Accelerated Optimization and Control Using Predefined-Time Convergent Dynamical Systems

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by

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Chapter 7

Conclusions and Future Perspectives

In this chapter, we bring closure to the comprehensive body of work presented in this thesis, emphasizing its notable contributions while acknowledging its inherent limitations. Additionally, we identify prospective avenues for further exploration and investigation.

7.1 Conclusions

The primary objective of this thesis is to contribute to the advancement of continuous-time optimization, a field that has gained significant attention in recent years due to its broad range of applications, including robot motion coordination, optimal power flow computation, and image processing. Each chapter of this thesis makes distinctive contributions, and we summarize the main findings below:

In the exploration detailed in Chapter 3, first, we introduced a predefined-time convergent dynamics to solve unconstrained optimization problems. This dynamics has the property that its equilibrium point is the optimal point of the optimization problem, and the optimal point is reached in a pre-specified time. As continuous-time dynamics to solve optimization problems are more amenable to mathematical analysis, we use the Lyapunov stability theory approach to rigorously study the predefined-time convergence of the proposed dynamics. Next, we proposed projected predefined-time convergent gradient flows to solve equality-constrained optimization problems. In order to formulate precisely the effect of perturbations on predefined-time gradient flows, we employ the formalism of ISS theory. Theoretical analysis and extensive simulation studies were conducted to validate the effectiveness and robustness of the proposed methodology.

Chapter 4 extended the proposed approach in chapter 3 to solve optimization whose objective function and constraints are varying with time. First, we introduced a Newton-like predefined-time stable dynamics to solve time-varying optimization problems, and its Lyapunov-based convergence analysis is discussed for both unconstrained as well as constrained case. A robust Newton-like predefined-time stable dynamics is proposed for the cases when we do not have the exact knowledge of the time derivative of the gradient of the objective function. Levenberg-Marquardt-like predefined-time stable dynamics is proposed for the cases when the Hessian of the objective function is not invertible, with the assumption of gradient dominance. The convergence analysis of this dynamics is studied in the sense of input-to-state stability (ISS), where the input is a perturbation from the true Hessian. Numerical examples for the unconstrained case and constrained case of TVO problems are demonstrated using the proposed method. We have also shown that the discretized implementation is conterminous with theoretical results.

Chapter 5 of the thesis set out to explore the real-world application of the proposed dynamics in Chapter 4. We demonstrate the application of the proposed approach in robot navigation with collision avoidance. We explore a situation where the robot is tasked with tracking a moving target in a priori chosen time. This chapter established a novel vector field-based strategy for predefined-time, collision-free navigation of disk-shaped robots in sphere worlds. We guaranteed convergence to a moving target from almost all initial positions by formulating the navigation task as a TVO problem with inequality constraints. The efficacy of our proposed predefined-time convergent dynamics was rigorously validated through simulations, demonstrating robust performance in complex, obstacle-filled environments.

Chapter 6 centers on the application of the proposed gradient flow strategy to analyze the passivity properties of a system, further facilitating controller design. This chapter presented a significant contribution to data-driven control: a methodology for determining passivity indices (IFP, OFP) and achieving stabilization without system models. Our approach formulates passivity indices as an optimization problem solvable via prescribed-time gradient flows (GFs), converging within a predefined time using gradients computed directly from optimal input-output samples. Furthermore, we established that these data-derived indices facilitate controller design—specifically, using IFP/OFP systems—to guarantee asymptotic and finite-time stability via feedback interconnections.

This represents the first integration of prescribed-time optimization with passivity theory for fully model-free control, effectively bypassing traditional modeling requirements.

7.2 Future Perspectives

Although this thesis has contributed to the field of continuous-time optimization, it is crucial to recognize that the proposed design comes with certain limitations, suggesting possibilities for further progress. As a result, the subsequent sections outline these challenges and pinpoint potential avenues for future research and development:

- While the current work establishes the ISS of the proposed gradient flows in the presence of perturbations, future work will pursue fundamentally stronger robustness guarantees. We propose to develop novel gradient flow algorithms inspired by the principles of sliding mode control (SMC). The core idea is to deliberately introduce carefully designed switching manifolds or discontinuous elements into the flow dynamics. This strategic modification is anticipated to confer insensitivity to matched perturbations, ensuring that the convergence trajectory reaches and remains precisely at the optimal solution point despite such disturbances. This represents a paradigm shift from merely bounding the error (as per ISS) to actively rejecting its influence for a specific, critical disturbance class.

- Extending the applicability of predefined-time gradient flows to tackle more complex and realistic distributed optimization problems. A critical direction involves handling non-smooth objectives, such as those with L_1-regularization, by integrating proximal operators or subgradient methods into the predefined-time convergence framework, particularly for composite objectives. Concurrently, enhancing robustness against real-world imperfections is paramount; this includes developing flows resilient to communication delays (potentially using Lyapunov-Krasovskii techniques) and stochastic noise affecting both gradients and inter-agent communications. Furthermore, the dynamics of real systems necessitate extensions to time-varying objectives and constraints, requiring adaptive mechanisms combined with predefined-time barrier functions to handle drifting optima and evolving feasible sets effectively.

- Tackling non-convex optimization problems, common in areas like distributed deep learning, demands research into how predefined-time flows can accelerate escape from saddle points and integrate effectively with stochastic gradient descent. Underpinning these advances, a more rigorous and unified theoretical foundation is needed, including the development of specialized Lyapunov theories for networked predefined-time systems and the characterization of robustness margins using concepts like input-to-state stability.