# Chapter 1

# Introduction

Correlations are commonly discussed in everyday life. In a scientific setting, particularly in the fields of statistics, classical, and quantum physics, a correlation analysis helps us in determining the degree of relationship between two or more than two variables [1, 2]. The explanation of a significant degree of correlation of any two variables may be due to any of the following two reasons: *(i)* Both the variables may be mutually influencing each other. For example, the relationship between price and demand, where demand increases price and vice versa. *(ii)* Both variables may be influenced by other variables; for example, the production of tea is correlated with land and is affected by the amount of rainfall. The significance of the correlation is that we can estimate the change in one of the variables, given the correlation of the two related variables.

Based on the space and time dependence, we can classify the correlation as spatial and temporal. If the observable at different locations are correlated irrespective of time dependence, they are spatially correlated. For example, Sheep are highly correlated with each other in their group; however, there is no correlation with another group far away from their group. If the correlation of the observable is taken with itself at a different position (spatial variation), then it is known as spatial autocorrelation. It can be positive or negative. However, in the case of temporal correlation, the correlation of two observables

takes place at different times without changing the position, for example, GDP and life expectancy, which means that improvement in GDP improves life expectancy over time. If the correlation of the observable is taken with itself at different times, then it is known as temporal autocorrelation. In the following section, we will discuss the classical correlation of a bipartite system in the context of classical correlation theory.

## 1.1 Classical correlation

Numerous measurements of correlations based on statistical analysis were proposed in Refs. [3, 4]. However, measuring the classical correlation in a bipartite system remains unclear. A measure of classical correlations between two different random variables $X$ and $Y$ is proposed in the field of classical information theory where information in an entity is defined by the amount of data that is required to describe it completely [5]. It is calculated by using mutual information, which is defined as [6]

$$H(X:Y) = H(X) + H(Y) - H(X,Y). \tag{1.1}$$

First and second term of Eq. (1.1) are known as Shannon entropy and defined as [6]

$$H(X) \equiv H(p) = -\sum_i p_i \log p_i, \quad H(Y) \equiv H(q) = -\sum_i q_i \log q_i. \tag{1.2}$$

Shannon entropy is used to find the information in a source, $X/Y$, that provides messages $x_i/y_i$ with probabilities $p_i/q_i$. Last term of Eq. (1.1) is known as joint entropy which is defined as

$$H(X,Y) = -\sum_{i,j} p_{ij} \log p_{ij}, \tag{1.3}$$

where, $p_{ij}$ is the probability of both outcomes $x_i$ and $y_j$. It is to be noted that correlation does not change with the change of the observables $X$ and $Y$ because it is, by definition, a property of the combined bipartite system rather than the property of either subsystem.

Correlations can also be discussed in terms of a bit which is the fundamental unit of classical computation. The state of the classical bit is either 0 or 1. A classical bit is similar to a coin: either tails or heads up. In the next section, we will discuss quantum correlation in terms of quantum bit, *i.e.,* qubit, which is a fundamental concept for quantum computation.

## 1.2 Quantum correlation

Like a classical bit, two possible states of a quantum bit are $|0\rangle$ and $|1\rangle$. In quantum mechanics, '$|\ \rangle$' represents a state in the form of Dirac notation. Other than $|0\rangle$ or $|1\rangle$ state, a qubit can be in a superposition state. In general, it is written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{1.4}$$

where, $\alpha$ and $\beta$ are complex numbers. When we measure a qubit outcome will be either 0, with probability $|\alpha|^2$, or 1, with probability $|\beta|^2$ and $\alpha$ and $\beta$ follow the condition $|\alpha|^2 + |\beta|^2 = 1$.

We will discuss quantum correlation in a composite quantum system made up of two or more distinct physical systems. For the sake of simplicity, we consider a two-qubit system. Corresponding to this system four computational basis states denoted as $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes 1\rangle$, $|1\rangle \otimes |0\rangle$, and $|1\rangle \otimes |1\rangle$. A pair of qubits can also exist in superpositions of these four states that is given as

$$|\psi\rangle = \alpha_{00}|0\rangle \otimes |0\rangle + \alpha_{01}|0\rangle \otimes |1\rangle + \alpha_{10}|1\rangle \otimes |0\rangle + \alpha_{11}|1\rangle \otimes |1\rangle. \tag{1.5}$$

Similar to the case for a single qubit, when we do a measurement on the state of two qubits $|xy\rangle = |x\rangle \otimes |y\rangle$, where $|x\rangle \in H_1$, $|y\rangle \in H_2$, and $|xy\rangle \in H_1 \otimes H_2$, where $H_1$ and $H_2$ are Hilbert spaces. The measurement result is $xy(= 00, 01, 10 \text{ or } 11)$ with probability $|\alpha_{xy}|^2$ and the probabilities add up to one, *i.e.,* $\sum_{x,y\in(0,1)} |\alpha_{xy}|^2 = 1$. Four perfectly correlated states of two qubits are defined by Bell, named Bell states, and given as

$$\begin{aligned}
|\phi^+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B), \\
|\phi^-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B), \\
|\psi^+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |0\rangle_A \otimes |1\rangle_B), \\
|\psi^-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |0\rangle_A \otimes |1\rangle_B),
\end{aligned} \tag{1.6}$$

where, "A" and "B" are acronyms of Alice and Bob. The meaning of expression $|\phi^+\rangle$ in Eq. (1.6) is that qubit held by Alice or Bob can be 0 as well as 1. Alice and Bob prepare a few copies of the $|\phi^+\rangle$ state and take a qubit each. Let us assume that Alice chooses the z-basis and measures her qubit. The measurement outcome (0 or 1) would be random with probability $1/2$. Subsequently, when Bob measures his qubit on the same z-basis, Bob's outcome would be the same result that Alice has already measured for all the copies of the $|\phi^+\rangle$ state prepared. If Alice and Bob communicate their results, it would be found that although the outcomes are seemingly random at each end, whence combined, they are perfectly correlated.

Let us discuss a generic experiment setup in which two parties, Alice and Bob, are a distance apart [7]. Charlie prepares two particles for the measurement and sends one to Alice and the second to Bob. Alice (or Bob) performs measurements on one system, but there is no effect on the result of Bob's (or Alice's) measurement. Let us consider two different realities. Corresponding to these realities, Alice and Bob have two outcomes. Outcome on Alice's side: $A^{'} = \pm 1$ or $A^{''} = \pm 1$, and Bob's side: $B^{'} = \pm 1$ or $B^{''} = \pm 1$.

Measurements are performed simultaneously by Alice and Bob. As Alice receives her particle, she performs a measurement on it. Suppose she has two different apparatuses for measurement to know the reality on her side. So she has two options to perform the measurements. These measurements are label as $P_{A'}$ and $P_{A''}$, respectively. In advance, Alice does not make sure which measurement should perform first. She either flips the coin or uses some random technique to do a measurement. For simplicity, consider each measurement to have one of two outcomes, either $+1$ or $-1$. Let Alice's particle has a value $A'/A''$ for the property $P_{A'}/P_{A''}$. Now, suppose Bob also has two operations for measurements, and these are labeled as $P_{B'}$ or $P_{B''}$. Consider each measurement has one of two outcomes, either $+1$ or $-1$. As Bob receives his particle, he randomly selects an operator and starts to measure. Since the experiments are performed by Alice and Bob simultaneously, therefore, the results of the measurements of Alice and Bob cannot disturb one another.

Let us discuss simple algebra of the quantity $S = A'B' + A''B' + A''B'' - A'B''$ which includes all the correlation between possible outcomes on Alice and Bob side. It can be written as

$$A'B' + A''B' + A''B'' - A'B'' = (A' + A'')B' + (A'' - A')B''. \tag{1.7}$$

Since, $A''$ and $A' = \pm 1$, we can see that either $(A' + A'')B' = 0$ or $(A'' - A')B'' = 0$. In both cases, it is easy to see from Eq. (1.7) that $A'B' + A''B' + A''B'' - A'B'' = \pm 2$.

Let $p(a', a'', b', b'')$ is the probability that the system is in a state $A' = a'$, $A'' = a''$, $B' = b'$, and $B'' = b''$ before the measurements are performed and $E(S)$ is the mean value of the quantity $S$. Then we have

$$\begin{aligned} E(A'B' + A''B' + A''B'' - A'B'') &= p(a', a'', b', b'')(a'b' + a''b' + a''b'' - a'b'') \\ &\leq p(a', a'', b', b'') \times 2 \text{ or } \geq p(a', a'', b', b'') \times (-2) \end{aligned}$$

Also

$$\begin{aligned} E(A'B' + A''B' + A''B'' - A'B'') &= p(a',a'',b',b'')a'b' + p(a',a'',b',b'')a''b' \\ &\quad + p(a',a'',b',b'')a''b'' - p(a',a'',b',b'')a'b'' \\ &= E(A'B') + E(A''B') + E(A''B'') - E(A'B''). \end{aligned}$$

Comparing the above equation, we get Bell inequalities.

$$-2 \leq E(A'B') + E(A''B') + E(A''B'') - E(A'B'') \leq 2 \tag{1.8}$$

Alice and Bob can determine $E(A'B')$, $E(A''B')$, $E(A''B'')$, and $E(A'B'')$ and repeat the experiment several times. After completing a series of tests, Alice and Bob meet to discuss and analyse their data. They examine each experiments where Alice measured $P_{A'}$ and Bob measured $P_{B'}$. They obtain a sample of values for $A'B'$ by collectively multiplying their tests' outcomes. They can estimate $E(A'B')$ by averaging over the sample. Likewise, they can make estimate $E(A'B')$, $E(A''B')$, $E(A''B'')$, and $E(A'B'')$.

All the classical experiments defined in the above manner follow Bell's inequality. Now we perform the expectation value calculations, using quantum mechanical state and the observables manifesting two different properties of the same system. For this purpose, let Charlie set up a two-qubit quantum system in the state,

$$|\psi^-\rangle = \frac{\alpha_{01}|0\rangle \otimes |1\rangle - \alpha_{10}|1\rangle \otimes |0\rangle}{\sqrt{\alpha_{01}^2 + \alpha_{10}^2}}. \tag{1.9}$$

He passes the first qubit to Alice to perform measurements of the observables

$$A' = Z_1, \quad A'' = X_1 \tag{1.10}$$

and passes the second qubit to Bob to perform measurements of the observables:

$$B^{'} = \frac{-Z_2 - X_2}{\sqrt{2}}, \quad B^{''} = \frac{Z_2 - X_2}{\sqrt{2}} \tag{1.11}$$

The expectation value of the observable $A^{'}B^{'}$ is

$$\langle A^{'}B^{'} \rangle = \left\langle \frac{\langle 01| + \langle 10|}{\sqrt{2}} \middle| Z_1 \frac{-Z_2 - X_2}{\sqrt{2}} \middle| \frac{|01\rangle + |10\rangle}{\sqrt{2}} \right\rangle = \frac{1}{\sqrt{2}} \tag{1.12}$$

Similarly, expectation value of the observables $A^{''}B^{'}$, $A^{''}B^{''}$, and $A^{'}B^{''}$ can be found to be:

$$\langle A^{''}B^{'} \rangle = \frac{1}{\sqrt{2}}; \ \langle A^{''}B^{''} \rangle = \frac{1}{\sqrt{2}}; \ \langle A^{'}B^{''} \rangle = \frac{1}{\sqrt{2}}. \tag{1.13}$$

Thus, quantity $S$ shall therefore be

$$S = \langle A^{'}B^{'} \rangle + \langle A^{''}B^{'} \rangle + \langle A^{''}B^{''} \rangle - \langle A^{'}B^{''} \rangle = 2\sqrt{2}. \tag{1.14}$$

In Eq. (1.8), we find that the $\langle A^{'}B^{'} \rangle + \langle A^{''}B^{'} \rangle + \langle A^{''}B^{''} \rangle - \langle A^{'}B^{''} \rangle$ can never exceed two. However, for a quantum mechanical state like Eq. (1.9), the sum of the expectation value is equal to $2\sqrt{2}$. Quantum mechanical states like Eq. (1.9) violate Bell's inequality. Therefore, the quantum mechanical states like Eq. (1.9) have a nonlocal correlation. In the next section, we will discuss temporal correlation, defined as the correlation of the observables at different times.

## 1.3 Temporal correlation

Temporal correlations are used in information-sharing processes. Depending upon the time-ordering, correlation is categorized into two parts:

(1) Time-ordered correlation

(2) Out-of-time-order correlation.

We will discuss these two cases independently in the following subsections.

### 1.3.1 Time-ordered correlation

Time-ordered correlation functions play essential role in many quantum dynamical problems. A general time-order correlation function for two observables, $\hat{W}_1(t_1)$ and $\hat{W}_2(t_2)$ is defined as $\langle \hat{W}_2(t_2)\hat{W}_1(t_1)\rangle$ at times $t_1 < t_2$, where $\langle \cdots \rangle$ is the quantum mechanical averages and for four observables is given as $\langle \hat{W}_4(t_4)\hat{W}_3(t_3)\hat{W}_2(t_2)\hat{W}_1(t_1)\rangle$, at times $t_1 < t_2 < t_3 < t_4$. The time-ordered correlation of four general observables at different times is shown in Fig. 1.1.

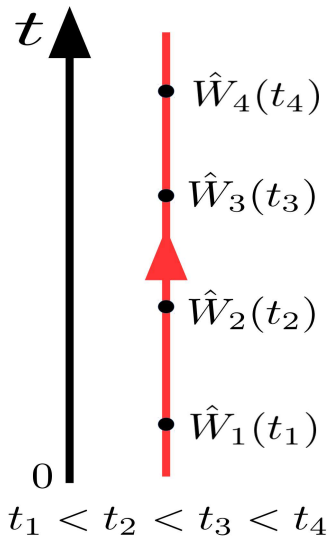


Fig. 1.1 Contour of time folded that is showing the temporal ordered correlation of the observables $\hat{W}_1, \hat{W}_2, \hat{W}_3$, and $\hat{W}_4$ with times $t_1 < t_2 < t_3 < t_3 < t_4$.

However, a special type of behavior in some quantum systems has recently drawn considerable attention among physicists. Small changes in the initial conditions lead to drastic changes in the time-evolved state. Such problems need to overlap two states of the system prepared by a successive backward and forward evolution of the observable

in time. In such cases, the correlation violates time ordering and is known as an out-of-time-order correlator (OTOC). This quantity has been found useful for determining the scrambling of information in quantum systems [8, 9, 10], and have been used as measures of thermalization and many-body localization [11, 12], chaos [8, 13, 14, 15], and entanglement [14, 16]. At the same time, several experimental protocols [16, 17, 18] have been proposed to measure the OTOCs. A brief discussion of OTOC is given below.

### 1.3.2 Out-of-time-order correlator

OTOC is a special type of four-point correlation that is not in time ordered. For the calculation of OTOC, we generate a correlation of two observables $\hat{V}(0)$ and $\hat{W}(t)$, where $\hat{V}$ at time $t = 0$ and another at $t$ time and it is defined as $\langle \hat{W}(t)\hat{V}(0)\hat{W}(t)\hat{V}(0)\rangle$ in the mathematical form. A schematic representation of OTOC is given in Fig. 1.2.

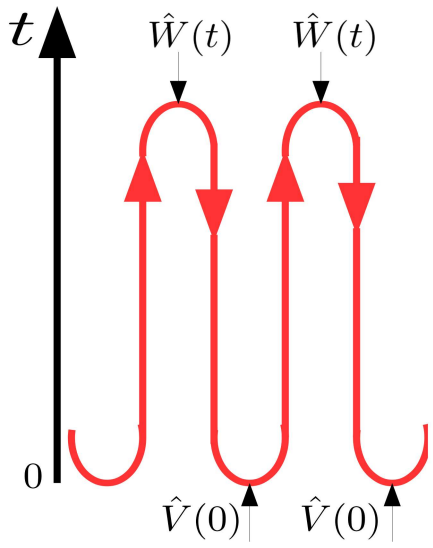


Fig. 1.2 Illustration of observable $\hat{W}$ at time $t$ and observable $\hat{V}$ at initial time $t = 0$ which are not in time ordered manner. Arrows indicate the direction of the correlation defined for the operators along the time axis.

Larkin and Ovchinnikov first introduced OTOC in 1969 [19]. After that, OTOC has been explored in many fields of quantum, and spin systems [14, 20, 21, 22, 23, 24, 25, 26,

27]. In the recent years, OTOC extensively used to indicate the chaos in the quantum and semiclassical systems [8, 13, 14, 15].

For two observables $\hat{W}$ and $\hat{V}$, OTOC is given as [19]

$$C(t) = \frac{1}{2}\left\langle \left[\hat{W}(t),\hat{V}(0)\right]^{\dagger}\left[\hat{W}(t),\hat{V}(0)\right]\right\rangle, \tag{1.15}$$

where parentheses $\langle\cdots\rangle$ denotes a quantum mechanical average. Both observables $\hat{W}$ and $\hat{V}$ commute with each other at $t = 0$ OTOC, defined by Eq. (1.15), is zero. At the time $t$, Heisenberg time evolution of $\hat{W}(0)$ is defined as $\hat{W}(t) = e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}$ and expansion of it is given by Baker-Campbell-Hausdorff formula that has the sum of products of many local observables given as in the mathematical form

$$\begin{aligned}\hat{W}(t) = \hat{W} + it[\hat{H},\hat{W}] &+ \frac{(it)^2}{2!}[\hat{H},[\hat{H},\hat{W}]] + \frac{(it)^3}{3!}[\hat{H},[\hat{H},[\hat{H},\hat{W}]]] \\ &+ \cdots\frac{(it)^k}{k!}[\hat{H},[\hat{H},\cdots[\hat{H},\hat{W}]]\cdots]_k + \cdots\end{aligned} \tag{1.16}$$

$\hat{V}$ does not commute with the higher-order term of $\hat{W}(t)$, which implies the nonzero value of OTOC. This noncommutative behavior of OTOC may indicate the chaotic nature of the system.

After doing some simplification in Eq. (1.15), we get

$$C(t) = \frac{1}{2}\left\{\langle\hat{V}\hat{W}(t)^2\hat{V}\rangle + \langle\hat{W}(t)\hat{V}^2\hat{W}(t)\rangle - \langle\hat{W}(t)\hat{V}\hat{W}(t)\hat{V}\rangle - \langle\hat{V}\hat{W}(t)\hat{V}\hat{W}(t)\rangle\right\}. \tag{1.17}$$

If observables $\hat{W}$ and $\hat{V}$ are Hermitian, then quantities $\langle\hat{V}\hat{W}(t)^2\hat{V}\rangle$ and $\langle\hat{W}(t)\hat{V}\hat{W}(t)\hat{V}\rangle$ are complex conjugate of quanties $\langle\hat{W}(t)\hat{V}^2\hat{W}(t)\rangle$ and $\langle\hat{V}\hat{W}(t)\hat{V}\hat{W}(t)\rangle$, respectively. If we consider only the real part of all the expectation values, then Eq. (1.17) simplifies in the form given as

$$C(t) = \Re[\hat{W}^2(t)\hat{V}^2(0)] - \Re[\langle\hat{W}(t)\hat{V}\hat{W}(t)\hat{V}\rangle]. \tag{1.18}$$

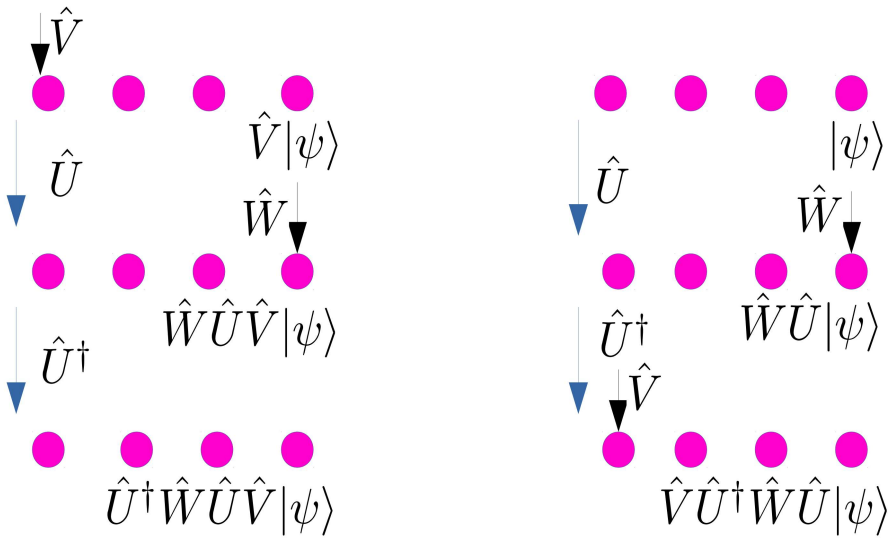


Fig. 1.3 (Left) We consider a state $|\psi\rangle$ and apply operator $\hat{U}^\dagger\hat{W}\hat{U}\hat{V}$ and generate a state $|\phi(t)\rangle = \hat{U}^\dagger\hat{W}\hat{U}\hat{V}|\psi\rangle$. (Right) We apply observable $\hat{V}\hat{U}^\dagger\hat{W}\hat{U}$ on state $|\psi\rangle$ and get state $|\xi(t)\rangle = \hat{V}\hat{U}^\dagger\hat{W}\hat{U}|\psi\rangle$. Inner product of $|\xi(t)\rangle$ and $|\phi(t)\rangle$ is equal to $F(t)$.

If observables $\hat{W}$ and $\hat{V}$ are Hermitian and unitary, then the first quantity of Eq. (1.18), *i.e.*, $\langle\hat{W}^2(t)\hat{V}^2(0)\rangle$ will be identity. Hence, Eq. (1.18) become

$$C(t) = 1 - \Re[\langle\hat{W}(t)\hat{V}\hat{W}(t)\hat{V}\rangle] = 1 - \Re[F_z^{l,m}(t)], \tag{1.19}$$

where, $\Re \rightarrow$ real number and $F_z^{l,m}(t) = \langle\hat{W}(t)\hat{V}\hat{W}(t)\hat{V}\rangle$.

OTOC can be defined in terms of the inner product of differently time evolved two wave functions $|\phi\rangle$ and $|\xi\rangle$. Let us consider an initial state wave function $|\psi\rangle$. For the generation of $|\phi\rangle$, the order of applied observables on state $|\psi\rangle$ is in the following manner: first, state $|\phi\rangle$ is perturbed by an observable $\hat{V}$ at initial time $t = 0$, after that, it gets evolved by unitary operator $\hat{U}$ till time $t$. At the time $t$, it gets perturbed by the observable $\hat{W}$ and gets evolved by $\hat{U^\dagger}$ in the backward direction on the time scale from $t$ to 0. Hence, wavefuction after doing time evolution is $|\phi(t)\rangle = \hat{U^\dagger}\hat{W}\hat{U}\hat{V}|\psi\rangle = \hat{W}(t)\hat{V}|\psi\rangle$. Generation of the wave function $|\xi\rangle$ has the following steps: first, evolve forward with unitary evolution $\hat{U}$ till time $t$, after that perturbed with $\hat{W}$ at time $t$, evolved backward

from $t$ to initial time $t = 0$ and again perturbed with $\hat{V}$ at $t = 0$. Hence, the wave-function is $|\xi(t)\rangle = \hat{V}\hat{U^{\dagger}}\hat{W}\hat{U}|\psi\rangle = \hat{V}\hat{W}(t)|\psi\rangle$. overlapping of $|\xi(t)\rangle$ and $\phi(t)\rangle$ is equivalent to $1 - C(t)$. A graphical representation of the above statement is given in Fig. 1.3.

In general, OTOC is a correlation of two observables in which one observable is evolving with time by Heisenberg time evolution, and another is independent of time. Hence, OTOC shows different behavior for different observables. In this thesis, we explore different sets of observables for the study of phase structure, quantum chaos, and rectification of magnon. In the following subsection, we will discuss OTOCs taking single spin observables and block spin observables.

### 1.3.3 OTOC using position-dependent single spin observable

Let us consider a spin chain in which spins interact in $x$ direction. Now, we consider a position-dependent pair of single spin Pauli observables at position $l$ and $m$ as $\hat{W}^l = \hat{\sigma}^l$ and $\hat{V}^m = \hat{\sigma}^m$. OTOC [Eq. (1.15)] of Hermitian and unitary Pauli operator will be

$$C^{l,m}(t) = 1 - \langle \hat{\sigma}^l(t)\hat{\sigma}^m(0)\hat{\sigma}^l(t)\hat{\sigma}^m(0)\rangle = 1 - \Re[F^{l,m}(t)], \tag{1.20}$$

where, $F^{l,m}(t) = \langle \hat{\sigma}^l(t)\hat{\sigma}^m(0)\hat{\sigma}^l(t)\hat{\sigma}^m(0)\rangle$. Separation between the observables, $\hat{W}^l$ and $\hat{V}^m$ is $\Delta l = l - m$. A graphical representation of the position-dependent observables is given in Fig. 1.4 in which observable $\hat{W}$ is at position 2 and $\hat{V}$ at position $N$. We can change the position of the observables. Depending upon the direction of the observables, we categorize into two parts:

1. Transverse magnetization OTOC (TMOTOC)
2. Longitudinal magnetization OTOC (LMOTOC)

1. **Transverse magnetization OTOC (TMOTOC):**

   If in OTOC, two Hermitian spin observables $\hat{W}$ and $\hat{V}$ at sites $l$ and $m$ be in the

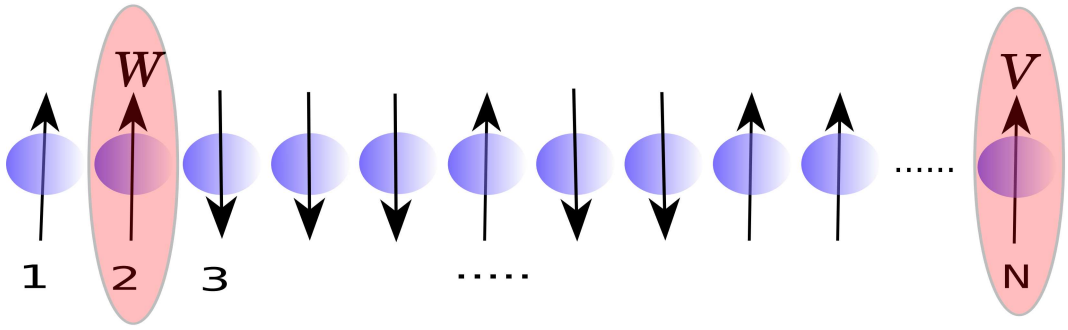


Fig. 1.4 Schematics of single spin observables. One spin is considered as observable $\hat{W}$ and another spin is considered as observable $\hat{V}$.

direction of the z-axis. , *i.e.*, $\hat{W} = \hat{\sigma}_z^l$ and $\hat{V} = \hat{\sigma}_z^m$ then we call it as TMOTOC and defined as:

$$C_z^{l,m}(t) = 1 - \Re[F_z^{l,m}(t)], \tag{1.21}$$

where, $F_z^{l,m}(t) = \langle \hat{\sigma}_z^l(t)\hat{\sigma}_z^m\hat{\sigma}_z^l(t)\hat{\sigma}_z^m\rangle$. In place of the quantum mechanical average, we consider a particular state as $|\phi_0\rangle = |\uparrow\uparrow\uparrow\cdots\uparrow\rangle$. $|\uparrow\rangle$ denotes eigenstate of $\hat{\sigma}_z$ with eigenvalue $+1$. Using a special state type makes numerical and analytical calculations easier.

2. **Longitudinal magnetization OTOC (LMOTOC) :**

   If in OTOC, two Hermitian spin observables $\hat{W}$ and $\hat{V}$ at sites $l$ and $m$ be in the parallel direction of the Ising axis (x-axis), *i.e.*, $\hat{W} = \hat{\sigma}_x^l$ and $\hat{V} = \hat{\sigma}_x^m$ then we call it as LMOTOC which is given as:

$$C_x^{l,m}(t) = 1 - \Re[F_x^{l,m}(t)], \tag{1.22}$$

   where, $F_x^{l,m}(t) = \langle\psi_0|\hat{\sigma}_x^l(t)\hat{\sigma}_x^m\hat{\sigma}_x^l(t)\hat{\sigma}_x^m|\psi_0\rangle$. In this case, initial state is defined as $|\psi_0\rangle = |\rightarrow\rightarrow\rightarrow\cdots\rightarrow\rangle$. $|\rightarrow\rangle$ denotes the eigenstate of $\hat{\sigma}^x$ with eigenvalue $+1$.

   In a closed chain Ising system, OTOC does not depend on $l$ and $m$ but depends on the distance between the spins *i.e.* $\Delta l = l - m$. However, in the open chain case, OTOC

depends on the $l$ and $m$ as well as the distance between the spins (*i.e.* $\Delta l = l - m$) because, in the open chain case, the last spins are connected with the environment.

### 1.3.4 OTOC using block observables

Let us consider a spin chain of the length $N$ and divide it into two blocks such as

$$\hat{W} = \frac{2}{N}\sum_{i=1}^{\frac{N}{2}} \hat{\sigma}_x^l \quad \text{and} \quad \hat{V} = \frac{2}{N}\sum_{l=\frac{N}{2}+1}^{N} \hat{\sigma}_x^l. \tag{1.23}$$

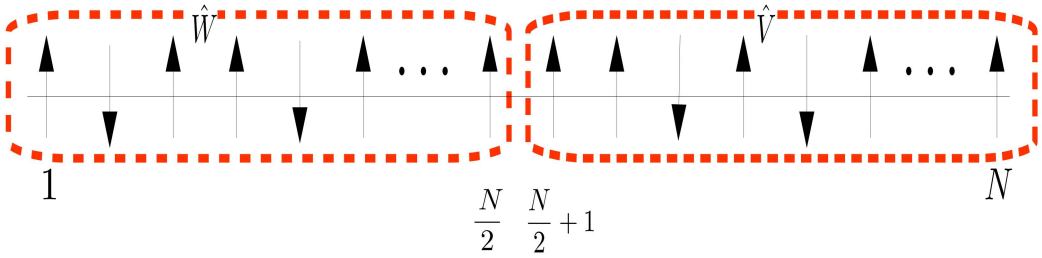


Fig. 1.5 Illustration of SBOs $\hat{W}$ and $\hat{V}$ represented by Eq. (1.24). The length of the chain is $N$ that should be even to be divided into two halved subsystems $\hat{W}$ and $\hat{V}$.

Observables $\hat{W}$ and $\hat{V}$ are defined as the first and second blocks of spins, respectively, known as spin block operators (SBOs). Graphical representations of the SBOs are given in Fig. 1.5. Since, observables $\hat{W}$ and $\hat{V}$ are Hermitian but not unitary, then OTOC Eq. (1.15), will be

$$C(t) = \langle \hat{W}^2(t)\hat{V}(0)^2\rangle - \langle \hat{W}(t)\hat{V}(0)\hat{W}(t)\hat{V}(0)\rangle = C_2(t) - C_4(t) \tag{1.24}$$

where, $C_2(t)$ is named as two-point correlation and $C_4(t)$ is named as four-point correlations, and defined as: $C_2(t) = \langle \hat{W}^2(t)V\hat{(0)}^2\rangle$ and $C_4(t) = \langle \hat{W}(t)\hat{V}(0)\hat{W}(t)V\hat{(0)}\rangle$.

In recent years, OTOCs have been used in many areas; one important use of it is to distinguish the chaotic and regular regimes in the semiclassical and quantum. The following section briefly discusses the chaos and chaotic systems.

## 1.4 Chaos

The word chaos comes from the Greek word “Khaos," meaning this is a “gaping void." Mathematicians find that it is easy to “recognize chaos when you see it," but no easy way exists to define it. In general, chaos is defined as a phenomenon where a small change in the input implies a large change in the output [28]. They find space in many disciplines such as physics, economics, philosophy, biology, and engineering [29]. The nature of the output of the chaos is highly complex, and it is not predictable [30]. Determining the output of the chaos is also a very complex process.

[31]. Chaos is found in driven simple pendulums and double pendulums [32]. In the study of chaos in the above system, It is found that chaos is present and have different type of behavior in different type of systems. Following, we will discuss the chaos in different systems, such as classical, quantum, and spin systems.

## 1.5 Chaotic Systems

A system is said to be chaotic whenever its evolution trajectory depends very strongly on the initial conditions. This property implies that even for two infinitesimally close initial conditions, the observed trajectories display large deviations that vary exponentially with time. Several natural phenomena can be recognized as chaotic and chaotic analysis can also be found in the solar system [33], meteorology, brain, and heart of living organisms [34], etc. The dynamic behavior of a chaotic system is very hard to predict because it involves various complicated mathematical equations. Solutions of mathematical equations

of chaotic systems are complex and cannot be easily extrapolated. Chaos in the classical and quantum systems is named classical chaotic system and quantum chaotic system, respectively. Recently, chaos has been studied in the spin system. Following, we will discuss the chaos in the systems.

### 1.5.1 Classical chaotic System

Classical regular and chaotic systems are properly understood, and lots of studies have been done on them [35, 36, 37, 38]. There is a common technique that is used for analyzing the dynamics of the system is known as Hamilton's equations of motion. If a system is described with $n$ degrees of freedom, then the classical dynamics of a system are described by using $2n$-dimensional phase space trajectory. Phase space is a multidimensional space in which axes are defined by position and conjugate momenta. Systems can be distinguished as integrable and nonintegrable systems by analyzing the constants of motion [32]. An integrable system has n independent constants of motion; however, the nonintegrable system has less than n constants of motion. Let us consider a system, a harmonic oscillator with one degree of freedom. Hamiltonian of it is given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$. Dimension of phase space is two in which one axis is position $(x)$, and the other is corresponding conjugate momenta $(p)$. In the harmonic oscillator, energy remains unchanged during the entire dynamics. Thus the number of constant quantities matches the degrees of freedom making the harmonic oscillator an integrable system. Nonintegrable systems often exhibit one of the most surprising properties, *i.e.,* unpredictability in evolution and under-applied perturbation. This unpredictability is due to exponential variation with changing the initial conditions. This property of the dynamical system is known as chaos.

Chaos is studied in many fields of classical physics, *e.g.,* dynamics of fluids, stars, and some biophysical models [35, 36, 37, 38]). Many tools, for example, level spacing distribution, amongst several others are used to distinguish the regular and chaotic classical

systems. Classical chaos can be diagnosed by exponential sensitivity with the initial condition. The exponent of the exponential is defined as the Lyapunov exponent (LE), and it is denoted by $\lambda_L$. The exponential growth of chaotic systems is known as the “butterfly effect" [8, 9, 10]. The butterfly effect serves as a diagnostic measure of chaos which is defined as small perturbations in the initial state leading to exponential growth.

Classical chaos in a system is dependent on initial conditions [36]. The exponential behavior of OTOC is also found in the infinitesimally small region surrounding critical points of the phase structure. The exponential growth of OTOC at a critical point is studied in the Lipkin-Meshkov-Glick (LMG) model [39]. This is a classical system having single-degree-of-freedom in nuclear physics [40], and it is realized with experiment by cold atoms [41, 42], and nuclear magnetic resonance [43].

Some systems show non-chaotic behavior in classical mechanics; however, they show chaotic behavior in quantum mechanics [27, 44]. This is due to the instability provided by quantum mechanics in a region where classical dynamics are stable. To understand the reason for instability in quantum mechanics, we will briefly discuss quantum mechanics concepts and quantum chaos.

### 1.5.2 Quantum chaotic system

Classical physics could not be used for the explanation of a few phenomena. Explanation of these phenomena led to the advent of ideas now known as quantum physics. Quantum theory depicts an evolving wave function in accordance with the linear Schrödinger equation, in contrast to the phase space evolution in classical physics. Here, variables of classical physics are replaced by Hermitian observables. Heisenberg’s uncertainty principle is applicable in quantum physics but it is not applied in classical physics. This principle is stated as the conjugate variables of any particle can not be determined simultaneously accurately. It fails to describe the quantum system by the phase space. In addition to this,

the linear Schrödinger equation does not provide any exponential variation of the wave function by using evolution. In quantum mechanics, the unitary property of the operator applies specific constraints under which the distance of two initial wave functions does not change under evolution. In the mathematical form, the above statement can be written as $\langle\psi_0|\hat{U}^{\dagger}(t)\hat{U}(t)|\psi_1\rangle = \langle\psi_0|\psi_1\rangle$ which is true for all $t$.

The system has some specific behavior in the quantum domain, such as wave-particle duality and the uncertainty principle. Such inherent properties change the appearance of the sharp features obtained in classical dynamics, such as the sensitive accordance with initial conditions, which are implied within the butterfly effect. In the chaotic system, this effect becomes crucial because butterfly effects are destroyed. In contrast, isolated systems experience the butterfly effect after a short period of semiclassical evolution. The short period depends on the system size in a logarithmic manner given as $t_E \equiv \log(N)$, where $t_E$ is defined as Ehrenfest time [45, 46]. Butterfly effect is recognized by the exponential growth of OTOC after the Ehrenfest time. Exponent of exponential growth is named as Lyapunov exponent.

In recent years, OTOC is also used for the discussion of the chaos and dynamics of the Ising spin systems. Researchers focused on the chaotic nature as well as the dynamics of the spin systems by OTOC. We will discuss the chaos in the spin systems. Before the discussion of chaos and dynamics of the spin systems, it is necessary to present a brief overview of spins, spin-spin interaction, and spin chains.

## 1.6 Spin-1/2

Spin is a purely quantum mechanical concept, and it is an intrinsic quantity [47]. A theoretical proposal of spin is given by Goudsmit and Uhlenbech to describe the vector atom model. After that, experimental verification was given by Stern and Gerlach in their experiment known as the “Stern-Gerlach experiment". The experimental arrangement is

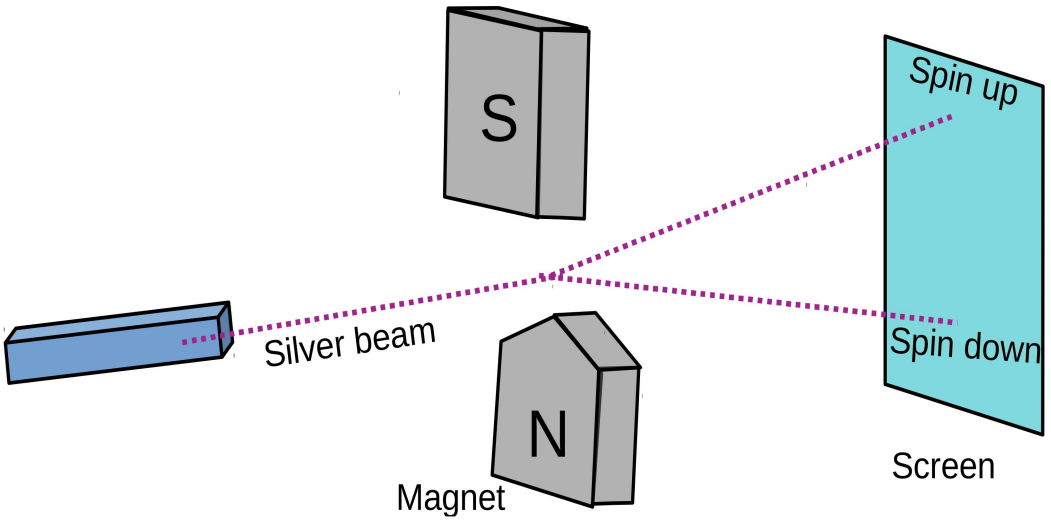


Fig. 1.6 Setup of Stern-Gerlach experiment.

illustrated in Fig 1.6. In the experiment, an oven produced a beam of the silver atom, which was passed through a nonuniform magnetic field. Two spots are observed on the screen, which is symmetric from the point of no deflection in the absence of the fields. The observation can be explained by the spin of the electron which gives rise to the magnetic moment of an atom **S**, *i.e.,* $\mu \propto \mathbf{S}$. The energy corresponding to the magnetic moment and the magnetic field is given by

$$E = -\vec{\mu}.\vec{B}. \tag{1.25}$$

A nonuniform magnetic field ($\mathbf{B} \equiv \vec{B}$) applies force to the silver atoms. Other components of **B** are ignored except the z-component because the atom is very heavy. Therefore, force on the atom due to the z-component of a magnetic field is given as

$$F_z = \frac{\partial}{\partial z}(\vec{\mu}.\vec{B}) \simeq \mu_z \frac{\partial B_z}{\partial z}. \tag{1.26}$$

The direction of the force on the atom depends upon the value of the z-component of magnetic moment ($\mu_z$). If $\mu_z > 0$, then a downward force is applied on the atom; however, if $\mu_z < 0$, then an upward force is applied in the atom. Hence, the beam of the silver atom

is split according to the value of $\mu_z$, and it was found that the silver atoms struck the plate only in two regions, symmetrically situated about the point of no deflection. The variation of the silver beam has only two components which dictate the magnetic moment vector of silver atoms must have only two orientations. The proportional condition of magnetic moment with spin implies that the z-component of spin also has two orientations. This confirms the theoretical proposal of spins. Now, let us discuss the interaction of two spins in the following subsection.

### 1.6.1 Spin-spin interaction

Let us consider two electrons with suppressed orbital degrees of freedom and having only spin degrees of freedom. The total spin operator of this system of two electrons is written as

$$\mathbf{S} = \mathbf{S_1} \otimes \mathbb{1} + \mathbb{1} \otimes \mathbf{S_2}, \tag{1.27}$$

where $\mathbb{1}$ is the identity operator of dimension $2 \times 2$, and it is placed at the spin space of electron $2$ $(1)$ in the first (second) term. The individual spins belong to the Hilbert space of two dimensions, and the complete system of two elections is described by Hilbert space of $2 \otimes 2$ i.e., 4 dimension. Commutation relations of spin operators at the same site and different sites are given as follows:

$$[S_{1x}, S_{1y}] = \mathrm{i}\hbar S_{1z},\ [S_{2x}, S_{2y}] = \mathrm{i}\hbar S_{1z}, \quad [S_{1x}, S_{2y}] = 0. \tag{1.28}$$

Operators $\mathbf{S^2} = (\mathbf{S_1} + \mathbf{S_2})^\mathbf{2}$, $S_z = S_{1z} + S_{2z}$, $S_{1z}$, and $S_{2z}$ has eigenvalues $s(s+1)\hbar^2$, $m\hbar$, $m_1\hbar$, and $m_2\hbar$, respectively. Ket vectors corresponding to the spin state of two electrons in terms of the Eigen kets of $\mathbf{S^2}$ and $S_z$ can be written as $|s = 1, m = \pm 1, 0\rangle$, $|s = 0, m = 0\rangle$, where $s = 1$ represent spin-triplet $(2s + 1 = 3)$ and $s = 0$ represent spin singlet $(2s + 1 = 1)$

state. Corresponding to spin-triplet ($s=1$), there are three basis vectors $|s=1,m=1\rangle, |s=1,m=0\rangle, |s=1,m=-1\rangle$, and corresponding to spin-singlet ($s=0$), there is one basis vector as $|s=0,m=0\rangle$. The interaction of two spins is given as

$$\hat{H} = J_{12}\mathbf{S_1}.\mathbf{S_2}, \tag{1.29}$$

where, $J_{12}$ is the interaction stength. Value of $\hat{H}$ depends on the $\mathbf{S_1}.\mathbf{S_2}$. Let us calculate the $\mathbf{S_1}.\mathbf{S_2}$ for singlet and triplet states. Since, $\mathbf{S^2} = \mathbf{S_1^2} + \mathbf{S_2^2} + \mathbf{2S_1}.\mathbf{S_2}$, therefore $\mathbf{S_1}.\mathbf{S_2} = (\mathbf{S^2} - \mathbf{S_1^2} - \mathbf{S_2^2})/2$. Value of $\mathbf{S_1}.\mathbf{S_2}$ for the singlet state ($s=0$) will be,

$$\begin{aligned}
\mathbf{S_1}.\mathbf{S_2} &= \frac{J_{12}}{2}[s(s+1) - s_1(s_1+1) - s_2(s_2+1)], \\
&= \frac{J_{12}\hbar^2}{2}\left[0(0+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right], \\
&= -\frac{3J_{12}\hbar^2}{4}.
\end{aligned}$$

Similarly, for triplet state ($s=1$), $\mathbf{S_1}.\mathbf{S_2} = \frac{J_{12}\hbar^2}{4}$. If $J_{12} > 0$, then the singlet state has lower energy than the triplet state, and the system is in a ferromagnetic state, however, if $J_{12} < 0$, then the triplet state has minimum energy, and the system is represented as an antiferromagnetic state. In the next section, we will discuss a system of $N (N > 2)$ interacting spins. Hamiltonian corresponding to such a system is given by Heisenberg, and the model is named the Heisenberg model.

## 1.7 Heisenberg model

Consider a lattice with $N$ site ($N \to \infty$) and at each lattice site $i$ a spin $\vec{\mathbf{S}}_\mathbf{i}$ is placed. The spin-spin interaction between the spins is given as $\vec{\mathbf{S}}_\mathbf{i} \cdot \vec{\mathbf{S}}_\mathbf{j}$. If we consider all the pairs of spin-spin interactions then

$$H = \frac{1}{2}\sum_{i \neq j} J_{ij}\vec{\mathbf{S}}_\mathbf{i}.\vec{\mathbf{S}}_\mathbf{j}, \tag{1.30}$$

where, indices $i$ and $j$ are run from site 1 to $N$ on a lattice. Coefficient $J_{ij}$ is exchange constants or interaction strength, and it is symmetric, *i.e.,* $J_{ij} = J_{ji}$. Interaction strength decreases as increases distance between indices $i$ and $j$. Value of $J_{ij}$ can be either positive or negative. Antiparallel alignment of spin favors a positive value of $J_{ij}$. It is the case of antiferromagnetic. Parallel alignment of the spins favors a negative value of $J_{ij}$, which is the case of ferromagnetic. Factor $\frac{1}{2}$ in the Hamiltonian avoids double-counting the bonds. Spin components at the same site follow commutation relations as

$$[S_j^{\alpha}, S_j^{\beta}] = i\varepsilon_{\alpha\beta\gamma} S_j^{\gamma} \qquad (\alpha, \beta, \gamma = x, y, z), \tag{1.31}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol, its value will be $+1(-1)$ if $\alpha$, $\beta$ $\gamma$ in cyclic (non-cyclic) order and 0 when at least any two variables ($\alpha$, $\beta$ $\gamma$) are same. However, spins at different sites commute with each other, *i.e.,* $[S_i^{\alpha}, S_j^{\beta}] = 0$.

## 1.8 Ising Model

Wilhelm Lenz first introduced the Ising model in the year 1922. He made the assumption that particles in a crystal structure can freely revolve around a given lattice point [48]. The Hamiltonian creates the complete model, which is a combination of two pieces, one representing the energy contribution from particle-particle interaction and the other representing the energy contribution from constraints on the system. In the Ising model, the constraint is applied from the magnetic field. The effect of the field on the quantum Ising system is to rotate the spins. To study the rotation of spins, apply anisotropic magnets to the quantum Ising chain in both transverse and longitudinal directions. The involvement of the field term in the Ising spin chain provides two terms in the Hamiltonian, one corresponding to the longitudinal field and another corresponding to the transverse field. Hence, the total

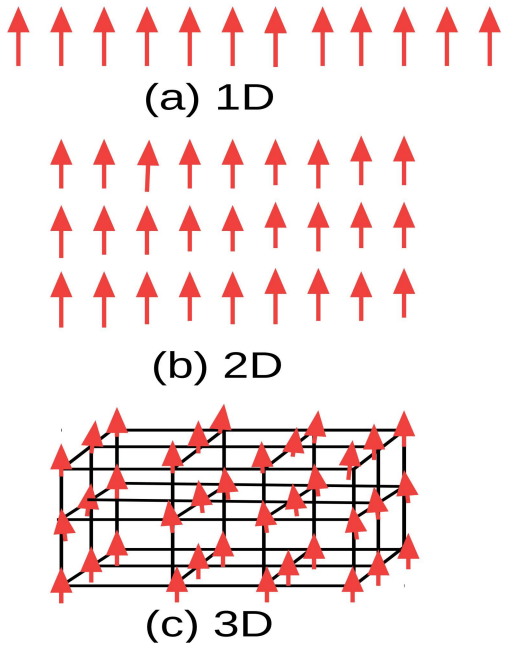


Fig. 1.7 Arrangement of spins at lattices in one, two, and three dimensions.

Hamiltonian will be

$$\hat{H} = J_x \hat{H}_{xx} + h_x \hat{H}_x + h_z \hat{H}_z, \tag{1.32}$$

where, $\hat{H}_{xx} = \sum_{l=1}^{N} \hat{\sigma}_l^x \sigma_{l+1}^x$, $\hat{H}_z = \sum_{l=1}^{N} \hat{\sigma}_l^z$ and $\hat{H}_x = \sum_{l=1}^{N} \hat{\sigma}_l^x$ ($\hat{\sigma}_l^x = \frac{\hbar}{2}\hat{S}_l^x$). Replace $J_{ij}$ by $J_x$ as we consider only nearest neighbor interaction in the x-direction. $h_x$ is the strength of the continuous and constant longitudinal magnetic field, and $h_z$ is the strength of the transverse magnetic.The dimension of the lattice could be one, two, or three. Corresponding to the lattice's one, two, and three dimensions, the Ising model is known as the one, two, and three-dimensional Ising model. A pictorial representation of it is given in Fig. 1.7. The following subsection discusses the boundary condition of the spin systems.

### 1.8.1 Boundary conditions

We consider a one-dimensional lattice with $N$ sites and spin-1/2 particles (say electron) situated on each site. Spins get interact with their neighbors. The effect of a spin at a site is determined by the interactions of spin with the other spins in the model. It is commonly taken as either nearest neighbor or next nearest neighbor because the effect of interaction decreases as the distance between the spins increases. A spin chain based on the nature of boundaries can be classified into

1. Periodic boundary condition
2. Open boundary condition

1. **Periodic boundary condition:**

   If both ends of the chain are connected, then a one-dimensional chain is called a closed chain. Periodic boundary condition means that system repeats after $N$th spin counting , *i.e.,* $\hat{\sigma}_{N+1} \equiv \hat{\sigma}_1$ as shown in Fig. 1.8 (Left). The coupling term in case of periodic boundary conditions will be

$$\hat{H}_{xx} = J_x \sum_{l=1}^{N} \hat{\sigma}_l^x \hat{\sigma}_{l+1}^x, \tag{1.33}$$

2. **Open boundary condition:**

   In an open chain case, both ends of the chain are not connected with each other, as shown in Fig. 1.8 (Right). In open boundary conditions, Hamiltonian is defined as

$$\hat{H}_{xx} = J_x \sum_{l=1}^{N-1} \hat{\sigma}_l^x \hat{\sigma}_{l+1}^x. \tag{1.34}$$

In the open chain case, one term $\hat{\sigma}_N^x \hat{\sigma}_1^x$ is absent from the Hamiltonian as compared to closed chain cases of the same system size.

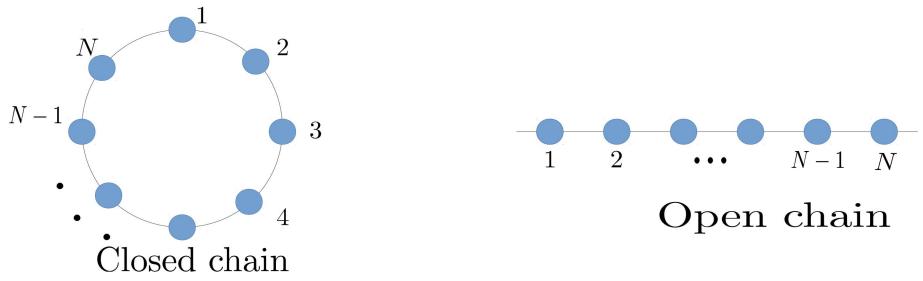


Fig. 1.8 One-dimensional lattice configuration with (Left) periodic boundary condition and (Right) open boundary condition. Solid lines denote interaction.

## 1.9 Transverse Ising model

The transverse Ising model is derived form the Ising model in which a constant magnetic field is applied in the coupling's transverse direction, and a longitudinal field is absent. It has long been an appreciated and well-studied model. It is dynamically interesting and can be easily implemented in quantum mechanics. The transverse Ising model has been studied in several contexts, including entanglement, state transport, and the quantum phase transition at zero temperature that separates ferromagnetic and paramagnetic phases. [49, 50, 51, 52]. It is integrable due to a mapping from interacting spins to a collection of noninteracting spinless fermions via the Jordan-Wigner transformation.

## 1.10 Floquet transverse Ising model

The Floquet spin model is a variant of the transverse Ising model. In this model, a periodic kicked transverse field is applied for the period $\tau_0$. A longitudinal constant field is applied for a period $\tau_1$. Hence total time period of the periodic field is $T = \tau_0 + \tau_1$ [Fig. 1.9]. The involvement of a time-periodic kicked field displays very interesting and peculiar behavior in the Ising system [25, 53, 54, 55]. A graphical representation of a periodic field in the form of delta pulses applied on the spin chain is given in Fig. 1.9.

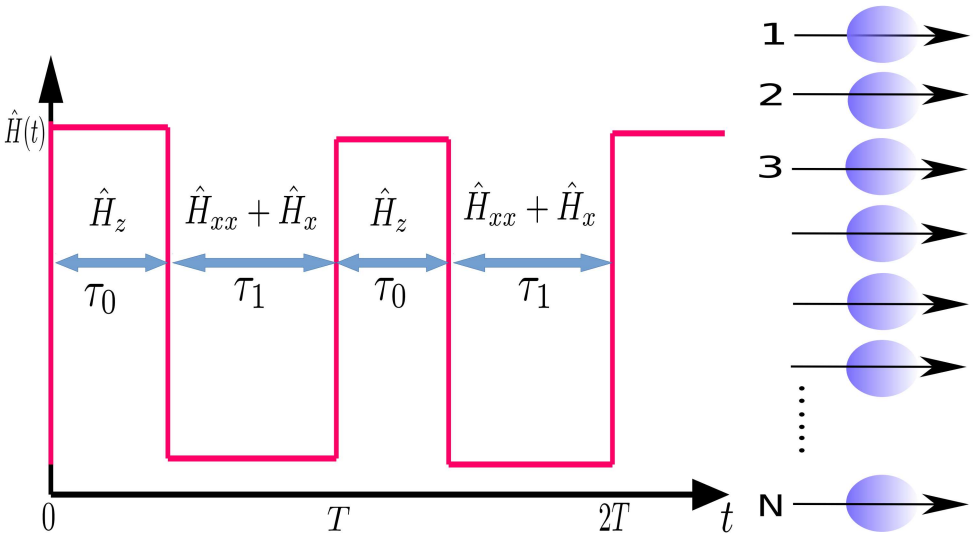


Fig. 1.9 Spin chain experience a periodic quench by non-commuting Hamiltonian functions $\hat{H}_z$ and $\hat{H}_x$ for durations $\tau_0$ and $\tau_1$, respectively. The complete system is periodic with period $T = \tau_0 + \tau_1$.

The Hamiltonian defined by Eq. (1.32) will take the form as

$$\hat{H}_F = J_x \hat{H}_{xx} + h_x \hat{H}_x + h_z \sum_{n=-\infty}^{\infty} \delta\left(n - \frac{t}{\tau_0}\right) \hat{H}_z. \tag{1.35}$$

Here, $\hat{H}_{xx} = \sum_{l=1}^{N-1} \hat{\sigma}_l^x \hat{\sigma}_{i+1}^x$ in the open chain, and $\hat{H}_{xx} = \sum_{l=1}^{N} \hat{\sigma}_l^x \hat{\sigma}_{i+1}^x$ in a closed chain, $\hat{H}_x = \sum_{l=1}^{N} \hat{\sigma}_l^x$ and $\hat{H}_z = \sum_{l=1}^{N} \hat{\sigma}_l^z$. When both longitudinal and transverse fields are present then the system will be nonintegrable; however, it will be integrable in the absence of any one field.

### 1.10.1 Floquet map

The wave function under time evolution is defined as follows

$$\psi(x,t) = \hat{U}(t)\psi(x,0), \tag{1.36}$$

where, $\hat{U}(t)$ is a time evolution operator that evolve the wave-function from $t = 0$ to $t$. Time-dependent Schrödinger equation is given as

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi. \tag{1.37}$$

Inserting Eq. (1.36) in Eq. (1.37), we get

$$\mathrm{i}\hbar\frac{\partial\hat{U}(t)}{\partial t} = \hat{H}\hat{U}(t). \tag{1.38}$$

Initially, at time $t = 0$, $\hat{U}(0) = 1$. Since Hamiltonian is Hermitian so $\hat{U}(t)$ should be unitary. For the proof we take the adjoint of Eq. (1.38),

$$-\mathrm{i}\hbar\frac{\partial\hat{U}^{\dagger}(t)}{\partial t} = \hat{U}^{\dagger}(t)\hat{H}, \tag{1.39}$$

Multiply $\hat{U}^{\dagger}(t)$ in both sides of Eq. (1.38) from the left and $\hat{U}(t)$ in both sides of Eq. (1.39) from the right and take the difference of it. We get

$$\frac{d(\hat{U}^{\dagger}(t)\hat{U}(t))}{dt} = 0, \tag{1.40}$$

From Eq. (1.40), it is obvious $\hat{U}^{\dagger}(t)\hat{U}(t)$ is constant. From the initial condition $\hat{U}(0) = 1$ it follows that the constant must be one,

$$\hat{U}^{\dagger}(t)\hat{U}(t) = 1. \tag{1.41}$$

With time $t = \tau$ Eq. (1.36) will take the form

$$\psi(x,\tau) = \hat{\mathscr{U}}_x(\tau)\psi(x,0), \tag{1.42}$$

Generalized form of the above equation is given as

$$\psi(x,n\tau) = [\hat{\mathscr{U}}_x(\tau)]^n \psi(x,0). \tag{1.43}$$

Hence, the time-evolution operator of the periodic system is

$$[\hat{U}(n\tau)] = [\hat{\mathscr{U}}_x(\tau)]^n. \tag{1.44}$$

Observation of a periodic system at arbitrary time $t = nT$ $(n = 1, 2\cdots)$, can be done by the knowledge of $\hat{\mathscr{U}}_x(\tau)$. In the Floquet system, steps like drive between Hamiltonians $\hat{H}_z$ of duration $\tau_0$ and $\hat{H}_x$ of duration $\tau_1$ are used. Corresponding to such type of drive, the propagator connecting states over a single time interval $T = \tau_0 + \tau_1$ is the Floquet operator, and this operator is denoted by $\hat{\mathscr{U}}_x$

$$\hat{\mathscr{U}}_x = \exp\left[-i\tau_1(J_x\hat{H}_{xx} + h_x\hat{H}_x)\right]\exp\left(-i\tau_0 h_z\hat{H}_z\right). \tag{1.45}$$

In the absence of longitudinal fields ($h_x = 0$), $\hat{\mathscr{U}}_x$ changes in $\hat{\mathscr{U}}_0$.

### 1.10.2 Dzyaloshinskii–Moriya interaction (DMI)

The DMI is a spin-spin interaction in a system that has no inversion symmetry. The concept of DMI comes into consideration after the two proposals; first, Dzyaloshinskii proposed that the low symmetry and spin-orbit coupling lead to an antisymmetric exchange interaction [56], and second by Moriya, who explained how to use a microscopic model to determine the antisymmetric exchange interaction for localized magnetic systems. [57]. Let us consider two magnetic spins $\sigma_i$ and $\sigma_j$. The total magnetic exchange interaction between these spins is known as DMI, and Hamiltonian corresponding to these terms can

be written as

$$H_{ij}^{DM} = \mathbf{D_{ij}}.\sigma_i \times \sigma_j, \tag{1.46}$$

where $\mathbf{D_{ij}}$ is a DM vector. Graphical representation of the interaction of spins and orientation of the DM vector are shown in Fig. 1.10.

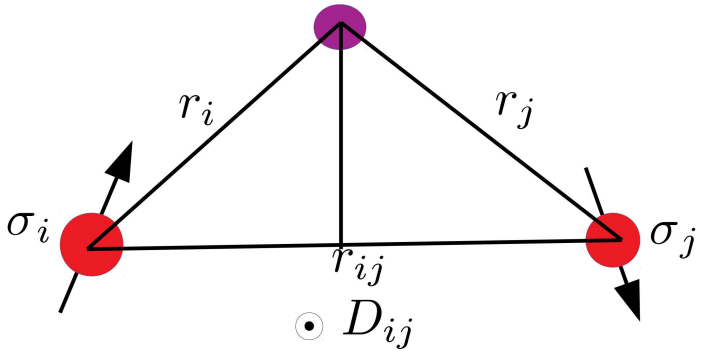


Fig. 1.10 Local geometry to determine the DM vector's orientation.

### 1.10.3 2D square-lattice system with DMI interacation

Let us consider a 2D square spin system and introduce ferroelectric polarization by an external electric field. Hamiltonian corresponding to the square lattice with applied electric field is given as

$$\hat{H}_s = J_1 \sum_{\langle n,m \rangle} \hat{\sigma}_n \hat{\sigma}_m + J_2 \sum_{\langle\langle n,m \rangle\rangle} \hat{\sigma}_n \hat{\sigma}_m - \mathbf{P} \cdot \mathbf{E}. \tag{1.47}$$

Here, we replace $J_{ij}$ by $J_1$ and $J_2$ for the nearest and second nearest-neighbor interaction strength, respectively. $\langle n,m \rangle$ and $\langle\langle n,m \rangle\rangle$) are the representation of the nearest and second nearest-neighbor interaction, respectively. $\mathbf{P} \cdot \mathbf{E}$ describes a coupling of the ferroelectric polarization $\mathbf{P} = g_{\mathrm{ME}} \mathbf{e}_{i,i+1}^{x} \times (\hat{\sigma}_i \times \hat{\sigma}_{i+1})$ with an applied external electric field and mimics an effective DMI term $D = E_y g_{\mathrm{ME}}$ breaking the left-right symmetry, where $g_{\mathrm{ME}}$ is the

magneto-electric coupling constant. This may be written as follows

$$-\mathbf{P}\cdot\mathbf{E} = D\sum_{n}(\hat{\sigma}_n \times \hat{\sigma}_{n+1})_z. \tag{1.48}$$

Here we consider only the nearest neighbor DMI and only in one direction. Hence, Hamiltonian will be

$$\hat{H}_s = J_1 \sum_{\langle n,m\rangle} \hat{\sigma}_n\hat{\sigma}_m + J_2 \sum_{\langle\langle n,m\rangle\rangle} \hat{\sigma}_n\hat{\sigma}_m - D\sum_{n}(\hat{\sigma}_n \times \hat{\sigma}_{n+1})_z. \tag{1.49}$$

In the next section, we will discuss the distinguisher of regular and chaotic spin systems and also discuss the dynamics of OTOC in the systems.

## 1.11 Chaos in spin system

Recently, OTOCs are explored rapidly in the spin systems to describe the dynamics and saturation behavior of the systems [58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69]. Some spin models such as Luttinger liquid model [67], XY model [66], XXZ model [68, 69], Sachdev-Ye-Kitaev (SYK) model [70], integrable quantum Ising spin model with constant magnetic field [58] and tilted magnetic field [62], XXZ spin model, Heisenberg spin model with random magnetic fields [62], and some other integrable and nonintegrable spin models [59, 60, 61, 63, 64, 65] are reported in the literature. In all the above studies, no exponential growth of OTOC in the dynamic region is found. Therefore, it is difficult to distinguish between the regular and chaotic systems using only OTOC. Usually, it is distinguished by spectral analysis of the systems.

In spin system, spectral properties of the systems either nearest neighbor spacing distribution (NNSD) of the energy spectrum [71, 72], or the local properties of energy eigenvectors [73, 74, 75] are used to distinguish integrable, chaotic, near-integrable, and

near-chaotic regimes. For the calculation of NNSD, initially, it is necessary to identify the system's symmetries. After that Hamiltonian generated by removing the symmetries is block diagonalized. Different spin systems have different symmetries. Here we will discuss symmetries in the Floquet system, which is studied in this thesis. The Floquet system has only a "bit-reversal" symmetry in the open boundary conditions and let's define the bit-reversal operator by $\hat{B}$. The operation of this operator is given as $\hat{B}|s_1, s_2, \cdots, s_N\rangle = |s_N, \cdots, s_2, s_1\rangle$, and it follow the commutation relation with Floquet map *i.e.,* $[\hat{U}, \hat{B}] = 0$. $|s_i\rangle$ represents a basis state in the basis of $S_z$. We collect complete basis sets into two groups. The first group contains the state that will not change by after the operator as $\hat{B}$ as $\hat{B}|s_1, s_2, \cdots, s_N\rangle = |s_1, s_2, \cdots, s_N\rangle$. This group is called Palindrome. The second group contains the state which gets reflection by applying operator $\hat{B}$ as $\hat{B}|s_1, s_2, \cdots, s_N\rangle = |s_N, \cdots, s_2, s_1\rangle$. This group is called non-palindrome. Since $\hat{B}^2 = 1$, the eigenvalues of bit reversal operator $\hat{B}$ are $\pm 1$. Corresponding to the eigenvalue $+1/-1$ of bit reversal observable $\hat{B}$, eigenstates can be classified as even/odd. All the palindromes come in the group of even states; however, non-palindromes contain half-even and half-odd states. The sum/difference of the non-palindrome state with its reflection provides even/odd states. The dimension of the odd subspace is equal to $\frac{1}{2}(2^N - 2^{N/2})$, while the even subspace is equal to $\frac{1}{2}(2^N + 2^{N/2})$.

NNSD is used as a distinguisher between chaotic and regular systems. If NNSD displays Wigner-Dyson distribution behavior, then the system is said to be a strongly chaotic system. [76, 77, 78] In mathematical form, the Wigner-Dyson distribution is defined as follows

$$P_W(s) = \frac{\pi s}{2} e^{-\pi s^2/4}, \tag{1.50}$$

If NNSD displays Poisson-type distribution, then the system is said to be a regular system. In the mathematical form, it is given as

$$P_P(s) = e^{-s}. \tag{1.51}$$

If the shape of the distribution lies near the Poisson type and near the Wigner-Dyson type, then it will be near-integrable and near chaotic, respectively.

OTOC can be utilized to count magnons that flow from magnonic crystals. Before discussing the flow of magnons, we will briefly discuss magnons and magnonic crystals in the following section.

## 1.12 Magnons and spin wave

In a magnetic material, the particle-like behavior of spin excitations is known as magnon; however, the wave-like behavior of spin excitation is called as spin wave. The movement of a magnon or spin wave in the magnonic crystal is referred to as the spin dynamics phenomenon. It has attracted considerable attention among researchers in recent years. For spin excitations, deposition and nanopatterning techniques are now used in ferromagnetic materials. Other techniques also used are: localization [79], spin wave quantization [80], and interference [81]. Spin waves have both quantum and classical properties of waves. They can tunnel through magnetic barriers and reflect when incident on magnetic potential wells. [82]. In the magnonic crystal, propagation of spin waves displays different behavior than in uniform media [83]. Propagation of spin waves does not display the band gap in uniform media but in the magnonic crystal. Spin waves can not propagate through the band gap.

## 1.13 Magnonic crystal

Magnonic crystals are synthetic magnetic materials whose magnetic characteristics exhibit regular spatial variation, *i.e.,* periodic variation in space. In such a periodic arrangement, the spin wave spectrum is affected by Bragg scattering, which causes band gaps. Magnonic crystals should have low-damping magnetic materials for the study of spin wave dynamics. Among all low-damping magnetic materials, mono crystalline YIG ($Y_3Fe_5O_{12}$) is the most useful material [84]. Spin waves can propagate to the centimeter distances in the YIG due to low damping. YIG-based MCs are characterized into two types on the basis of the characteristics of the transmission.

1. Simplest design of MC is one-dimensional grooved structures in which grooves are drawn on the MC to make spin-wave waveguides with periodically changing thickness [85].

2. This type of magnonic crystal is controlled by current, and it has specific properties such as gradual tuning and modifying crystal characteristics quickly [86].

In the transmission band, there is only one rejection band in the case of current-controlled; however, in the case of grooved MC, which has many rejection bands. This rejection band means the region of frequency where propagation of spin waves is prohibited [85, 86]. The size of the rejection band can be adjusted and [87]. It is also possible to control the number of rejection bands [88], which allows the creation of microwave filters with a single or multiple bands. Micrometer and sub-micrometer size YIG-based grooved MCs with desired band gap characteristics are used for the study of spin wave dynamics.

### 1.13.1 Structure of magnonic crystal

A grooved MCs can be fabricated from an YIG film. YIG poses a cubic crystal structure of dimension 12.376 Å. Each unit cell contains eighty atoms Grooves were deposited on the

YIG film using a lithography procedure in a few nanometer steps. A prepared grooved thin film of YIG works as a spin wave waveguide. YIG film is deposited on a substrate. For the propagation of a spin wave, the substrate should have a similar lattice constant of YIG. The lattice constant of Gallium gadolinium garnet (GGG) is $(12.383\ \text{Å})$ and is exactly matched with the lattice constant of YIG. It is used in the production of flawless films. However, YIG can be lightly doped with gallium or lanthanum to produce the optimum matching.

A magnonic transistor has been proposed using a YIG magnonic crystal with periodic modulation thickness [89]. Similar to the electronic transistor, a magnonic transistor has a source, drain, and gate antennas. A gate antenna injects magnons of a frequency $\omega_G$ into the crystal that matches the magnonic crystal band gap. The gate magnons may acquire a high density in the crystal. Magnons emitted from a source with wave vector $\mathbf{k}_s$ flow in the direction of the drain. Interaction between the source magnons and the magnonic crystal magnons is a Four-magnon scattering process. Due to the scattering, the source magnon current attenuates in the magnonic crystal therefore weak signal arrives at the drain. The relaxation process is swift if the following condition holds [89, 90]

$$k_s = \frac{m_0 \pi}{a_0}, \tag{1.52}$$

where $m_0$ is the integer, and $a_0$ is the crystal lattice constant. Fig. 1.11 provides a schematic illustration of the magnonic transistor. The primary component of it is a YIG film with several parallel grooves on its surface. Microstrip antennas are used to inject magnons from a source terminal and detect them from a drain terminal. Magnon was injected from the gate terminal to control the magnon current flowing through the source-to-drain. Each groove reflects the propagating magnons around one percent. Only those magnons will be scattered back, wavelengths of which satisfy the Bragg condition $k_s = \frac{m_0 \pi}{a}$, leading to produce rejection bands (band gaps) in a system.

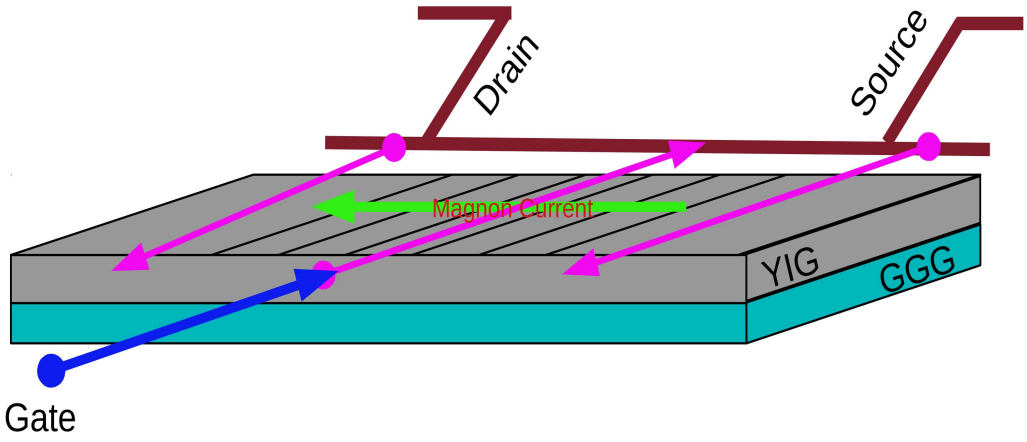


Fig. 1.11 Pictorial representation of magnonic transistor in which YIG film with grooves is deposited on GGG substrate. Three terminals of the microstrip antenna, such as the source, drain, and gate, are added to inject and measure the magnons.

According to the transistor's operating principle, magnons are injected into its source at a frequency that falls inside the magnon transmission band. The S-magnons propagate almost distortion-free toward the drain when no magnon is in the gate of the transistor. Magnons are injected into the gate region to influence the flowing magnon through the source to drain. To confine the magnon within the magnonic crystal, the frequency of the G-magnon should be in the center of the band gap of the magnonic crystal. The G-magnon concentration can be greatly increased because of this confinement. Injected S-magnons into the source region are scattered as they pass through the G-magnon-populated transistor gate; therefore, only partially reach the drain terminal.

## 1.14 Outline of the thesis

In chapter 2, we do analytical calculations to find the formula of TMOTOC. We will do a comparative study of the revival time speed of correlation propagation in TMOTOC and LMOTOC. After that, we will verify the phase structure of the Floquet system in $\tau_0 - \tau_1$ parameter space, numerically.

In chapter 3, we will discuss three different regions of OTOC named characteristic (OTOC remain zero), dynamic (OTOC grow), and saturation (OTOC start to saturate) regimes of the LMOTOC and TMOTOC in the integrable and nonintegrable Floquet system. We will present a comparative study of LMOTOC and TMOTOC in all the regions.

Further, in chapter 4, we use symmetric spin block observables instead of local spin observables to study OTOC in spin chains. We chose the block-spin observables to calculate OTOC in pre-scrambling and post-scrambling time regimes and analyzed the growth of OTOC, and replaced spin block observables with random block observables to analyze the saturation behavior of OTOC. We will show the averaged OTOC over random Hermitian observables is exactly the same as operator entanglement entropy.

Finally, in chapter 5, we utilize OTOCs as a quantifier for quantum information currents in a 2D Heisenberg spin system with Dzyloshinski Moriya interaction. we provide a concept of quantum information diode based on magnonic crystals.

In chapter 6, we will summarize the results. We will also discuss future plans briefly.