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APPENDIX A

DERIVATION OF MATRIC SUCTION FOR STEADY-STATE FLOW AND GOVERNING DIFFERENTIAL EQUATION FOR TRANSIENT FLOW

A.1. Deriving the matric suction equation

Darcy's linear flow law establish the following relationship between discharge rate and matric suction (ψ):

$$q = k(\psi)\left(\frac{1}{\gamma_w}\frac{d\psi}{dz} - 1\right) \tag{A.1}$$

Gardner's (1958) one parameter HCF model is written as:

$$k(\psi) = k_s \exp(-\alpha\psi) \tag{A.2}$$

Substituting the expression of $k(\psi)$, Eq. (A.1) can be recast as:

$$q = k_s \exp(-\alpha\psi)\left[\frac{1}{\gamma_w}\left(\frac{d\psi}{dz}\right) - 1\right] \tag{A.3}$$

$$\Rightarrow Q = \exp(-\alpha\psi)\frac{1}{\gamma_w}\left[\left(\frac{d\psi}{dz}\right) - \gamma_w\right] \quad \text{where, } Q = q/k_s \tag{A.4}$$

$$\Rightarrow \gamma_w[\exp(-\alpha\psi) + Q]dz = \exp(-\alpha\psi)d\psi \tag{A.5}$$

$$\Rightarrow dz = \frac{\exp(-\alpha\psi)d\psi}{\gamma_w[\exp(-\alpha\psi) + Q]} \tag{A.6}$$

Upon integrating both sides, the relationship between ψ and z in the vadose zone (the zone bounded by the water table and the ground surface) can be expressed as follows:

$$\int_0^z dz = \int_0^\psi \frac{\exp(-\alpha\psi)d\psi}{\gamma_w[\exp(-\alpha\psi) + Q]} \tag{A.7}$$

The right hand side of the Eq. (A.7) can be solved by using method of substitution which is demonstrated as follows:

Let, $u = \exp(-\alpha\psi) + Q$ (A.8)

After differentiating both sides of Eq. (A.8) with respect to ψ, following expression can be obtained.

$$\begin{aligned} du &= -\alpha \exp(-\alpha\psi)d\psi \\ \Rightarrow -\frac{1}{\alpha}du &= \exp(-\alpha\psi)d\psi \end{aligned} \tag{A.9}$$

The lower limit and upper limit of the definite integral which is depicted at the right hand side of the Eq. (A.7) can be changed in terms of *u*. The limit transformation is presented as follows:

Table A.1 Integration Limit transformation for ψ and *u*

ψ	0	ψ
u	$1+Q$	$\exp(-\alpha\psi)+Q$

Substituting Eq. (A.8) and Eq. (A.9) into Eq. (A.7) and transforming limits, following expression can be presented.

$$\int_0^z dz = -\frac{1}{\alpha\gamma_w}\int_{1+Q}^{\exp(-\alpha\psi)+Q}\frac{du}{u} \tag{A.10}$$

$$\begin{aligned} &\Rightarrow -\alpha\gamma_w z = \ln[\exp(-\alpha\psi)+Q] - \ln(1+Q) \\ &\Rightarrow -\alpha\gamma_w z = \ln\frac{\exp(-\alpha\psi)+Q}{1+Q} \\ &\Rightarrow \exp(-\alpha\gamma_w z) = \frac{\exp(-\alpha\psi)+Q}{1+Q} \\ &\Rightarrow \exp(-\alpha\psi)+Q = (1+Q)\exp(-\alpha\gamma_w z) \\ &\Rightarrow -\alpha\psi = \ln[(1+Q)\exp(-\alpha\gamma_w z) - Q] \end{aligned}$$

$$\Rightarrow \psi = -\frac{1}{\alpha}\ln\left[(1+Q)\exp(-\alpha\gamma_w z) - Q\right] \tag{A.11}$$

A.2. Deriving the governing differential equation

Richards' equation: $\frac{\partial}{\partial z}\left[k(h_m)\left(\frac{\partial h_m}{\partial z}+1\right)\right] = \frac{\partial\theta}{\partial t}$ (A.12)

Gardener's one parameter HCF model: $k = k_s \exp(\alpha h_m)$ (A.13)

Gardener's SWCC model: $\Theta_n = S_e = \frac{\theta-\theta_r}{\theta_s-\theta_r} = \exp(\alpha h_m)$ (A.14)

Eq. (A.14) can be further simplified as follows:

$$\theta = \theta_r + (\theta_s - \theta_r)\exp(\alpha h_m) \tag{A.15}$$

Performing partial differentiation of Eq. (A.15) with respect to time (*t*), following expression can be obtained:

$$\frac{\partial\theta}{\partial t} = \alpha(\theta_s-\theta_r)\exp(\alpha h_m)\frac{\partial h_m}{\partial t} \tag{A.16}$$

Substituting Eq. (A.12) and Eq. (A.16) into Eq. (A.12), following differential equation can be obtained:

$$k_s\left[\exp(\alpha h_m)\frac{\partial^2 h_m}{\partial z^2} + \alpha\exp(\alpha h_m)\left(\frac{\partial h_m}{\partial z}\right)^2\right] + \alpha k_s\exp(\alpha h_m)\frac{\partial h_m}{\partial z} = \alpha(\theta_s-\theta_r)\exp(\alpha h_m)\frac{\partial h_m}{\partial t} \tag{A.17}$$

$$\Rightarrow k_s\exp(\alpha h_m)\left[\left\{\frac{\partial^2 h_m}{\partial z^2} + \alpha\left(\frac{\partial h_m}{\partial z}\right)^2\right\} + \alpha\frac{\partial h_m}{\partial z}\right] = \alpha(\theta_s-\theta_r)\exp(\alpha h_m)\frac{\partial h_m}{\partial t} \tag{A.18}$$

$$\Rightarrow\left[\frac{\partial^2 h_m}{\partial z^2}+\alpha\left(\frac{\partial h_m}{\partial z}\right)^2\right]+\alpha\frac{\partial h_m}{\partial z}=\frac{\alpha(\theta_s-\theta_r)}{k_s}\frac{\partial h_m}{\partial t}\quad\text{(Obtained from GD-SWCC)}\tag{A.19}$$

vG-SWCC model: $$\Theta_n=S_e=\frac{\theta-\theta_r}{\theta_s-\theta_r}=\left[\frac{1}{1+(\alpha h_m)^n}\right]^m\tag{A.20}$$

Eq. (A.20) can be further simplified as follows:

$$\theta=\theta_r+(\theta_s-\theta_r)\left[\frac{1}{1+(\alpha h_m)^n}\right]^m\tag{A.21}$$

Executing partial differentiation of Eq. (A.21) with respect to time (*t*), following expression can be obtained:

$$\frac{\partial\theta}{\partial t}=-mn\alpha(\theta_s-\theta_r)\left[1+(\alpha h_m)^n\right]^{-(m+1)}(\alpha h_m)^{n-1}\frac{\partial h_m}{\partial t}\tag{A.22}$$

Substituting Eq. (A.13) and Eq. (A.15) into Eq. (A.12), following differential equation can be obtained:

$$\begin{aligned}&k_s\left[\exp(\alpha h_m)\frac{\partial^2 h_m}{\partial z^2}+\alpha\exp(\alpha h_m)\left(\frac{\partial h_m}{\partial z}\right)^2\right]\\&+\alpha k_s\exp(\alpha h_m)\frac{\partial h_m}{\partial z}=-mn\alpha(\theta_s-\theta_r)\left[1+(\alpha h_m)^n\right]^{-(m+1)}(\alpha h_m)^{n-1}\frac{\partial h_m}{\partial t}\end{aligned}\tag{A.23}$$

$$\Rightarrow k_s\exp(\alpha h_m)\left[\left\{\frac{\partial^2 h_m}{\partial z^2}+\alpha\left(\frac{\partial h_m}{\partial z}\right)^2\right\}+\alpha\frac{\partial h_m}{\partial z}\right]=-mn\alpha(\theta_s-\theta_r)\frac{(\alpha h_m)^{n-1}}{\left[1+(\alpha h_m)^n\right]^{(m+1)}}\frac{\partial h_m}{\partial t}\tag{A.24}$$

$$\Rightarrow\left[\frac{\partial^2 h_m}{\partial z^2}+\alpha\left(\frac{\partial h_m}{\partial z}\right)^2\right]+\alpha\frac{\partial h_m}{\partial z}=-\frac{mn\alpha(\theta_s-\theta_r)(\alpha h_m)^{n-1}}{k_s\exp(\alpha h_m)\left[1+(\alpha h_m)^n\right]^{(m+1)}}\frac{\partial h_m}{\partial t}\quad\text{(vG-SWCC)}\tag{A.25}$$

A.3. Numerical Scheme for solving the IVP

Governing differential equation:

$$\left[\frac{\partial^2 h_m}{\partial z^2}+\alpha\left(\frac{\partial h_m}{\partial z}\right)^2\right]+\alpha\frac{\partial h_m}{\partial z}=\lambda\frac{\partial h_m}{\partial t} \tag{A.26}$$

For GD SWCC: $\lambda=\lambda^{GD}=\dfrac{\alpha(\theta_s-\theta_r)}{k_s}$

For vG SWCC: $\lambda=\lambda^{vG}=-\dfrac{mn\alpha(\theta_s-\theta_r)(\alpha h_m)^{n-1}}{k_s\exp(\alpha h_m)\left\{1+[\alpha h_m]^n\right\}^{m+1}}$

Further, Eq. (A.15) can be represented in the following form:

$$\left[(1-\theta)\frac{\partial^2 h_m}{\partial z^2}\bigg|_{t+\Delta t}+\theta\frac{\partial^2 h_m}{\partial z^2}\bigg|_{t+\Delta t}+\alpha\left(\frac{\partial h_m}{\partial z}\bigg|_{t}\right)^2\right]+\alpha\frac{\partial h_m}{\partial z}\bigg|_{t}=\lambda\frac{\partial h_m}{\partial t} \tag{A.27}$$

In terms of Crank-Nicolson finite difference terms (θ =0.5), the generalized equations for any arbitrary i^{th} node can be written as:

$$\left[\frac{h_{m,i-1}^{t+\Delta t}-2h_{m,i}^{t+\Delta t}+h_{m,i+1}^{t+\Delta t}}{2\Delta z^2}+\frac{h_{m,i-1}^{t}-2h_{m,i}^{t}+h_{m,i+1}^{t}}{2\Delta z^2}+\alpha\left[\frac{h_{m,i+1}^{t}-h_{m,i}^{t}}{\Delta z}\right]^2\right]+\alpha\left[\frac{h_{m,i+1}^{t+\Delta t}-h_{m,i}^{t+\Delta t}}{\Delta z}\right]=\lambda\frac{h_{m,i}^{t+\Delta t}-h_{m,i}^{t}}{\Delta t} \tag{A.28}$$

$$\Rightarrow -m_1h_{m,i-1}^{t+\Delta t}+\left(1+2m_1+m_2\right)h_{m,i}^{t+\Delta t}-\left(m_1+m_2\right)h_{m,i+1}^{t+\Delta t}=h_{m,i}^{t}+m_1\left(h_{m,i-1}^{t}-2h_{m,i}^{t}+h_{m,i+1}^{t}\right)+\alpha m_1\left(h_{m,i+1}^{t}-h_{m,i}^{t}\right)^2$$

Here, $h_{m,i+1}^{t}, h_{m,i}^{t}$, and $h_{m,i-1}^{t}$ are the matric suction head at time t corresponding to i+1, i, and i-1 grid-points.

$$m_1=\frac{\Delta t}{2\lambda\Delta z^2},\ m_2=\frac{\alpha\Delta t}{\lambda\Delta z}$$

Top boundary constraint yields the following algebraic relation:

$$k_s\exp\left(\alpha h_m\right)\left(\frac{\partial h_m}{\partial z}+1\right)\bigg|_{@(n+1)^{\text{th}}\text{ node}}=q$$

$$\Rightarrow\left(\frac{\partial h_m}{\partial z}\bigg|_{n+1}+1\right)=\frac{q}{k_s}\exp\left(-\alpha h_{m,n+1}\right)\Rightarrow\frac{\partial h_m}{\partial z}\bigg|_{n+1}=Q\exp\left(-\alpha h_{m,n+1}\right)-1$$

$$\Rightarrow \frac{h_{m,n+2}^{t+\Delta t}-h_{m,n}^{t+\Delta t}}{2\Delta z}=Q\exp\left(-\alpha h_{m,n+1}^{t}\right)-1\text{; } Q \text{ (flow ratio)} = q/k_s\text{ ;}$$

$$\Rightarrow h_{m,n+2}^{t+\Delta t}=h_{m,n}^{t+\Delta t}+2\Delta zQ\exp\left(-\alpha h_{m,n+1}^{t}\right)-2\Delta z \tag{A.29}$$

The simultaneous set of linear equation can be rewritten as: $\boldsymbol{AH}_m^{t+\Delta t}=\boldsymbol{B}^t$ (A.30)

$$\boldsymbol{H}_m^{t+\Delta t}=\begin{bmatrix}h_{m,1}^{t+\Delta t} & h_{m,2}^{t+\Delta t} & \dots & \dots & h_{m,i-1}^{t+\Delta t} & h_{m,i}^{t+\Delta t} & h_{m,i+1}^{t+\Delta t} & \dots & \dots & h_{m,n}^{t+\Delta t} & h_{m,n+1}^{t+\Delta t}\end{bmatrix}_{1\times(n+1)}^{T} \tag{A.31a}$$

$$\boldsymbol{B^t}=\begin{bmatrix}B_1 & B_2 & \dots & \dots & B_{i-1} & B_i & B_{i+1} & \dots & \dots & B_n & B_{n+1}\end{bmatrix}_{1\times(n+1)}^{T} \tag{A.31b}$$

$$B_1=0\ ;\ \ B_i=h_{m,i}^t+m_1\left(h_{m,i-1}^t-2h_{m,i}^t+h_{m,i+1}^t\right)+\alpha m_1\left(h_{m,i+1}^t-h_{m,i}^t\right)^2;$$

$$B_n=h_{m,n}^t+m_1\left(h_{m,n-1}^t-2h_{m,n}^t+h_{m,n+1}^t\right)+\alpha m_1\left(h_{m,n+1}^t-h_{m,n}^t\right)^2$$

$$B_{n+1}=h_{m,n+1}^t+p\left(m_1+m_2\right)+m_1\left(2h_{m,n}^t-2h_{m,n+1}^t+p\right)+\alpha m_1\left[h_{m,n}^t+p-h_{m,n+1}^t\right]^2$$

$$p=2\Delta\text{z}\left(Q\exp\left(-\alpha h_{m,n+1}^t\right)-1\right)$$

$$\boldsymbol{A}=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0\\
-m_1 & \left(1+2m_1+m_2\right) & -\left(m_1+m_2\right) & 0 & 0 & 0 & 0\\
\dots & \dots & \dots & \dots & \dots & \dots & \dots\\
0 & 0 & -m_1 & \left(1+2m_1+m_2\right) & -\left(m_1+m_2\right) & \dots & 0\\
\dots & \dots & \dots & \dots & \dots & \dots & \dots\\
\dots & \dots & \dots & \dots & \dots & \dots & \dots\\
\dots & \dots & \dots & \dots & \dots & \dots & \dots\\
0 & 0 & 0 & 0 & -m_1 & \left(1+2m_1+m_2\right) & -\left(m_1+m_2\right)\\
0 & 0 & 0 & 0 & 0 & -\left(2m_1+m_2\right) & \left(1+2m_1+m_2\right)
\end{bmatrix}_{\left(\substack{n+1\\ \times n+1}\right)} \tag{A.32}$$

Eq. (A.30) is further solved by employing Gauss Elimination scheme.

$$\begin{array}{rcl}
\boldsymbol{AH}_m^{t+\Delta t} & = & \boldsymbol{B}^t\\
\boldsymbol{AH}_m^{t+2\Delta t} & = & \boldsymbol{B}^{t+\Delta t}\\
\dots & \Bigg\downarrow & \dots\\
\dots & & \dots\\
\dots & & \dots\\
\boldsymbol{AH}_m^{t+n\Delta t} & = & \boldsymbol{B}^{t+(n-1)\Delta t}
\end{array}$$

DERIVATION OF MUALEM'S AND BURDINE'S HCF MODELS

The van Genuchten's (1980) SWCC model is expressed as follows:

$$\Theta_n=\frac{1}{\left[1+(\alpha\psi)^n\right]^m}$$

The above expression can be easily inverted as $\psi=f(\Theta_n)$

$\Rightarrow\psi=\frac{1}{\alpha}\left(\Theta^{-1/m}-1\right)^{1/n}$ where, ψ =matric suction; Θ_n= normalized water content

k can be expressed as a function of Θ_n.

Table B.1 Derivation of Mualem's HCF and Burdine's HCF models

Mualem's HCF model (1976)	Burdine's HCF model (1953)
$K_r=\frac{k(\Theta_n)}{k_s}=\Theta_n^{1/2}\left[\int_0^{\Theta_n}\frac{1}{\psi(x)}dx\Big/\int_0^{1}\frac{1}{\psi(x)}dx\right]^2$ $\Rightarrow K_r=\Theta^{1/2}\left[\int_0^{\Theta}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{1/n}dx\Big/\int_0^{1}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{1/n}dx\right]$ $\Rightarrow K_r=\Theta^{1/2}\left[\frac{f(\Theta)}{f(1)}\right]^2$ $f(\Theta)=\int_0^{\Theta}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{1/n}dx$; Let, $x^{1/m}=y\Rightarrow dx=my^{m-1}dy$ $f(\Theta)=m\int_0^{\Theta^{1/m}}\left(\frac{y}{1-y}\right)^{1/n}y^{m-1}dx$ $\Rightarrow f(\Theta)=m\int_0^{\Theta^{1/m}}y^{m-1+1/n}(1-y)^{-1/n}dy$ To obtain analytical solution assume, $m-1+1/n=R_M$ (B.1)	$K_r=\frac{k(\Theta_n)}{k_s}=\Theta_n^2\left[\int_0^{\Theta_n}\frac{1}{\psi^2(x)}dx\Big/\int_0^{1}\frac{1}{\psi^2(x)}dx\right]$ $\Rightarrow K_r=\Theta^2\left[\int_0^{\Theta}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{2/n}dx\Big/\int_0^{1}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{2/n}dx\right]$ $\Rightarrow K_r=\Theta^2\left[\frac{f(\Theta)}{f(1)}\right]$ $f(\Theta)=\int_0^{\Theta}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{2/n}dx$ Let, $x^{1/m}=y\Rightarrow dx=my^{m-1}dy$ $f(\Theta)=m\int_0^{\Theta^{1/m}}\left(\frac{y}{1-y}\right)^{2/n}y^{m-1}dx$ $\Rightarrow f(\Theta)=m\int_0^{\Theta^{1/m}}y^{m-1+2/n}(1-y)^{-2/n}dy$ To obtain analytical solution let, $m-1+2/n=R_B$ (B.2)
Note: K_r= relative hydraulic conductivity; k_s= saturated hydraulic conductivity. R_M and R_B vary from 0 to 5 with an interval of 1.	

APPENDIX C

DERIVATION OF MATRIC SUCTION AND SUCTION STRESS BY USING TWO PARAMETER HCF MODEL

By substituting Gardner’s TPM (Eq. 8.6) into Darcy’s linear flow (Eq. A.1) the following expressions as follows:

$$dz = -\frac{d\psi}{\left(1+\frac{q}{k_s}\right)+\frac{q\alpha}{k_s}\psi^{\xi}} = \frac{d\psi}{A+B\psi^{\xi}} \quad \text{; where, } A=1+Q;\ B=Q\alpha;\ Q=q/k_s \qquad \text{(C.1)}$$

Table C.1 Derivation of matric suction and suction stress for different values of fitting parameter (ξ)

ξ=1	$\int_0^z dz = -\int_0^\psi \frac{d\psi}{A+B\psi} \Rightarrow z = \frac{1}{B}\ln\frac{A}{A+B\psi} \Rightarrow z = \frac{1}{Q\alpha}\ln\frac{1+Q}{(1+Q)+Q\alpha\psi} \Rightarrow \frac{1+Q}{(1+Q)+Q\alpha\psi} = \exp(Q\alpha\, z) \Rightarrow Q\alpha\psi = (1+Q)[\exp(-Q\alpha z)-1]$ $\boxed{\psi = \frac{1+Q}{Q\alpha}[\exp(-Q\alpha z)-1]} \Rightarrow \boxed{\sigma^s = -\gamma_w \frac{1+Q}{Q\alpha}[\exp(-Q\alpha z)-1] \times \frac{1}{\left(1+\left\{-\alpha\gamma_w \frac{1+Q}{Q\alpha}[\exp(-Q\alpha z)-1]\right\}^n\right)^m}}$ (C.2)
ξ=2	$\int_0^z dz = -\int_0^\psi \frac{d\psi}{A+B\psi^2} \Rightarrow z = -\frac{1}{\sqrt{AB}}\tan^{-1}\left(\sqrt{\frac{B}{A}}\psi\right) \Rightarrow \sqrt{\frac{B}{A}}\psi = \tan\left(-\sqrt{AB}z\right) \Rightarrow \psi = \sqrt{\frac{A}{B}}\tan\left(-\sqrt{AB}z\right) \Rightarrow \boxed{\psi = \sqrt{\frac{1+Q}{Q\alpha}}\tan\left[-\sqrt{(1+Q)(Q\alpha)}z\right]}$ $\boxed{\sigma^s = -\gamma_w\sqrt{\frac{1+Q}{Q\alpha}}\tan\left[-\sqrt{(1+Q)(Q\alpha)}z\right] \times \frac{1}{\left(1+\left\{\alpha\gamma_w\sqrt{\frac{1+Q}{Q\alpha}}\tan\left[-\sqrt{(1+Q)(Q\alpha)}z\right]\right\}^n\right)^m}}$ (C.3)
ξ=3	$\int_0^z dz = -\int_0^\psi \frac{d\psi}{A+B\psi^3} \Rightarrow z = -\frac{1}{A^{2/3}B^{1/3}}\left[\frac{1}{3}\log\left\|1+\sqrt[3]{\frac{B}{A}}\psi\right\| - \frac{1}{6}\log\left\|\left(\frac{B}{A}\right)^{2/3}\psi^2 - \left(\frac{B}{A}\right)^{1/3}\psi+1\right\| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}\left(\sqrt[3]{\frac{B}{A}}\psi-\frac{1}{2}\right)\right)\right]_0^\psi$ $\boxed{z = -\frac{1}{A^{2/3}B^{1/3}}\left[\frac{1}{3}\log\left\|1+\sqrt[3]{\frac{B}{A}}\psi\right\| - \frac{1}{6}\log\left\|\left(\frac{B}{A}\right)^{2/3}\psi^2 - \left(\frac{B}{A}\right)^{1/3}\psi+1\right\| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}\left(\sqrt[3]{\frac{B}{A}}\psi-\frac{1}{2}\right)\right)\right] - \frac{1}{A^{2/3}B^{1/3}}\times\frac{\pi}{6}}$ (C.4)

ξ=4	$\int_0^z dz = -\int_0^\psi \frac{d\psi}{A+B\psi^4} \Rightarrow \boxed{z = \frac{1}{\sqrt[4]{A^3B}}\left[\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{B}{A}}\psi^2 - 1}{\sqrt{2}\sqrt[4]{\frac{B}{A}}\psi}\right) - \frac{1}{4\sqrt{2}}\log\left\lvert\frac{\sqrt{\frac{B}{A}}\psi^2 - \sqrt{2}\sqrt[4]{\frac{B}{A}} + 1}{\sqrt{\frac{B}{A}}\psi^2 + \sqrt{2}\sqrt[4]{\frac{B}{A}} + 1}\right\rvert\right]}$	(C.5)
ξ=5	$\int_0^z dz = -\int_0^\psi \frac{d\psi}{A+B\psi^5} \Rightarrow \boxed{z = -\frac{1}{A^{4/5}B^{1/5}}(D-E+F+G-H+I) + \frac{1}{A^{4/5}B^{1/5}}(J+L)}$ where, $D = \frac{1}{5}\log\left\lvert 1 + \sqrt[5]{\frac{B}{A}}\psi\right\rvert$; $E = \frac{1}{20}\log\left\lvert\left(\frac{B}{A}\right)^{4/5}\psi^4 - \left(\frac{B}{A}\right)^{3/5}\psi^3 - \left(\frac{B}{A}\right)^{1/5}\psi + 1\right\rvert$ $F = \frac{1}{4\sqrt{5}}\log\left\lvert\left(\frac{B}{A}\right)^{2/5}\psi^2 + \left(\frac{\sqrt{5}-1}{2}\right)\times\left(\frac{B}{A}\right)^{1/5}\psi + 1\right\rvert$; $G = 0.475\tan^{-1}\left(\frac{4\left(\frac{B}{A}\right)^{\frac{1}{5}}\psi + \sqrt{5} - 1}{\sqrt{10+2\sqrt{5}}}\right)$; $H = \frac{1}{4\sqrt{5}}\log\left\lvert\left(\frac{B}{A}\right)^{2/5}\psi^2 - \left(\frac{\sqrt{5}+1}{2}\right)\times\left(\frac{B}{A}\right)^{1/5}\psi + 1\right\rvert$; $I = 0.083\tan^{-1}\left(\frac{4\left(\frac{B}{A}\right)^{\frac{1}{5}}\psi - \sqrt{5} - 1}{\sqrt{10-2\sqrt{5}}}\right)$; $J = 0.475\tan^{-1}\left(\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)$; $L = 0.083\tan^{-1}\left(\frac{-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)$	(C.6)

Note: From Eqs. C.4 to C.6, ψ is calculated by using Newton- Raphson method as well as Gauss-Newton iteration technique and corresponding σ^s is evaluated from Eq. (6.8b).