of the research in this direction and proposes several open problems for further investigation.

I hope that the results presented in this thesis will not only enhance the understanding of coding theory over finite non-chain rings but also inspire further research into the direction of generalizations of cyclic codes, their study over finite rings and construction of quantum codes from them.