CHAPTER – IV

IMPROVED ANALYTICAL DESIGN OF FRACTIONAL ORDER PI CONTROLLER FOR NON-MONOTONIC PHASE SYSTEMS

INTRODUCTION

The concept of extending an integer order calculus to non-integer order calculus by no means it is a new topic. Fractional calculus having a history of more than 300 years, but its research and applications in physics and engineering has been developed in recent years largely. By early sixties besides the theoretical research in the field of fractional derivatives and integrals, there is growing number of application in the various fields of control engineering. Oustaloup et al (1991) is the first person to introduce the fractional order controller in feedback control systems. He developed the so called commande robuste d ordre non entier (CRONE) controller which is used in different fields of control systems. In recent years podlubny et al (1999) proposed a generalization of PID controller called fractional $PI^{\lambda}D^{\mu}$ controller, involving λ (integration action of fractional order) μ (differentiation action of fractional order). In fractional $PI^{\lambda}D^{\mu}$ controller the fractional I and D actions being fractional have wider scope of design. In the literature Padula et al (2011), Valerio et al (2005c), Wang et al (2009), Yeroglu et al (2011), Zhang et al (2012), Luo et al (2010), Luo et al (2010) several attempts has been found on tuning of the fractional order controller for specific class of plants. The main principle behind the frequency response compensation of an LTI system is that all amplified frequencies in open loop transfer function can make a control action in closed loop system with negative feedback Skogestad et al (2005).

For a closed loop system the distance between the open loop phase and -180^{o} line, at the frequency where the gain crosses 0 db line is the phase margin. According to the Bode stability criterion, if this phase margin is positive then the system is stable. Thus the phase margin is an important robustness indicator which shows how much the open loop phase may vary while the open loop system remains stable Ogata et al (2009). Also, it is related to the damping ratio of a second order system. Higher the damping ratio is, smaller the peak overshoot will be. The block diagram of the plant with the controller is shown in the Figure 4.1. Here the control system is unitary and negative feedback system. The bandwidth here will be the frequency range between from zero to the gain crossover frequency. The classical frequency response control design defines that the phase margin is always at the gain crossover frequency, however practically it is not valid since if the left half plane zero is present near the left half plane dominant pole then the phase curve will exhibit a non-monotonic behavior in the frequency response.

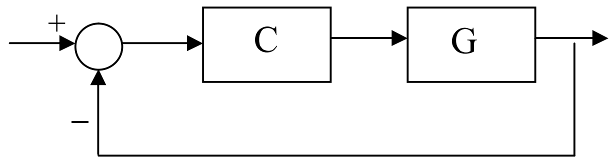


Figure 4.1 Block diagram of unity negative feedback control system

Monotonicity means consistently increasing and never decreasing or consistently decreasing and never increasing in value. So a non monotonic system is that which has increasing phase somewhere and decreasing phase somewhere. In the frequency response of the non-monotonically decreasing phase system we will observe a valley in the phase curve close to the undamped frequency, which is a point of minima in the bandwidth. This is the worst case phase of the system and the frequency where this point of minima is located is the worst case phase frequency. This type of phase results from the fact that the left half plane zero is not located far from the left half pole (de Paula et al (2012)).

This worst case frequency is always smaller than the gain crossover frequency since it is in the bandwidth of the system and also the worst case phase margin is always less than the phase margin. So if this worst case phase margin is positive then the phase margin is also positive. In this chapter, a fractional order proportional integral controller has proposed and designed to control a monotonic and non-monotonic phase system.

The remaining part of the chapter is organized as follows: in Section 4.1, the description of a non-monotonic system is given In Section 4.2 the proposed method has been discussed. In section 4.3, a FOPI controller and IOPID controller has been designed for non-monotonic system. The results has been discussed in section 4.4. Finally, the conclusions are drawn in section 4.5.

4.1 OBJECTS OF STUDY

Consider the following equation which will be used for the modeling of the continuous LTI dynamical system that will be studied in this brief.

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_1 \dot{u}(t) + b_0 u(t) \tag{4.1}$$

The systems that can be modeled by above differential equation are, an LC low pass filter, or a two ambient-coupled heat tank. A linearized model of synchronous buck regulator is considered. The buck regulator is a combination of the power stage, which is given by an LC low pass filter and a pulse width modulation (PWM) controller (International Rectifiers (2002)).The transfer function of the considered buck converter is

$$G(s) = \frac{V_{IN}}{V_{OSC}} \frac{1 + sR_C C}{LCs^2 + s(R_C C + \frac{L}{R}) + 1} \tag{4.2}$$

Where

C=output capacitance

L=output inductance

R= load resistance

R_C= output Capacitor intrinsic resistance

V_{OSC}= PWM oscillator reference

V_{IN}= power stage input voltage

A buck regulator which is suggested in International Rectifiers (2002) yields the following transfer function with R_C= 40-mΩ and other corresponding values

$$G(s) = \frac{4(1 + 1.2 \times 10^{-5} s)}{3 \times 10^{-9} s^2 + 3.6 \times 10^{-5} s + 1} \tag{4.3}$$

Here the undamped frequency is approximately 18 krad/sec and the damping ratio is 0.33. A left half plane zero is located at 83krad/sec which is almost five times larger than the undamped frequency. The Bode plot of this system is shown in the Figure 4.2 where we can see a valley in the phase curve. The occurrence of this valley is due to the fact that the left half plane zero due to the output capacitor intrinsic impedance is not far from the poles. Thus, if there is any zero frequency after the bandwidth, it would lead to a non-monotonic type of compensation. The transfer function would still remain minimum phase and the Bode stability criterion could be used. The Bode plot of the system is shown in the Figure 4.2.

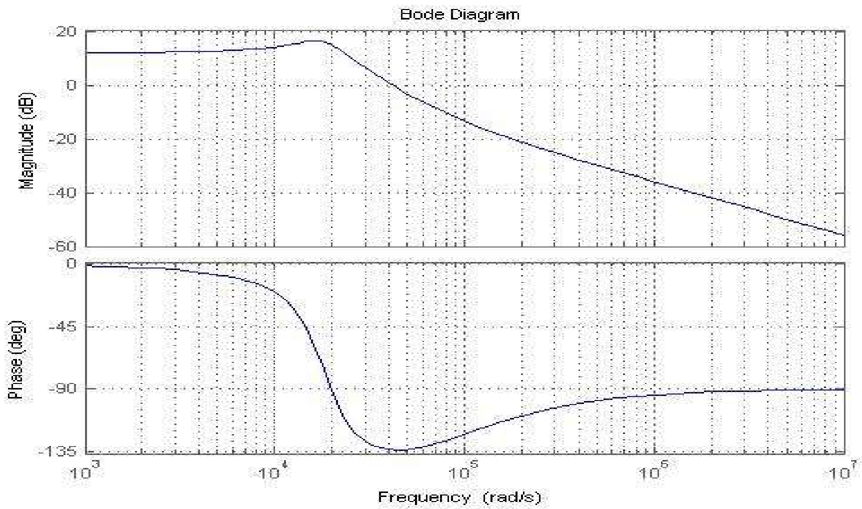


Figure 4.2- frequency response of the plant

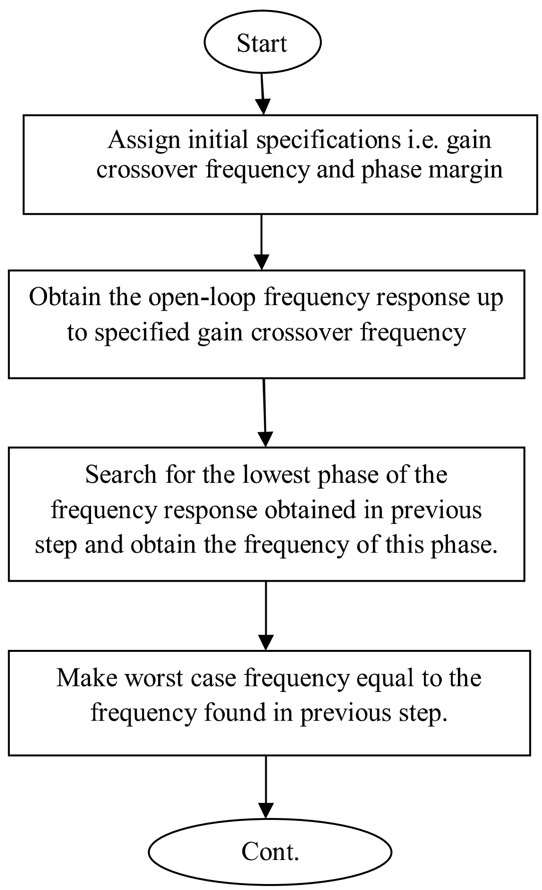
4.2 PROPOSED METHOD

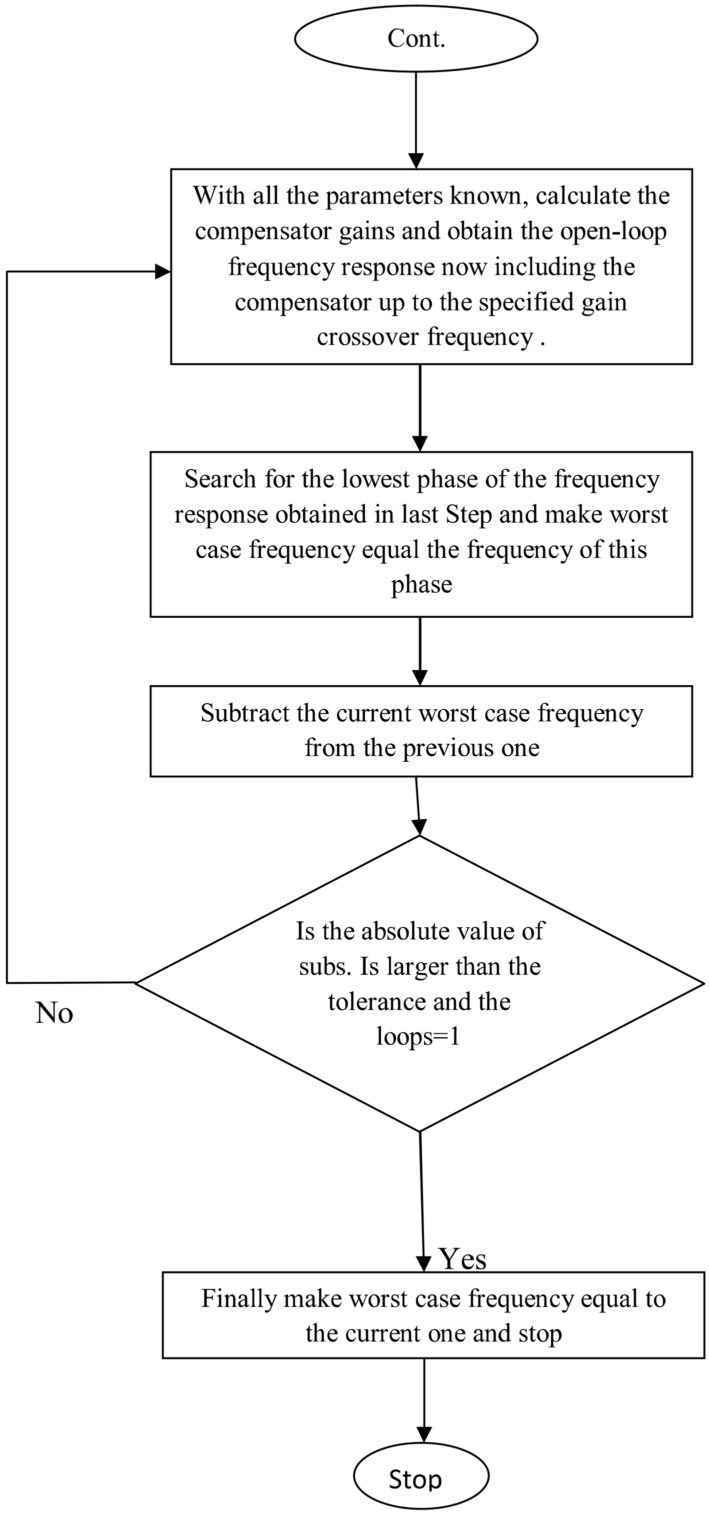
The proposed method is an analytical design procedure for the fractional order PI controller that will give a designer specified worst case phase margin and gain crossover frequency for a stable and minimum phase open loop control system. As discussed earlier the phase margin and the gain crossover frequency is a powerful robustness measure and is closely related to the damping ratio and the undamped frequency. In the proposed method we assume that the controller shapes the negative feedback control system into a low pass behaviour in the open loop frequency response.

For this method we need to define four parameters, i.e., gain crossover frequency(ω_u), phase margin(ϕ_u), worst case frequency(ω_m) and the worst case phase margin(ϕ_m). The gain crossover frequency(ω_u) is the characteristic equation solution in the frequency domain, phase margin(ϕ_u) is the distance between the phase curve and the -180^o line at the gain crossover frequency. The worst case frequency(ω_m) is the frequency where the system exhibits the lowest phase inside the bandwidth and the worst case phase(ϕ_m) is the distance between the -180^o line and the phase curve at the worst case frequency. The worst case frequency is smaller than or equal to the gain crossover frequency, and thus the worst case phase margin is equal to or smaller than the phase margin. So if worst case phase margin is positive the phase margin will also be positive and the system will be stable.

$$\phi_m \leq \phi_u \tag{4.4}$$

$$\omega_m \leq \omega_u \tag{4.5}$$




Cont.
With all the parameters known, calculate the compensator gains and obtain the open-loop frequency response now including the compensator up to the specified gain crossover frequency .
Search for the lowest phase of the frequency response obtained in last Step and make worst case frequency equal the frequency of this phase
Subtract the current worst case frequency from the previous one
Is the absolute value of subs. Is larger than the tolerance and the loops=1
No
Yes
Finally make worst case frequency equal to the current one and stop
Stop

The gain crossover frequency and the worst case phase margin will be free specifications but the worst case frequency has to be found in a recursive way, since the compensator gains will modify the open loop system. To find out the worst case frequency a search procedure has to be followed de Paula et al (2012). This search procedure is explained by the help of a flowchart

The characteristic equation of the plant is given by

$$|K(j\omega_u)G(j\omega_u)|\angle K(j\omega_u)G(j\omega_u) = 1\angle -180^0 + \phi_u \quad (4.6)$$

This equation has a magnitude part and a phase part. This equation defines the phase margin and the gain crossover frequency. As we have discussed that ω_u is always greater than ω_m, and also ϕ_u is greater than ϕ_m. So if we make ϕ_m positive then ϕ_u will also be positive. Thus, in the phase part of the equations we can replace ω_u by ω_m which will give the system the worst case stability and the system will be stable throughout the bandwidth.

$$|K(j\omega_u)| = \frac{1}{|G(j\omega_u)|} \quad (4.7)$$

$$\angle K(j\omega_u) = -180^0 + \phi_m - \angle G(j\omega_m) \quad (4.8)$$

4.3 DESIGNING OF THE FOPI AND IOPID CONTROLLER

For fair comparison between the conventional method and the proposed method, we need to design the FOPI controller Luo et al (2009) and IOPID for the buck regulator by both methods. The transfer function for the FOPI controller and IOPID controller is

$$C_1(s) = K_p\left(1 + \frac{K_i}{s^{\lambda}}\right) \quad (4.9)$$

$$C_2(s) = K_p + \frac{K_i}{s} + K_d s \quad (4.10)$$

To design the fractional order controller, first of all we have to find the worst case frequency. As discussed in the previous section, we first assign the initial specification i.e. the gain crossover frequency and the desired phase margin. The desired phase margin is 40°. As seen from the transfer function, we observe that the undamped frequency is approximately 18krad/sec and the left half plane zero is approximately 83krad/sec. The switching frequency of the converter is equal to 200 kHz (approximately 1250 krad/sec). So, it is suggested that the gain crossover frequency should be approximately equal to one fifth of this value. So the gain crossover frequency is 250krad/sec.

Now, with these initial specifications, we start the search for worst case frequency following the flowchart given in the previous section. When the search is complete the worst case frequency comes out to be 40 krad/sec. For designing the FOPI and IOPID controller we need three specifications since there are three variables in the transfer function of the controller. The three specifications that we will be considering are

- ✓ *Phase Margin Constraint*

$$\arg(G(j\omega_c)) = \arg(C(j\omega_c)P(j\omega_c)) = \pi + \phi_m \tag{4.11}$$

- ✓ *Gain crossover frequency constraint*

$$\left|G(j\omega_c)\right|_{dB} = \left|C(j\omega_c)P(j\omega_c)\right|_{dB} = 1 \tag{4.12}$$

- ✓ *Robustness to loop gain variation constraint*

$$\left|\frac{d(\arg(G(j\omega)))}{d\omega}\right|_{\omega=\omega_c} = 0 \tag{4.13}$$

Now, we design the FOPI controller firstly by the monotonic method, or the conventional method. The steps are given below.

I. Given, the $\omega_c = \omega_u$ =250 krad/sec and ϕ_m = 40°

II. With these values we write Equation (4.11) and (4.13), it will give us two equations with two unknowns i.e. λ and K_i.

III. Find out the values of these two parameters by using fsolve command in MATLAB. The values comes out to be, λ=0.4224 and K_i=852.6067

IV. Put these two values in Equation (4.12) to find out the value of K_p. The value of K_p comes out to be 2.79545.

V. With the obtained values of lambda, K_i and K_p, obtain the transfer function of the FOPI controller.

VI. Approximate the FOPI controller to integer order by using oustapp command in fomcon toolbox of MATLAB (take frequency band 0.0001 to 1000000).

VII. With this integer order controller, we get the response of the plant and the frequency response of the open loop plant

This is the monotonic method of the designing of the FOPI controller. Now, we design the IOPID controller by same monotonic method, or the conventional method. The steps are given below.

I. Given, the $\omega_c = \omega_u$ =250 krad/sec and $\phi_m = 40^o$

II. With these values we write Equation (4.11) ,Equation (4.12) and (4.13), it will give us three equations with three unknowns i.e. K_p , K_i and K_d

III. Find out the values of these three parameters by using fsolve command in MATLAB. The values comes out to be, K_p =10 ,K_i=2.07*10^6 and K_d=0.01*10^{-3}

IV. With the obtained values of K_p, K_i and K_d obtain the transfer function of the IOPID controller.

This is the monotonic method of the designing of the FOPI and IOPID controllers. Now, we will design the FOPI controller by the non-monotonic method, by using the same procedure but with worst case values.

I. Given, the $\omega_c = \omega_m$ =40 krad/sec and $\phi_m = 40^o$.

II. With these values we write Equation (4.11) and (4.13), it will give us two equations with two unknowns i.e. λ and K_i.

III. Find out the values of these two parameters by using fsolve command in MATLAB. The values comes out to be, λ=0.6862 and K_i=194.6612.

IV. Put these two values in Equation (4.12) to find out the value of K_p. The value of K_p comes out to be 14.489.

V. With the obtained values of lambda, K_i and K_p, obtain the transfer function of the FOPI controller.

VI. Approximate the FOPI controller to integer order by using oustapp command in fomcon toolbox of MATLAB (take frequency band 0.0001 to 1000000).

With this integer order controller, we get the response of the plant and the frequency response of the open loop plant. Now, we design the IOPID controller by same non-monotonic method, or the proposed method. The steps are given below.

I. Given, the $\omega_c = \omega_u$ =250 krad/sec and $\phi_m = 40^o$

II. With these values we write Equation (4.11) ,Equation (4.12) and (4.13), it will give us three equations with three unknowns i.e. K_p , K_i and K_d

III. Find out the values of these three parameters by using fsolve command in MATLAB. The values comes out to be, K_p =15.7 ,K_i=0.208*10^6 and K_d=0.04*10^{-4}

IV. With the obtained values of K_p, K_i and K_d obtain the transfer function of the IOPID controller.

Thus, by this procedure we are able to design controllers for both monotonic and non-monotonic method.

4.4 RESULTS

By the procedure given in previous section the controllers by a monotonic and non-monotonic method can be designed. Now to compare the performance FOPI and IOPID controllers we will firstly draw the bode of the open loop plant

Table 4.1 shows Performance of the monotonic compensation and non-monotonic compensation of FOPI and IOPID controllers

Compensation type	Values of controller parameters	Peak overshoot	Settling time
Monotonic Compensation based FOPI	K_p= 2.795, K_i=852.60, λ=0.4224	1.32	58µsec
Non-Monotonic Compensation based FOPI	K_p=14.48, K_i=194.66, λ=0.6862	1.11	31µsec
Monotonic Compensation based IOPID	K_p=10, K_i=2.07*10^6, K_d=0.01*10^{-3}	1.43	86µsec
Non-Monotonic Compensation based IOPID	K_p=15.7, K_i=0.208*10^6, K_d= 0.04*10^{-4}.	1.12	32µsec

Table 4.1 Performance of the monotonic compensation and non-monotonic compensation of FOPD and IOPID controllers

By the Figure 4.3 we can observe that, the bandwidth is almost same for both FOPI and IOPID controllers. The non-monotonic of FOPI compensation assures 40^o of phase margin throughout the bandwidth to all frequencies where as IOPID controller approximately having 40^0 phase margin. while the monotonic method is unable to provide. In the bode plot we can see that the monotonic compensation based FOPI and IOPID controllers has crossed the stability limit of -180^o two times, thus its stability can't be analyzed by the bode plot, so we will draw its nyquist plot to analyze the stability of both the compensation.

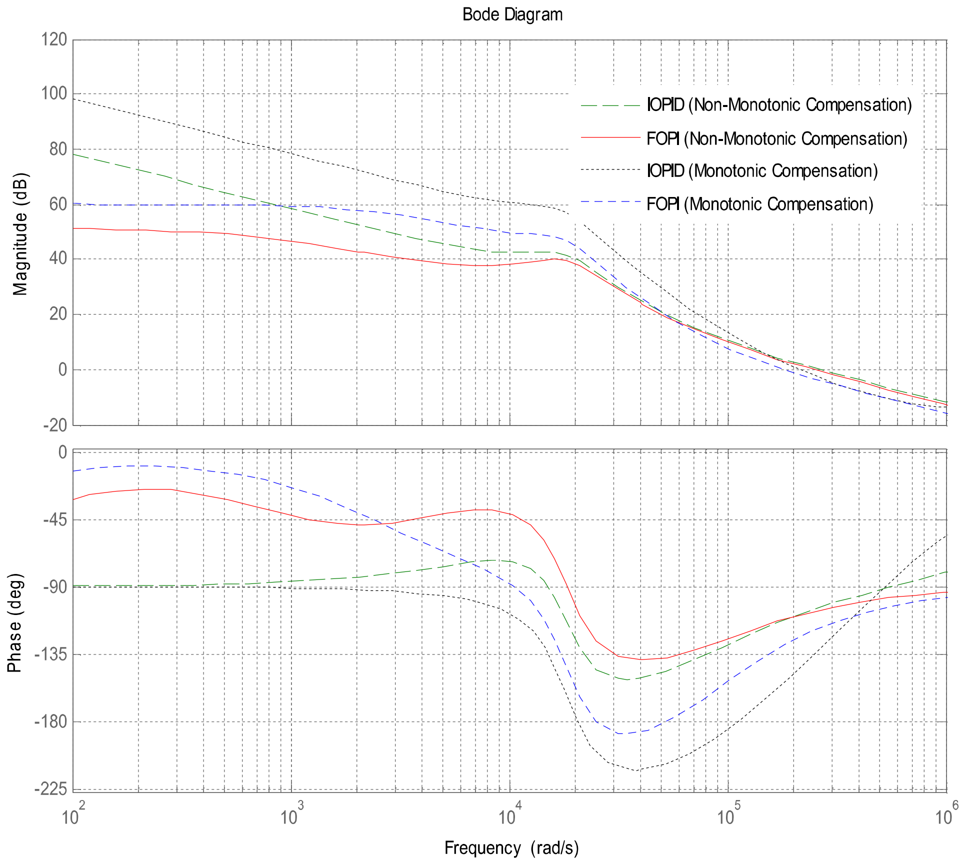


Figure 4.3-Bode plots of monotonic compensation and non-monotonic compensation.

From the Figure 4.4, it can be concluded that the monotonic compensation based FOPI and IOPID controllers does not guarantee the stability margins, as there is one encirclement of the critical point i.e. (-1,0). But in the non-monotonic based FOPI and IOPID controllers we can see that there is no encirclement of the critical point, thus it can be concluded that the system is stable in non-monotonic compensation.

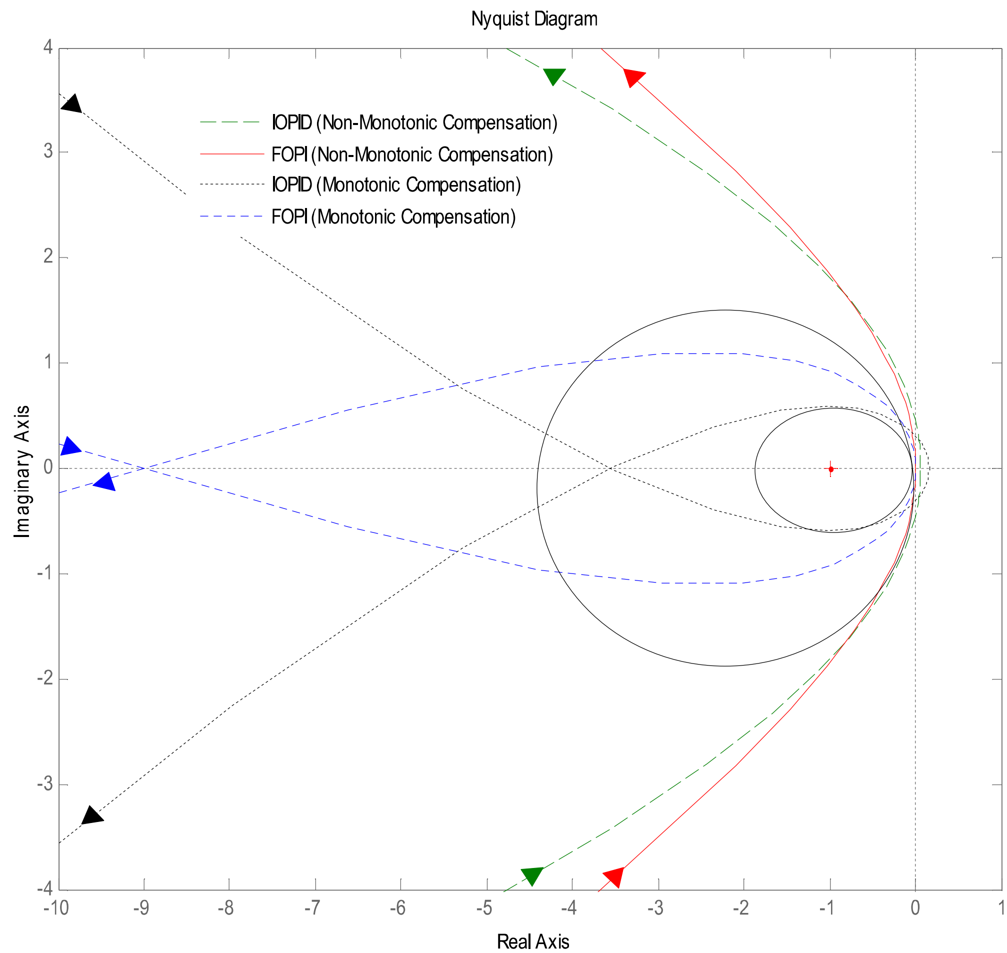


Figure 4.4-Nyquist plot of monotonic compensation and non-monotonic compensation.

To compare the robustness measure we draw the sensitivity circle and the complementary sensitivity circle with Ms = 2 and M_t = 1.25. from the Figure 4.4 it is clear that both the techniques is robust considering the sensitivity circle but when we consider the complementary sensitivity circle, only the non-monotonic compensation is robust, since the complementary sensitivity circle is outside the nyquist plot of the monotonic compensation plot. Thus, from this we can conclude that non-monotonic compensation presents an adequate shape to fulfil all the robustness measures.

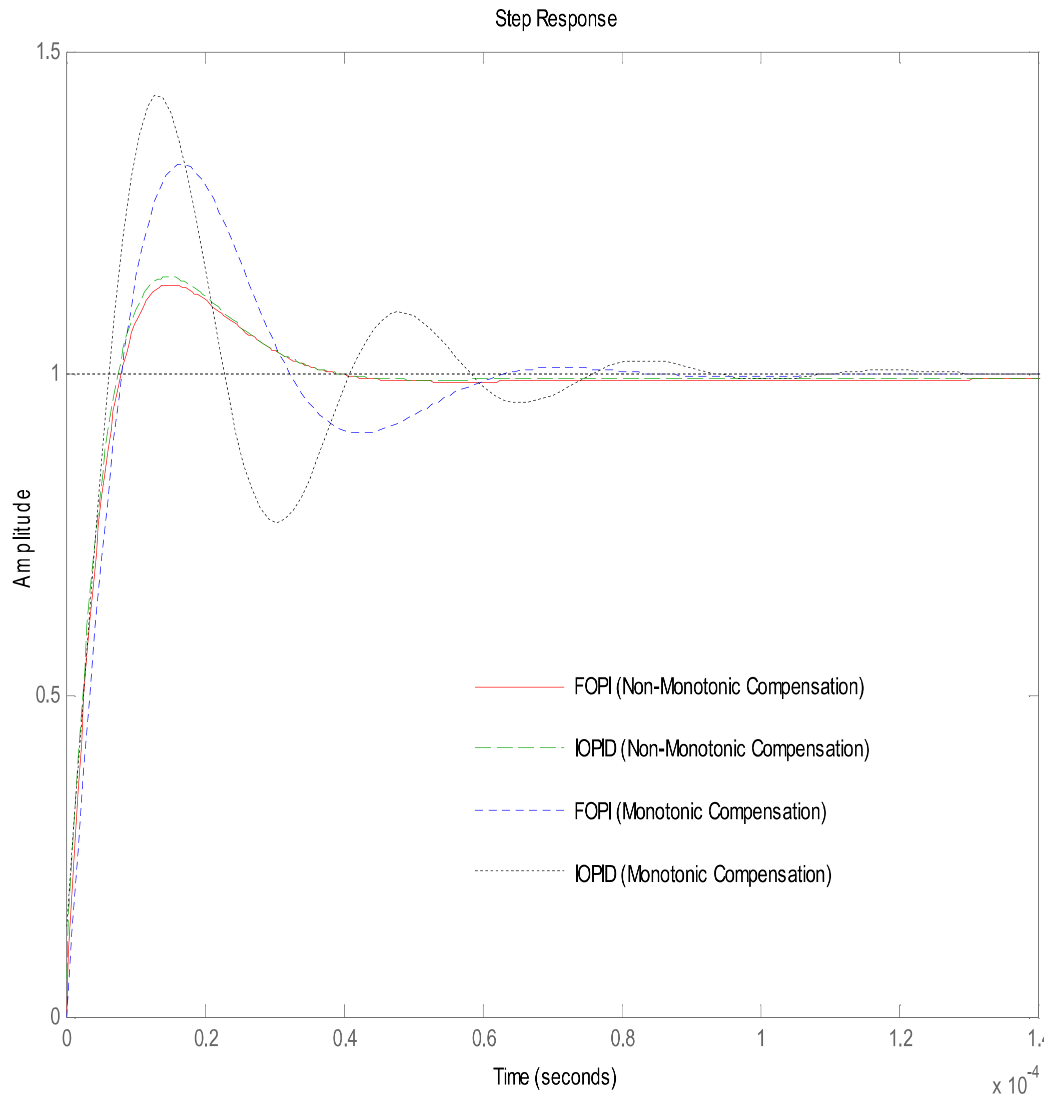


Figure 4.5-step response of monotonic compensation and non-monotonic compensation.

The Figure 4.5 Shows that the settling time for the monotonic compensation based FOPI and IOPID controllers are 58 micro seconds and 86 micro seconds respectively. In case of non-monotonic compensation based FOPI and IOPID controllers the settling time is about 32 micro secs for both controllers. Also the peak overshoot of the monotonic compensation based FOPI and IOPID controllers are1.43 and 1.32 respectively. For non-monotonic compensation based FOPI and IOPID controllers it is about 1.12 for both controllers. Thus it is clear from the response of both the systems that peak overshoot of the non-monotonic system is considerably smaller than the monotonic system. This is because the system assures a minimum phase margin throughout the bandwidth. Also, the smaller overshoot of the non-monotonic compensation system is a result of its robustness which has been discussed in the nyquist plot. So, the proposed non-monotonic method of designing of the FOPI and IOPID controllers offers much improved design than the conventional monotonic method.

4.5 CONCLUSION

In this Chapter, a much better technique of designing of the FOPI controller has been discussed for non-monotonically decreasing system. The proposed method offers some desirable improvement which has not been achieved by the classical monotonic method. The proposed method can also be used for the systems whose process plant is monotonic but the open loop is not. The designing has been done in the frequency domain by the means of gain crossover frequency and the phase margin. The concept of the phase margin has been redefined. The proposed method has some improvements such as a guaranteed minimum phase margin, better closed loop performance, and superior robustness properties. The peak overshoot of the system is considerably smaller and the steady state comes much sooner than the classical method. The bandwidth for the proposed technique is almost the same as the classical method but the system is more stable and robust. From the simulation results it is evident that the Non-Monotonic compensation based FOPI controller outperforms the Non-Monotonic compensation based IOPID controller in terms of fulfilling the design specifications i.e., phase margin, gain cross over frequency and robustness. The results provide the fact that fractional controller outperforms integer controller for same set of tuning constraints.