Chapter 2

A review of literature on atmospheric vortices and objective of the thesis

Atmospheric vortices, including dust devils, tornadoes, and tropical cyclones, are complex phenomena driven by thermal, mechanical, and turbulent processes within the atmosphere. From early idealised analytical solutions to advanced numerical simulations and laboratory investigations, mathematical modelling has been essential to understanding their formation, structure, and evolution. This chapter reviews the key contributions to the modelling of atmospheric vortices, organized by vortex type and different modelling approach. It provides an extensive overview of foundational theories, analytical models, and experimental validations.

2.1 Early Analytical Models of Vortices

Early analytical models of atmospheric vortices established the foundation for our current understanding of rotating fluid dynamics. William John Macquorn Rankine presented the

Rankine (1882) vortex in 1882. It was the first classical analytical model for vortices. An inviscid, incompressible, axisymmetric, and constant flow was supposed to have a two-region structure: a solid-body rotation core inside the radius of maximum wind (RMW), where tangential velocity was $v = \frac{\Gamma r}{2\pi r_m^2}, \quad$ for radius $r \leq r_m$, and a potential flow region outside, where $v = \frac{\Gamma}{2\pi r}, \quad r > r_m$, with Γ as circulation and r_m as RMW. The model provided a maximum tangential velocity at the RMW and guaranteed continuity of velocity at $r = r_m$. Although it provided a fundamental framework for studying fluid dynamics and vortex kinematics, it oversimplified real atmospheric vortices by neglecting boundary layer effects, pressure fields, radial and vertical motions, and viscosity.

The viscous vortex models of Oseen (1911) and Lamb (1932) improved upon Rankine's idealized formulation by incorporating the effects of viscosity. These models described a two-dimensional vortex that decayed diffusively over time, with the tangential velocity expressed as $v = \frac{\Gamma}{2\pi r}\left(1 - e^{-r^2/4\nu t}\right)$, ν, t being kinematic viscosity and time respectively. The model smoothed the discontinuity at the RMW and realistically represents vorticity spreading. However, it remained limited by its two-dimensional formulation. The lack of radial and vertical components, and constant viscosity assumption made it unsuitable for highly dynamic or turbulent systems.

Burgers (1948) vortex model, introduced by J. M. Burgers incorporated axial stretching and a steady-state framework to balance viscous diffusion. Its velocity field included as $u = -\frac{\alpha r}{2}, v = \frac{\Gamma}{2\pi r}\left(1 - e^{-r^2/r_0^2}\right), w = \alpha z$ respectively, where $r_0 = \sqrt{4\nu/\alpha}$, α being axial stretching rate. This model captured a realistic three-dimensional vortex structure with secondary circulation but neglected transient evolution, asymmetries, and complex boundary interactions.

Robert D. Sullivan's model (Sullivan, 1959) extended the Burgers vortex by introducing a two-cell structure to capture complex vertical and radial circulations in strong vortices. The tangential velocity approximated the Burgers profile, $v \approx \frac{\Gamma}{2\pi r}\left(1 - e^{-r^2/r_0^2}\right)$, modified to allow core outflow and outer inflow. This structure aligned with observed tornado dynamics and produced more accurate vertical velocity and pressure distributions.

The cyclostrophic balance model, developed conceptually in early meteorology Ferrel (1889), focused on the balance between centrifugal and pressure gradient forces, neglecting Coriolis effects for small-scale systems. It expressed tangential velocity as $v = \sqrt{\frac{1}{\rho}\frac{\partial p}{\partial r}r}$. Unlike Rankine model, this linked velocity with pressure, enabling estimation of wind speed from pressure fields. It was particularly useful for tornadoes and dust devils, but it remained limited by its steady, two-dimensional formulation, lack of viscosity, and dependency on empirical pressure data.

Fiedler and Rotunno (1986) simulated a vortex confined between a surface and an upper lid, enhanced rotation via vertical constraint and convergence. It improved on Davies-Jones by including end-wall effects but remains limited by idealized, steady, and axisymmetric assumptions.

Vatistas et al. (1991) proposed a generalized kinematic vortex model offered a flexible tangential velocity profile $v = \frac{\Gamma}{2\pi r}\left(\frac{r^n}{r^n+r_0^n}\right)^{1/n}$, where n shapes the velocity curve and r_0 was the core radius. This formulation accommodated a range of observational data by interpolating between models like Rankine and Lamb-Oseen. Though versatile in empirical fitting, it lacked a dynamic basis, omitted vertical and radial motions, and assumes inviscid flow outside the core.

Davies-Jones (1995) model was introduced with a near-surface frictional boundary layer to improve upon Sullivan (1959) model. By considering surface-driven inflow and turbulence, the model more accurately represented the convergence and vertical motion observed in tornadoes. Though it added realism near the ground, it also increased complexity and still assumed axi-symmetry and a steady state, which limited its ability to capture asymmetries and upper-level dynamics

2.2 Dust Devils

Dust devils are small-scale atmospheric vortices formed by the intense heating of earth's surface which creates pressure gradient. Due to this gradient, the air movement starts that sometimes leads to rotating vortex. Important research in this field using analytical, laboratory, and experimental techniques has greatly improved our knowledge of their genesis, behaviour, and broader significance for planetary and terrestrial science.

Theoretical and numerical approaches have significantly advanced the understanding of dust devil formation and dynamics. Dust devils were frequently reported in arid places with strong solar heating under hot, dry, and clear weather conditions in north-west Libya, according to McGinnigle (1966). He discovered favourable conditions that allow the upper soil layers to heat quickly while reducing energy loss, such as low background winds, surfaces with low thermal conductivity and high thermal capacity, and uneven surface heating. His research also examined how air stability and geography affect the size, duration, and behaviour of vortices. Bretherton and Turner (1968) conducted experiments on angular momentum mixing in stirred, rotating fluids, exploring how stirring intensity, rotation rate, and boundary effects influence turbulence and flow structure. Their findings demonstrate the redistribution of angular momentum, offering insights into the rotational dynamics relevant to dust devils and other geophysical systems. Sinclair (1969, 1973) conducted seminal field research in the Avra Valley and Tucson Basin (1960–1962), describing dust devils as thermal phenomena in which a convective updraft generates a vortex. They measured warm boundary-layer air drawn radially inward near the ground, heating the vortex core with temperature perturbations of 4–8 K above ambient levels. Willis and Deardorff (1979) hypothesized that the tallest dust devils form in narrow updrafts at the intersections of convective cells–a concept likely tested through laboratory simulations of convective processes. These experiments simulate the uneven surface heating and thermally driven circulations observed in natural settings, providing a controlled analogue to field conditions. Snow and McClelland (1990) surveyed dust devils in the New Mexico desert (1986–1987), finding peak activity over flat, cleared areas due to rapid surface heating and thermal generation. They also noted a shift in peak activity between years, possibly linked to higher precipitation in 1986, which may have altered surface thermal properties.

Shapiro and Kogan (1994) and Cortese and Balachandar (1993) modelled dust devil vortical structures, emphasising the convective processes that lead to their development. Rennó et al. (1998) constructed a thermodynamic framework for the formation of dust devils, highlighting the role of both horizontal and vertical temperature differences in producing the convective updrafts necessary for the formation of vortices. Buoyancy is the main component within the convective boundary layer of dust devils that are produced by solar heating of the surface proved by this study. Kanak et al. (2000) addresses the role of vorticity in thermal plumes within turbulent Rayleigh-Bénard convection. The results reveals a strong correlation between narrow regions of ascending hot fluid and local vertical vorticity. Zhao et al. (2004) uses Large Eddy Simulation (LES) in a convective boundary layer to study the dynamics and production of dust devils. According to the study, dust devils have vertical structures that closely align with observational data. It is demonstrated that environmental factors such as surface heating and ambient wind shear have a major impact on the vortex's intensity and structure, with low wind shear and substantial surface heating being especially advantageous.

Similarly, Kanak (2005) employed large eddy simulations to analyse azimuthal velocity profiles, demonstrating that the Burgers-Rott model better matches observed dust devil dynamics compared to the Rankine combined vortex model.

Onishchenko et al. (2014a) investigated the mechanisms behind dust devil generation through thermal and mechanical processes, using theoretical and numerical models to explore the interaction between the surface boundary layer and the overlying atmosphere. Their work highlighted the importance of shear flows and thermodynamic instabilities in initiating and sustaining these vortices. Building on this, Onishchenko et al. (2019) analysed velocity profiles–comprising radial inflow, tangential rotation, and axial updrafts–incorporating centrifugal forces, pressure gradients, and turbulence to describe height-dependent dynamics. On Mars, Michaels and Rafkin (2004) and Fenton and Lorenz (2015) observed dust devils reaching several kilometres in height, attributed to a convective boundary layer about three times deeper than Earth's. Pandey and Maurya (2017) proposed a model for dust devil formation, using radial and vertical pressure gradients to explain the development and persistence of a low-pressure vortex core that drives dust uplift. Complementing this, Pandey and Maurya (2018a) derived exact analytical solutions for unsteady axisymmetric vortex motions, providing precise velocity and pressure fields to describe the temporal evolution of atmospheric vortices. Together, these studies established a strong theoretical framework for understanding small-scale vortex phenomena.

Data from NASA missions, as explored by Ellehoj et al. (2010), Jackson et al. (2020), and Hueso et al. (2023), highlight their critical role in the Martian dust cycle, drawing parallels and contrasts with terrestrial counterparts. Further, Onishchenko et al. (2021b) developed a theoretical model for stationary concentrated vortices, examining the interplay of tangential rotation, radial inflow, and axial velocity while conserving vorticity. This model offers a robust framework for understanding small-scale vortices like dust devils. Intense dust devils, ascending to altitudes of 1–2 km, were documented by Hess and Spillane (1990), while Balme and Greeley (2006a) and Ito and Niino (2014) used field measurements–including particle image velocimetry–to confirm that vertical velocity is approximately a quarter of the maximum rotational velocity.

Franzese et al. (2021) studied Saharan dust devils using field surveys and remote sensing, noting peak midday activity due to intense heating and their significance in dust transport and atmospheric processes. These experimental findings, supported by earlier observations from Battan (1958), Ryan and Carroll (1970), and Oke et al. (2007), characterize dust devils as swirling, rising plumes originating in super-adiabatic near-surface air. Collectively, these studies provide critical insights into the spatial and temporal dynamics of dust devils on Earth and beyond, emphasizing their relevance to meteorology and planetary science.

2.3 Tornadoes

Tornadoes are the rapid rotating column of air that are associated with thunderstorm. They exhibit high-vorticity cores and destructive dynamics that have been extensively studied through various models. Important research in this field that uses analytical, laboratory, and experimental models to demonstrate the complex processes controlling tornado genesis, structure, and behaviour.

Analytical modelling has played a important role in understanding tornado formation and behaviour, emphasising connections between atmospheric conditions and vortex features. The earliest investigations by Ying and Chang (1970) conducted laboratory experiments using a rotating cylindrical screen and exhaust fan to study tornado-like vortices. Their measurements showed that radial velocity in the boundary layer scaled with circulation strength, while vertical velocity followed a Gaussian profile ($v_z \propto \exp(-r^2/R_0^2)$). This model also revealed the presence of a reverse flow zone influenced by outlet geometry, provide a better understanding on tornado structure.

Kuo (1971) explored three-dimensional flow inside boundary layer flow for steady-state and axisymmetric vortex like tornado and cyclone. It focuses on a vortex formation constructed of an exterior region with minimal vorticity surrounding a core region of high vorticity. The study also identified oscillatory vertical motion within the core, resembling behaviour typical of an Ekman layer. The study emphasises the role of surface friction and its impact on the inflow and vertical motion near to the vortex centre by obtaining analytical solutions to study the radial and tangential velocity profiles within the boundary layer.

Lewellen (1993) developed a theoretical framework for axisymmetric, steady-state flows in tornado. Analytical solutions for the radial and tangential velocity components, highlighting the role of surface friction and its influence on radial and vertical motion near the vortex core. Davies-Jones (1995) examined the mechanics of tornado development and evolution, emphasising the interplay among ambient wind shear, mesocyclones, and updrafts. The study also emphasised the parameters required for tornadogenesis and the complex nature of vortex disintegration. While Larcheveque and Chaskalovic (1994) introduced an analytical model for tornadoes that discusses the formation of vortex lines in tornadoes, aligning with meteorological mechanisms and captures the genesis of these vortex structures. Lewellen et al. (2000) introduced the concept of the local corner flow swirl ratio (S_c), demonstrating via high-resolution numerical simulations that S_c—which depends on near-surface flow features—governs transitions between vortex structures, including breakdown phenomena, thereby influencing tornado intensity and near-ground dynamics.

Kosiba et al. (2008) employed mobile Doppler radar to analyse high-resolution wind fields of tornadoes in the field, revealing detailed patterns of inflow, updraft, and rotational structure, which informed forecasting models. Natarajan and Hangan (2012) gave a computational model using large eddy simulations (LES) to reveal the impact of surface roughness and translational motion. The model focuses on environmental factors affect the vortex dynamics near the ground, such as changes in wind speed profiles and vortex intensity.

Davies-Jones (2015) summarised decades of tornado research in a more general framework, highlighting the increasing understanding regarding the significance of swirl ratios, boundary layer dynamics, and atmospheric thermodynamics in influencing tornado formation and evolution.

Baker and Sterling (2017) advanced tornado research by conducting laboratory simulations of wind fields and debris motion. Their physical model integrated vortex dynamics and turbulence to represent three-dimensional tornado structures and debris paths. Tang et al. (2018) in Boundary-Layer Meteorology studied tornado-like vortices in a Ward-type simulator, analysing velocity and pressure fields. It showed vortex structure shifts from single- to dual-celled with increasing swirl ratio, producing stronger tangential velocities and pressure deficits that match real tornado data, supporting improved structural design despite turbulence replication limits. Elsaesser et al. (2018) created an analytical model for tornado-like vortices based on cyclostrophic balance in cylindrical dimensions. In order to more accurately depict the complex physics seen in tornadoes, they concentrated on improving the Vortex Sink with Axial Flow (VSAF) model. For the tangential wind, they used a Rankine-type profile, which is defined by a $1/r$ decrease beyond the radius of maximum wind (RMW) and a linear increase with radius close to the vortex core ($v \propto r$). The radial velocity component ($u \propto -1/r$) was derived using mass continuity, indicating inward flow. For the vertical motion, they assumed a Gaussian distribution ($w = V_0 \exp(-r^2/R_v^2)$), which reaches its maximum at the vortex centre and decreases with radial distance.

More recently, Zhang et al. (2023) constructed a 3D analytical model that took into consideration the interaction of Coriolis effects, centrifugal force, and pressure gradients. In order to better represent the structure that real tornado flows, this model uniquely takes into account the impacts of centrifugal force, pressure gradients, and viscous forces.

Lastly, progress is still being driven by continuous improvements in observational technology. For example, research has combined high-resolution LES with data collected by satellite and radar platforms, and new machine learning methods are being investigated to better predict tornadoes and model storm environments (Dawson and Marquis, 2020; Zhou et al., 2022; Wu et al., 2024).

2.4 Tropical Cyclones

Tropical cyclones are rapid rotating air columns that are accompanied by strong winds, and heavy rain and fuelled by warm ocean water. They have complex lifecycle dynamics, structure, and intensification. A comprehensive understanding of these powerful systems has been provided by the various laboratory studies, theoretical frameworks, and numerical simulations that have been validated by field data.

A thermodynamics approach was presented by Emanuel (1986) to explain how tropical cyclones retain their intensity and how air-sea interactions contribute to cyclone maintenance. By investigating convective processes and angular momentum exchanges within the eyewall, Emanuel (1997) expanded on this model by linking thermodynamic efficiency to vortex contraction and intensification rates. Bloor and Ingham (1987) uses both theoretical and numerical methods to study the internal flow characteristics of industrial cyclone separators. In industrial settings, cyclones are frequently employed to extract particles from fluids using centrifugal force.

Experimental studies, encompassing numerical simulations and field-validated models, have significantly advanced tropical cyclone forecasting and understanding, as your submitted text thoroughly described. Ooyama (1969) created and applied one of the first three-dimensional numerical models to model a tropical cyclone's whole life cycle, from formation to intensification and demise. The model incorporates the thermodynamics of the atmosphere, including surface fluxes, latent heat release, and large-scale environmental flow. He use of a vortex-following coordinate system, which makes it possible to simulate cyclone movement and structure. Kurihara and Tuleya (1974) used a 3D primitive equation model to simulate a cyclone over a week with a fixed sea surface temperature of 302K, revealing a central pressure drop to 940mb, a warm core, maximum winds 60 km from the centre, and outward-propagating spiral bands as internal gravity waves.

Willoughby (1990a) analysed the temporal evolution of primary circulation, focusing on tangential wind fields and radial structure changes. The study highlighted eyewall replacement cycles, radial inflows and outflows, and energy exchanges, offering insights into lifecycle fluctuations and intensity variations.

Kepert (2001) studied boundary layer jets using linear and non-linear theories, explaining jet formation and asymmetry due to surface friction, radial inflow, and pressure gradients. Willoughby and Rahn (2004) evaluated the Holland (1980) model, described the radial structure of wind and pressure in the primary vortex of tropical cyclones. When compared to documented hurricane wind profiles, the analysis reveals systematic errors in the Holland (1980) model. In particular, it has a tendency to overestimate wind speeds at larger radii, that are farther from the storm centre, and underestimate wind speeds near to the radius of maximum wind (RMW), where the highest winds occur.

Majumdar (2003) developed a nonlinear analytical model for early development of cyclone, by solving Navier-Stokes equation for axisymmetric, incompressible, and viscous flow. The main result of the study was the low-pressure core was essential for the formation. Xiang and Lee (2005) examined the relationship between cyclone height and the internal flow field in cyclone separators using computational fluid dynamics (CFD). The simulations showed a unique flow pattern with an inner upward vortex and an outer descending vortex that were both impacted by variations in the height of cyclone. Although this resulted in a greater pressure drop within the system, the flow became more fully formed as the height increased, exhibiting longer residence periods and improved whirling motion. Vyas and Majdalani (2006) derived analytical solutions for bidirectional vortex flow in a laboratory context, using a setup that captures radial symmetry and bidirectional dynamics—axial inflow and outflow—with velocity components derived from the Navier-Stokes equations. This work, while analytical in derivation, bridges to laboratory applications by providing a testable framework for vortex behaviour applicable to cyclone studies. Kieu and Zhang (2009) gave an analytical model for rapid intensification for crucial phase when storms strengthen quickly over a short period focusing on vortex stretching, latent heat release, and moisture convergence in the inner core. Wood and White (2011); Wood et al. (2013) proposed a parametric model with five parameters—maximum tangential wind, radius of maximum wind (RMW), and three power-law exponents—to represent wind profiles across vortex types. This model accurately captures RMW and outer wind decay, aligning with idealized profiles like Rankine and Burgers-Rott, with one exponent controlling peak sharpness in the annular zone.

Bryan and Rotunno (2009) evaluated the maximum potential intensity (MPI) framework against high-resolution simulations, validating its thermodynamic basis—linking sea surface temperature, efficiency, and maximum winds—while noting limitations in simplified assumptions. Smith et al. (2014) investigated steady-state behaviour under idealized conditions, challenging traditional assumptions about boundary layer and upper-level dynamics while emphasizing eyewall energy exchange. Peng et al. (2018) assessed a time-dependent intensification model, highlighting the roles of surface heat fluxes and boundary layer dynamics in driving growth rates and structural evolution, with comparisons to simulated outcomes.

Majdalani (2022) advanced this by solving the Navier-Stokes equations for Beltramian motion in conical cyclones, detailing helical and self-similar vortex dynamics.

These analytical efforts, which you compiled in the text, collectively provide a robust theoretical foundation for understanding tropical cyclone structure and intensification.

2.5 Objective of the Thesis

The primary objective of this thesis is to construct analytical models that can capture the full lifecycle of atmospheric vortices, ranging from small-scale phenomena such as dust devils to large-scale like tropical cyclones. Existing models often have limitations; e.g., velocities independent of axial coordinates, unbounded velocity components, undefined velocities at the vortex-centre, either very large or virtually infinite vortex width or over-simplified flow assumptions. The thesis plans to bridge the gaps by formulating consistent, bounded, and analytically feasible models. In order to depict the structure and behaviour of these vortices more accurately, actual spatial dependencies and viscous effects are emphasised, especially in the axial and radial directions.

A key focus of the work is to explore how geometric parameters—such as core radius and eye size influence the distribution of velocity components and pressure gradients. The models are constructed within the framework of incompressible, axisymmetric, and steady or time-dependent viscous flows in cylindrical coordinates. The scope of the research is further broadened by including time-dependent analysis, which makes it possible to examine the evolution of vortices through phases of formation, intensification, and decay. The thesis seeks to advance theoretical understanding and create the foundation for improved atmospheric vortex analysis and prediction by integrating temporal and spatial dynamics into a unified analytical framework.

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