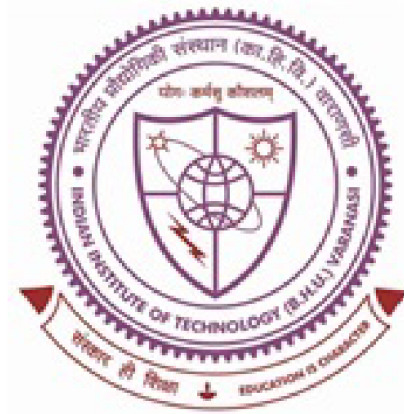


# Fractal Approximation on the Sierpiński Gasket and Fractal Surfaces



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*by*

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# Chapter 6

## Conclusions and Future Directions

In this chapter, we will conclude this thesis by providing an overall conclusion. Additionally, we will explore potential directions for future research.

### 6.1 Conclusions

We have studied  $\alpha$ - fractal function  $f^\alpha \in \mathcal{C}(SG)$ , which is a fractal perturbation of germ function  $f \in \mathcal{C}(SG)$ . Furthermore, a fractal operator that yields the fractal perturbation form of the germ function is defined, and some of its properties have been mentioned. With the help of graphical representation of the graph of the  $\alpha$ - fractal function, we have attempted to observe its continuous dependence on parameters. Bounds of the fractal dimension of the graph of the  $\alpha$ - fractal function are also estimated. Furthermore, we have extended our efforts by establishing identical results on  $SG$  within the  $\mathcal{L}^p$  space, using a self-similar measure. Some properties of multi-valued maps are also discussed. Next, we have shown that  $\alpha$ -fractal functions retain some germ function characteristics by imposing specific

restrictions on parameters. Furthermore, we have determined the bounds of the graph of  $\alpha$ -fractal function using the method of oscillation of functions on  $SG$ .

In the later part of this thesis, we have discussed dimension preserving approximations for bivariate  $\alpha$ -fractal functions on a rectangular domain. Additionally, we presented some results associated with multi-valued fractal operators. Finally, we have calculated the bounds for the box dimension of the graph of bivariate  $\alpha$ -fractal functions by considering appropriate assumptions on the IFS. Overall, this thesis can be viewed as a contribution to the field of fractal geometry and operator theory.

In what follows, we present certain questions for which our current literature is insufficient, requiring further exploration. These can be considered for future research initiatives.

- Construction of  $\alpha$ -fractal function on  $n$ -fold products of  $SG$ .
- Construction of  $\alpha$ -fractal function on a fractal domain such as product of Koch Curve.
- Dimensional results and other properties for the class of fractal rational polynomials defined on  $SG$ .
- Exact Hausdorff dimension for bivariate  $\alpha$ -fractal functions.
- Dimension preserving approximation of set-valued mappings.

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