Chapter 1

Introduction

Control system theory is a foundational discipline that plays a pivotal role in regulating and managing the behavior of dynamic systems across a diverse range of applications. It encompasses a comprehensive set of principles, methodologies, and techniques designed to ensure the stability, performance, and robustness of systems subject to external influences, disturbances, or uncertainties. At its core, control system theory is concerned with understanding, analyzing, and designing systems that can automatically maintain desired outputs or trajectories. Whether applied in engineering, physics, economics, or biological systems, control theory provides a systematic framework to model, analyze, and optimize the behavior of dynamic systems.

Key concepts within control system theory include feedback, where the system output is compared to a reference signal, and adjustments are made to minimize the discrepency or error. This feedback loop allows for continuous monitoring and correction, ensuring that the system responds appropriately to changing conditions. The study of control systems is broadly categorized into two main types: continuous-time and discrete-time systems. Continuous-time systems deal with signals that vary smoothly over time, while discrete-time systems operate with signals that are sampled at discrete intervals. Both types find applications in various domains, from electrical and mechanical engineering to aerospace, chemical processes, and beyond.

Control system theory involves the use of mathematical models, often in the form of differential equations or difference equations, to represent the dynamics of the system. Stability analysis is a critical aspect, assessing the system's tendency to return to a desired state after perturbations. Additionally, control design methodologies aim to determine the optimal parameters for controllers that govern the system's behavior. The development and application of various control strategies, such as proportional-integral-derivative (PID) controllers, state-space control, and optimal control, further contribute to the versatility and efficiency of control systems.

Further, control theory can be classified into conventional control, predominantly characterized by its continuous nature, and non-conventional control, exemplified by variable structure control (VSC), which incorporates some form of structural discontinuity. Conventional control and variable structure control represent distinct methodologies within the realm of control systems, each characterized by its unique principles and applications. Conventional control, rooted in frequency domain analysis, employs tools like Bode plots and Nyquist diagrams to assess stability, and often relies on proportional-integral-derivative controllers for system regulation. It is particularly well-suited for linear time-invariant systems and finds applications in areas such as process control, automotive engineering, etc. In contrast, variable structure control introduces a paradigm shift by utilizing discontinuous control actions and switching between different control structures. Sliding mode control (SMC), a common technique in VSC, constrains the system trajectory to a sliding surface, making it adept at handling nonlinearities and uncertainties. Stability analysis in VSC often involves Lyapunov stability theory, emphasizing robustness in the face of uncertainties. The choice between conventional control and variable structure control depends on the nature of the controlled system, with conventional control excelling in linear systems and variable structure control offering robust solutions for nonlinear and uncertain systems, such as those encountered in robotics and complex systems.

Sliding mode control is a robust and effective control strategy widely employed in the field of control systems. At its core, SMC operates by creating a sliding surface, a predefined subspace within the system's state space, on which the system's trajectory is forced to converge, thereby inducing a sliding motion. This sliding motion ensures that the system remains on the surface, minimizing the impact of uncertainties, disturbances, and variations. One of the key strengths of sliding mode control is its inherent ability to handle nonlinearities and external disturbances, making it particularly suitable for systems where uncertainties are prevalent. The control law in sliding mode control is typically discontinuous, introducing a switching element that facilitates robustness and stability.

SMC has found applications in diverse fields, including aerospace, robotics, power systems, and automotive control, owing to its ability to provide reliable performance in the face of dynamic uncertainties and varying operating conditions. Despite its effectiveness, the chattering phenomenon, a high-frequency oscillations around the sliding surface, can be a challenge in practical implementations, prompting ongoing research to refine and enhance the methodology for improved real-world applicability.

Owing to the availability of microcontrollers and computers has paved the way for the development of discrete-time sliding mode control (DSMC). Discrete-time sliding mode control is an extension of the well-established sliding mode control methodology tailored for discrete-time systems. In DSMC, the control law is designed to ensure that the system trajectory remains on a predefined sliding surface during each sampling interval. This approach is particularly valuable in applications where the system dynamics are inherently discrete or when dealing with digital control systems. DSMC does not inherit the complete disturbance rejection characteristics of its continuous-time counterpart, making it open for exploration and enhance the degree of robustness. The discrete nature of DSMC aligns with the practical constraints of digital control implementations, offering advantages in terms of computational efficiency and ease of implementation in discrete-time systems. We could commonly apply DSMC in fields such as digital control systems, embedded systems, and networked control systems where discrete-time dynamics play a crucial role. The design and analysis of discrete-time sliding mode control involve considerations of the system's discrete-time model, discretization techniques, and the impact of the sampling interval on the overall performance and stability of the control system. A crucial challenge arising from discretization is the loss of the ideal sliding mode behavior. This implies that the sliding variable no longer precisely adheres to the sliding manifold, giving rise to what is known as quasi-sliding mode (QSM). The corresponding region around the sliding manifold, known as the quasi-sliding mode band (QSMB), encapsulates the proximity within which the sliding variable operates.

One way to reduce the size of aforementioned quasi-sliding mode band width is by the use of higher-order discrete sliding modes. Higher-order discrete sliding mode control represents an advanced approach within the realm of discrete-time control systems, specifically tailored for systems influenced by bounded perturbations. This methodology involves increasing the relative degree of the sliding variable, enabling enhanced control over the system's behavior. By doing so, higher-order discrete sliding mode control aims to reduce the size of the quasi-sliding mode band, consequently improving robustness and performance. The twisitng algorithm and super-twisting algorithm are very popular second-order sliding mode in continuous-time counterpart. Over the years, many researchers have explored the discretized versions of aforementioned algorithm's continuous-time counterpart and reported the benefits and properties it retained. This pursuit not only enhances the system's robustness but also contributes to a more precise and efficient control scheme, making higher-order discrete sliding mode control a significant area of interest in advancing control system theories and applications.

1.1 Motivation

Control systems are instrumental in a multitude of engineering applications, spanning domains such as industrial automation, aerospace, transportation, robotics, and energy sectors. Their overarching goal is to influence the behavior of dynamic systems to attain specific performance objectives. Classical control methods like PID controllers and model-based control have undergone thorough examination and found widespread use in diverse fields. Nevertheless, with the progression of technology and the increasing complexity of systems, traditional control approaches are encountering constraints in fulfilling the requirements of contemporary applications. The evolving landscape of engineering challenges necessitates innovative control strategies that can effectively address the intricacies and demands posed by advanced systems and emerging technologies.

Addressing the demand for swift and precise responses within a finite-time framework represents a significant challenge in the design of control systems. Numerous real-world systems necessitate control strategies that not only ensure stability and accuracy but also achieve these objectives within a finite time duration. Classical control methods, such as PID control, typically achieve tracking or stabilization asymptotically, implying that the system never precisely reaches the desired trajectory. In industrial manufacturing, for instance, where precision is crucial, control systems must adhere to stringent timing constraints to optimize production efficiency and uphold quality standards. These instances underscore the imperative for control methodologies capable of ensuring finite-time convergence while meeting specific performance criteria. The quest for control strategies that combine speed, accuracy, and finite-time guarantees reflects the evolving landscape of control system design in response to the demands of diverse and dynamic applications.

This thesis is motivated by the aspiration to delve into and advance the domain of discrete-time sliding mode design based on difference equations with minima for dynamical systems. The impetus behind this exploration stems from the recognition of limitations inherent in classical and conventional control techniques, prompting a quest for methodologies that can effectively meet the evolving demands of modern applications. The overarching goal of this thesis is to introduce innovative approaches capable of facilitating time based and precise control, even in the presence of uncertainties and exogenous perturbations. Through the development of sophisticated discrete control strategies designed to ensure finite-time convergence, this research endeavors to elevate the capabilities and operational efficiency of diverse engineering systems, thereby contributing to the ongoing evolution of control methodologies in the face of contemporary challenges.

To be more specific, the exploration of discrete-time sliding mode is primarily driven by the pervasive uncertainties present in real-world systems. These uncertainties may stem from inherent factors within the system, such as parameter variations and modeling errors, or they may manifest externally in the form of disturbances. The influence of uncertainties poses a significant challenge to control performance in practical systems. Traditional control methods often struggle, especially when confronted with time-varying non-vanishing perturbations, leading to suboptimal performance or even instability. In response to these challenges, discrete-time sliding mode control methodologies emerge as a promising avenue. By ensuring convergence within a finite time frame, irrespective of the effects of uncertainties and perturbations, these methodologies present a robust solution for enhancing control performance in the face of real-world complexities.

The majority of existing methodologies in discrete-time sliding mode control are grounded in the reaching law approach, incorporating both switching and non-switching terms. Notably, reaching laws based on switching functions often exhibit a significant reaching phase, particularly noticeable for large initial conditions, accompanied by chattering and the presence of a quasi-sliding mode band. The widely adopted Gao's reaching law, utilizing a switching term approach, explicitly introduces chattering, leading to trajectories residing in an ultimate band during steady-state. This approach also suffers from a drawback of a substantial reaching phase for exceptionally large initial conditions.

Conversely, the equivalent control-based reaching law, initially proposed by Utkin, takes a non-switching approach. While it results in a dead-beat type response with no reaching phase, it demands a considerable control action, presenting a challenge in practical applications. These fundamental issues persist unresolved, indicating a pressing need for comprehensive solutions to address the challenges associated with discrete-time sliding mode control methodologies.

Furthermore, the sliding mode community has dedicated extensive efforts to the development of discrete-time twisting and super-twisting algorithms tailored for discrete-time dynamical systems. The super-twisting algorithm (STA) stands out as a celebrated second-order sliding mode algorithm. While the theoretical appeal of the continuous-time super-twisting algorithm is evident, its practical implementation faces challenges, particularly in directly extending the concept to discrete-time systems. This challenge arises due to the non-continuous nature of the sliding variable and its higher-order differences in DTS. Various existing designs employ Euler discretization in both explicit and implicit forms, with implicit Euler-discretized versions demonstrating superior chattering suppression compared to explicit Euler-discretized methods. Importantly, implicit Euler-discretized-based STAs exhibit unconditional stability, enabling them to handle a broader range of problems without the risk of numerical instability. In contrast, explicit Euler-discretized approaches are conditionally stable and may be prone to numerical instabilities. Additionally, research indicates that implicit Euler-discretized-based STAs showcase increased robustness in the presence of noise or uncertainties, enhanced accuracy with larger sampling times, and improved handling of nonlinearities. It is crucial to note that the discretization of continuous-time sliding mode algorithms may lead to periodic behavior or, in more severe cases, an unstable closed-loop system. Consequently, the justified necessity to design these algorithms separately in the discrete domain warrants further attention.

As far as the implementation of the discrete-time sliding mode algorithm is concerned, it is greatly influenced by the rapid advancement of computing technologies and the availability of increasingly powerful hardware. These developments have ushered in new opportunities for the practical realization of sophisticated control algorithms. In contrast to continuous-time sliding mode control strategies, which often demand extremely high switching frequencies and were historically challenging to implement, discrete-time sliding mode control presents a more feasible and straightforward option. The finite switching frequency requirement in discrete-time sliding mode control aligns well with the capabilities of modern cost-effective digital processors, making the implementation process more accessible and efficient. This convergence of technological progress and algorithmic requirements has facilitated the adoption and integration of discrete-time sliding mode control in a variety of applications.

1.2 Literature Review

In recent years, the field of control systems has experienced notable progress as researchers have investigated diverse methodologies to address the complexities presented by real-world systems, characterized by plant uncertainties and external disturbances. This literature review offers a comprehensive overview of current research in discrete-time sliding mode control, encompassing reaching law-based approaches, equivalent control-based approaches, higher-order sliding mode control, discrete-time twisting algorithm, discrete-time super-twisting algorithm, and related areas. It aims to illuminate the key contributions and emerging trends within this dynamic field.

1.2.1 Sliding Mode Control

Sliding mode control (SMC) originating in the 1950s for continuous-time systems (CTS), stands out as a well-established and robust technique, [1–5]. Renowned for its effectiveness in handling nonlinearities, time-varying dynamics, and uncertainties, SMC has demonstrated robustness against matched disturbances and offers efficient implementation [6–8, 13–18]. In various control engineering applications [10] requiring desired performance despite disparities between mathematical models and actual systems, along with external disturbances, sliding mode control emerges as a promising technique [4, 6, 9]. Its application extended naturally to discrete-time systems (DTS), propelled by the prevalence of digital platforms such as microcontrollers. The core idea of SMC involves selecting a sliding surface to constrain the state's motion, fundamentally shaping the dynamic behavior of the closed-loop system [4, 19]. One of the standout features of SMC is its inherent robustness against matched bounded disturbances. This means that even in the presence of matched disturbances, sliding mode control can force the system to follow the desired trajectory on the sliding surface. This robustness property makes SMC particularly appealing for applications where system dynamics are subject to external perturbations or uncertainties [4, 7].

Despite its effectiveness, sliding mode control is not without challenges. The phenomenon of chattering [11, 13], characterized by high-frequency oscillations around the sliding surface, is a well-known issue. Researchers are actively engaged in developing techniques to mitigate chattering and improve the practical applicability of sliding mode control [8]. The occurrence of chattering is primarily attributed to the idealized nature of the sliding mode. In an ideal scenario, the system state should precisely follow the sliding surface, experiencing instantaneous and abrupt control inputs to maintain this ideal motion. However, in practical systems, achieving such instantaneous changes is not feasible due to physical limitations, actuator dynamics, and the discrete nature of control implementations. Over the past years, so many methods have been developed to curb the chattering effects [12, 61].

1.2.2 Discrete-Time Sliding Mode Control

With the view of practical implementation, in this era of digital computers and microcontrollers, the discrete-time sliding mode control (DSMC) has been explored by many authors [20–31], just to name a few. The fundamental distinction between discrete-time sliding mode control and continuous-time sliding mode control (CSMC) lies in DSMC's finite switching frequency, negating the invariance property which is inherent in CSMC. In most part of the design, discrete sliding mode control has been further explored along two main directions. First, the reaching law-based DSMC which incorporates the switching term explicitly, resulting in unavoidable chattering [25, 32]. Secondly, equivalent control-based DSMC, as investigated by Utkin [22,33] and others [34–36], which omits the reaching phase but may necessitate substantial control effort. Moreover, [27] also explored non-switching type reaching law but without the need for excessive control effort. Further, the notion of quasi-sliding mode (QSM) was proposed for DTS since the ideal sliding mode could not occur for such systems. Likewise many properties of CSMC for DTS could not propagate that well as it was for CTS. Furthermore, finite-time stability (FTS) was not so obvious for the systems employing DSMC and has been given less attention until the past few years. Recently, a minima based notion to achieve finite-time stability for DTS is discussed in [37–39], and its application in optimization in [40], where system state converges to the origin in finite time instead of staying in an ultimate band. Later, this theory was further explored to achieve discrete sliding mode [41] and discrete super-twisting like algorithm in [42].

In the reaching law approach [25–28,46,54,55,57–60], the desired value of the switching variable for the next time step is determined by the reaching law, which is based on the current switching variable. This methodology finds its suitability for discrete-time systems, ensuring a guaranteed sliding mode. Originally introduced for continuous-time systems in [32], this approach was extended to DTS in [25]. To address non-ideal sliding motion in DTS, the concept of quasi-sliding mode (QSM) was introduced and further investigated in [26], incorporating additional conditions for its existence. Gao's reaching law has maintained its popularity for almost three decades, becoming a convenient choice for researchers in various applications [47]. Despite its widespread use, Gao's RL does have limitations, notably the fixed coefficient in the proportional term leading to a high rate of change of the sliding variable for large initial conditions. This can result in an excessively large control action, potentially violating constraints on the closed-loop signals. To address these issues, modifications to Gao's RL have been proposed, as evidenced in works such as [28, 46, 60]. These variations primarily concentrate on refining the proportional term in Gao's RL, ensuring that the rate of change of the sliding variable is bounded by design parameters, providing a solution desired by users.

Various approaches have been explored to mitigate chattering in discrete-time sliding mode control, as evidenced by several studies. In [27], the discussion centers around quasi-sliding mode control, with the goal of maintaining states near the switching manifold without the need for recrossing and, consequently, avoid chattering. The work in [43] introduces a robust discrete sliding mode algorithm designed to ensure chattering-free operation while effectively handling parameter uncertainties and external disturbances. The study in [44] delves into non-smooth DSMC as a strategy to address chattering induced by reaching law-based control. Furthermore, the merits of chattering-free DSMC are exemplified through a significant case study involving missile guidance systems, as demonstrated in [45].

Over the past decade, extensive research has been conducted in the realm of discrete-time sliding mode control, encompassing a range of areas [46–50]. Notable contributions include investigations into discrete-time integral SMC tailored for sampled-data systems [51], the exploration of discrete-time terminal SMC [52], and its practical application in addressing perturbations in automotive electronic throttle systems [53]. The advancement of DSMC methodologies includes the development of generalized reaching law-based DSMC incorporating variable functions [54] and exploring different convergence rates for them [55, 56]. Additionally, studies have delved into band approach analysis, considering predefined band sizes [57], robustness enhancement through relative degree two switching variables [58], its generalization for arbitrary relative degrees [59], and the implementation of reference switching profile-based DSMC [60]. These diverse research endeavors contribute significantly to the evolving landscape of DSMC applications and methodologies.

1.2.3 Discrete-Time Twisting Algorithm

The twisting algorithm, akin to the super-twisting algorithm, has found widespread use in control applications owing to its straightforward parameter configuration and robustness to model uncertainties. Its well-posedness, stability, and various qualitative properties, such as tracking accuracy and convergence rate, have garnered significant attention [61]. In comparison to the super-twisting algorithm, the numerical chattering of the twisting algorithm becomes more pronounced due to its composition of two cascaded signum functions and the absence of an integration operation. Efforts to mitigate this numerical chattering have been explored in the literature, encompassing discrete-time approaches [44, 62–64] and continuous-time methods, including the utilization of higher-order sliding mode (HOSM) [61]. As emphasized in [65–67], implicit-Euler methods show great promise in mitigating numerical chattering when compared to conventional discrete-time implementations based on explicit Euler methods. In [68], the authors introduced an implicit-Euler-based approach to alleviate numerical chattering in the twisting algorithm. The method's stability and robustness in both perturbed and unperturbed cases were analyzed using the discrete-time Lyapunov method. However, this approach exclusively considers the ideal discretization with the implicit-Euler method, neglecting the zero-order hold (ZOH) effect introduced by analog-to-digital converter devices in practical applications. Recognizing the impact of the ZOH effect, [69] proposed an implicit-Euler implementation incorporating ZOH discretization. Nevertheless, this algorithm necessitates numerical solvers for the affine variational inequality to compute the control input for the twisting algorithm. This requirement may pose challenges for its application in computation-limited embedded systems and could compromise real-time response properties. Further, to solve this problem, an implicit Euler zero-order hold discretization without numerical solvers was proposed in [70]. It is important to mention that all of these works have focussed on the discretization of their continuous-time counterpart with the intention of implementing the continuous-time twisting algorithm with the help of digital controllers.

1.2.4 Discrete-Time Super-Twisting Algorithm

The super-twisting algorithm (STA) [61], introduced by Levant in 1993, stands out as a widely adopted algorithm in the realm of robust nonlinear control and observation. Operating as a second-order sliding mode algorithm, it is designed for systems with relative degree one concerning a specified sliding variable, typically the output. Notably, one of its outstanding theoretical features is the capability for finite-time convergence of the output even in the presence of matched Lipschitz perturbations. In comparison to first-order sliding mode controllers, the STA exhibits the advantage of generating a continuous control signal, a desirable trait in many applications. From a practical perspective, the super-twisting algorithm offers the benefit of straightforward implementation in a digital environment. In most of the existing literature, Euler-discretization with implicit as well as explicit form is used to develop discrete-time higher-order sliding mode [52, 62, 63, 67, 68, 70–75]. The most commonly used method to implement STA digitally is by the use of explicit Euler-discretization method [36, 72, 74, 75]. With explicit Euler-discretization, the closed-loop system maintains the asymptotic accuracies observed in the continuous-time system. Out of these works, the implicit Euler-discretization based STAs [63, 67, 68], have shown better suppression of the chattering as compared to the explicit Euler-discretization methods. Moreover, implicit Euler-discretization based STA is unconditionally stable, i.e., it can handle a wider range of problems without the risk of numerical instability. On the contrary, explicit Euler-discretization is conditionally stable and may lead to numerical instabilities [67]. Further, it has also been shown that implicit Euler-discretization based STA is more robust in the presence of noise or uncertainties, is more accurate with large sampling time and can handle nonlinearities comparatively well [68]. Moreover, in contrast to explicit Euler-discretization, the implicit approach is exact in the unperturbed case, steering the variable to be controlled precisely to zero. Even in the presence of perturbations, trajectories maintain the optimal asymptotic accuracies observed in the continuous-time system. An important advantage of the implicit scheme is that the precision achieved is not compromised by an increase in control gains. Although, it is worth mentioning that the continuous-time sliding mode algorithm's discretization could culminate in periodic behavior [76–78], or, worse, unstable closed-loop system [79, 80].

1.3 Contributions

This thesis stands as a notable contribution to the expansive domain of control systems, presenting an innovative approach through the introduction and application of sliding mode based on difference equations with minima for dynamical systems. The profound impact of this research extends across various dimensions, with key contributions manifesting in the following facets:

- At first, our exploration introduced two new reaching laws specifically tailored for discrete-time systems, utilizing a difference equation with minima. These reaching laws not only eliminate chattering effectively, ensuring smoother operation, but also enhance system robustness. This improvement is realized by strategically reducing the width of the quasi-sliding mode band without imposing excessive stress on the control effort. The nuanced balance achieved by these reaching laws demonstrates their efficacy in addressing challenges within discrete-time systems. The proposed methodology's effectiveness and robustness were validated through theoretical analysis and extensive simulation studies, demonstrating its ability to achieve sliding mode for the unperturbed case and quasi-sliding mode for the perturbed case.

- Next, we extended the discussion on reaching laws based on the difference equation with minima, introducing a shift to rate-regulatory function-based reaching laws. The primary objective was to leverage the benefits associated with the variable rate of change of the sliding variable, facilitated by the inclusion of a variable gain function in the proportional term. This modification led to a significantly accelerated rate of change of the sliding variable. Importantly, we demonstrated that this rate of change is effectively upper-bounded by the design parameters, introducing an additional advantageous feature. We conducted simulations on a numerical example and a pendulum system. In the case of the pendulum system, a comparative analysis among various methods revealed that the proposed method consistently outperformed others.

- Further, this thesis delved into the Lyapunov characterization of discrete-time sliding mode based on a difference equation with minima. To achieve this, we formulated Lyapunov conditions for the finite-time input-to-state stability of perturbed discrete-time systems, shedding light on the settling time function. Moreover, we also discussed the Lyapunov stability of the closed-loop system. Additionally, we conducted a comparative study through simulations, emphasizing the superior performance of the proposed laws compared to existing ones. An intriguing observation from this analysis is the attainment of the least ultimate bound for the sliding variable trajectory with satisfactory transient behavior.

- Additionally, our focus shifted to the development of discrete-time twisting-like and super-twisting-like algorithms, deeply rooted in the framework of a difference equation with minima. A departure from the common practice of Euler-discretization in conventional discrete-time algorithms, our proposed designs displayed notable distinctiveness. Leveraging these innovative designs, we successfully achieved the convergence of system states to an equilibrium point in the case of unperturbed systems and to an invariant set for perturbed systems, all within a finite time. Furthermore, we expanded the application of the discrete-time super-twisting-like algorithm by employing it to design a super-twisting observer. This observer, in a noteworthy turn of events, robustly estimated unknown state variables for a perturbed pendulum system, demonstrating its efficacy within a finite timeframe.

- Finally, we aimed towards the application of the proposed reaching law based on a difference equation with minima in a benchmark practical setup, delving into the intricacies of the applied reaching law. This real-time realization offers a comprehensive analysis of the reaching law's performance within the context of the couple tank system. The results not only demonstrate the robustness of the proposed approach but also underscore its adaptability to real-world experimental conditions.

We successfully implemented the proposed law in the couple tank system, presenting the outcomes through a combination of simulation and experimentation. The level control of the tank system was meticulously conducted under both unperturbed and perturbed conditions. In both scenarios, the successful attainment of the desired water level serves as a testament to the efficacy of the proposed laws.

1.4 Organization of the Thesis

This thesis comprises a total of eight chapters.

In the inaugural chapter of the thesis, Chapter 1 offers a comprehensive introduction to the conducted work. This includes a nuanced exploration of the motivation driving the research, an in-depth literature survey to discern the existing research gap, and a meticulous delineation of the principal contributions embedded within the thesis. This chapter not only establishes the foundation for the subsequent sections but also strategically defines the research objectives. Furthermore, it underscores the significance and relevance of the proposed work in the broader context of the academic and research landscape.

Chapter 2 serves as a comprehensive exploration of the mathematical preliminaries essential for grasping the intricacies presented throughout the thesis. Within this chapter, we delve into key notions, definitions, and concepts that lay the groundwork for a better understanding of the subsequent chapters. This encompassing overview includes critical topics such as comparison functions, the concepts of sliding mode and quasi-sliding mode, and stability notions framed within the discrete-time systems paradigm. By delving into these foundational mathematical elements, this chapter serves as a pivotal resource, establishing the necessary framework for engaging with the advanced concepts and methodologies expounded upon in the subsequent sections of the thesis.

Chapter 3 embarks on a detailed exploration where we introduce two reaching laws crafted for discrete-time systems, leveraging the difference equations with minima. The definitions of sliding mode and quasi-sliding mode are provided. To validate their effectiveness and robustness, the chapter engages in theoretical analysis and conducts extensive simulation studies. The methodology's prowess is thereby confirmed, demonstrating its adeptness in achieving sliding mode for the unperturbed case and quasi-sliding mode for the perturbed case.

In Chapter 4, we delve deeper into the realm of reaching laws grounded in the difference equation with minima, expanding our exploration by introducing rate-regulatory function-based reaching laws. The desired properties of the rate-regulatory function are highlighted. To validate and showcase the efficacy of this approach, simulations were conducted on both a numerical example and a pendulum system. Moreover, in the case of the pendulum system, a comparative analysis among various methods was undertaken to observe the performance of all these methods together.

Moving on to Chapter 5, the thesis delves into the intricacies of the Lyapunov characterization of discrete sliding mode based on a difference equation with minima. In the pursuit of this objective, we meticulously laid out the Lyapunov conditions, unraveling the finite-time input-to-state stability of perturbed discrete-time systems, thereby shedding light on the nuances of the settling time function. Beyond theoretical exploration, we engaged in a comparative study through simulations to observe the performance of the proposed law in contrast to the existing methodologies.

Chapter 6 immersed itself in the design of a new type of discrete-time twisting-like and super-twisting-like algorithms, firmly grounded in the framework of a difference equation with minima. We provided the rigorous theoretical analysis to develop the stability conditions of the proposed algorithms by using Lyapunov theory. Additonally, we also provided simulation results to understand the behaviour of the algorithms by means of graphical illustration. Moreover, we expanded the utility of the discrete-time super-twisting-like algorithm to design a discrete super-twisting observer. This observer is implemented for the states estimation problem of a perturbed pendulum system in a finite-time framework.

Chapter 7 is dedicated to the practical application of the proposed reaching law based on difference equation with minima. This segment provides a detailed analysis of the reaching law's performance within the realm of the couple tank system. We have conducted simulation study as well as experimental study for the tank system. This study is performed for the level control of the tank system which is scrutinized under both unperturbed and perturbed conditions.

Finally, the concluding chapter encapsulates the essential discoveries and contributions outlined in the thesis, offering a comprehensive overview. It delves into the implications of the research, acknowledges its limitations, and outlines potential paths for future research and practical applications. This chapter serves as the culmination of the thesis, providing a sense of closure to the overarching study.