

## PREFACE

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Fractional calculus deals with the derivatives and integrals of arbitrary order (real or complex order). Integer order derivatives are local in nature, whereas fractional order operators are global operators that have a memory of past events. So, fractional calculus stands out in modeling problems that involve concepts of nonlocality and memory effects, which are not well explained by integer order calculus. Some commonly known applications are found in the fields of viscoelasticity, diffusion procedures, relaxation vibrations, electrochemistry, electromagnetics, etc., which are successfully described by differential equations involving derivatives of fractional order. The increasing interest of fractional calculus applications has motivated the development and investigation of numerical methods to solve fractional differential equations. In this thesis, fractional advection–diffusion equations and one and two-dimensional nonlinear fractional reaction–diffusion equations are considered, and high-order numerical methods are investigated to solve them.

Advection–diffusion equations arise in many physical processes; for instance, transport occurs in fluids through a combination of advection and diffusion. Fractional advection–diffusion equations in which the time derivative is of fractional order arise from power law particle residence time distribution and describe particle motion with memory in the lane. The nonlinear reaction–diffusion equations have found numerous applications in pattern formation in many branches of biology, chemistry, and physics. In an application to population biology, the reaction term models growth, and the diffusion term accounts for migration. Reaction–diffusion equations are often applied to represent a lot of applications like the study of chemical reactions, propagation phenomena, transport systems, pattern formation processes, chaos in finance, and spatio-temporal distribution of species, etc.

This thesis mainly focuses on the fractional partial differential equations (PDEs) generated by time fractional derivatives such as Caputo–Prabhakar derivative and Caputo derivative. Prabhakar fractional derivatives and integrals involve three parameters Mittag-Leffler (ML) function in the kernel and have importance in traveling problems of fractional relaxation and oscillation. They also have applications in fractional poison processes and the fractional Maxwell model in linear viscoelasticity.

Due to the nonlocal nature of fractional derivatives, it is pretty challenging to solve the fractional PDEs analytically, which signifies the importance of efficient numerical methods to solve such fractional PDEs. Here, we aim to devise high-order numerical methods to approximate Caputo–Prabhakar and Caputo type fractional derivatives and use these approximation methods in solving time fractional PDEs namely, time fractional advection–diffusion equations and time fractional reaction–diffusion equations in one and two-dimensions. The finite difference method is used to maintain simplicity and provide proficiency of the scheme. In addition to the development of high-order numerical methods, the scope includes the study of stability, solvability, convergence and to provide the experimental analysis to support the theoretical analysis.