CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Mathematical models on nonlinear dynamical systems are used to describe complex systems that display nonlinear behaviour such as chaos and bifurcation. Different from linear systems in which the linear equations characterize the relationship between the input and output, nonlinear systems involve nonlinear equations, making their behaviour more intricate and often harder to predict. These systems are widely used in many scientific and engineering fields such as physics, biology, economics, chemistry, control systems, population dynamics, and many other disciplines. The mathematical description of nonlinear systems are expressed in the form of the differential equations. In numerous dynamical systems, there are instances when external forces occur for a very short duration of time. Examples of such occurrences include natural disasters affecting population dynamics, vaccination during disease epidemics, cryptographic schemes in secure communication, and air traffic control. These external forces are not continuous in time and time duration is very less as compared to the entire process duration. These transient effects are referred to as impulsive effects, and the mathematical models that incorporate these effects are termed as differential systems with impulsive effects or simply called impulsive differential systems.

In the theory of dynamical systems, the primary objective is to study the dynamical behaviour of the system states when they are subjected to certain rules that predict the future states based on the current state. The stability analysis is one of the crucial aspects of studying the qualitative behaviour of dynamical systems. It helps us to determine whether the system states are stable or unstable. To study the analysis of the stability of dynamical systems, Alexander M. Lyapunov formulated his theory of stability of motion for the system of ordinary differential equations. Lyapunov introduced two methods, often referred to as the "indirect method" and the "direct method" to analyze the solution's stability for systems of ordinary differential equations. The difference between them is based on the fact that the "indirect method" depends on finding an approximate solution to the differential equation. But, in the "direct method", no such knowledge of solution is required. In the Lyapunov theory, the asymptotic stability refers to the convergence of the trajectories to the equilibrium when time tends to infinity. However, in many real-world applications, it would be better if dynamical systems converge to equilibrium in finite-time or fixed-time rather than stabilizing dynamical systems asymptotically. The finite-time stability is characterized by an upper bound of convergence time that varies with the initial condition whereas fixed-time stability is defined by an upper bound on convergence time that remains constant regardless of the initial condition. The main goal of this thesis is to investigate the nonlinear impulsive dynamical systems with different kinds of impulse sequences in finite-time and fixed-time. The impulsive sequence is taken into consideration separately to discuss stabilizing impulses and destabilizing impulses. Further, we tried to extend the proposed theoretical results of nonlinear impulsive systems to the synchronization of various types of neural networks.

1.1 Impulsive Differential Equation

The theory of impulsive differential equation is a mathematical framework used to describe systems that undergo sudden changes in the state amidst continuous development. The mathematical description of these systems consists of a combination of differential equations, which describe the continuous evolution of their state, and the conditions that define the discontinuities of the first kind, or "impulses" of the solution at certain moments [2, 3, 4]. Numerous real-world problems involve external forces that abruptly alters the system dynamics. One practical illustration of an impulsive differential equation is the modeling of medicine intake, in which the user is required to take regular doses of the medicine intake, resulting in abrupt changes in the quantity of medicine present in their body to control the disease or make it disappear. Some other real-world application problems of impulsive differential equations in science and engineering can be found in frequency modulated systems, and optimal control models in economics and pharmacokinetics. The evolution process of system state as described by an ordinary differential equation

$$\dot{z}(t) = g(t, z),$$

where $t \in \mathbb{R}_+$, $z \in \Omega \subset \mathbb{R}^n$, $g : \mathbb{R}_+ \times \Omega \to \mathbb{R}^n$, $\mathbb{R}^n$ is n-dimensional euclidean space, $\mathbb{R}_+$ denotes the set of non-negative real numbers. The impulsive moments for the solution $z(t)$ occurs at $t = t_l$, where $l \in \mathbb{N}$. Define $I(t, z) : \mathbb{R} \times \Omega \to \Omega$, where

$$(t, z) \to (t, z + I(t, z)),$$

is the transfer of the solution from the impulsive moments $z(t_l^-)$ to the impulsive effects $z(t_l^+)$. Then

$$\Delta z(t_l) = I(t_l, z(t_l)),$$

where $\Delta z(t_l) = z(t_l^+) - z(t_l^-)$.

According to the characteristics of an impulsive differential equation, there are two types of impulsive differential equation: system with impulses at fixed-times and impulses at variable times.

1.1.1 System with Impulses at Fixed-Times

Here the system is of the form

$$\begin{cases} \dot{z}(t) = g(t,z), & t \neq t_l, \\ \Delta z(t_l) = I_l(z(t)), & t = t_l, l \in \mathbb{N}, \end{cases} \tag{1.1.1}$$

where $I_l : \Omega \to \Omega$, $g : \mathbb{R}_+ \times \Omega \to \mathbb{R}^n$; t_l is a impulsive moment in $\mathbb{R}_+$ at which the system state $z(t)$ has a first kind of jump discontinuity. In particular, $z(t_l^-) = \lim\limits_{h\to 0^+} z(t_l - h) = z(t_l)$ but $z(t_l^+) \neq z(t_l)$ so that $\Delta z(t_l) = z(t_l^+) - z(t_l^-)$. The impulsive sequence $\{t_l\}$ is strictly increasing and unbounded above, i.e., $t_0 = 0 < t_1 < t_2 < \ldots < t_l$ and $\lim\limits_{l\to\infty} t_l = \infty$.

Suppose $z(t)$ is the solution of the continuous part of the system (1.1.1) with the starting point (t_0, z_0). The evolution process of the solution $z(t)$ of the system (1.1.1) is elaborated as follows: the point $Q_t = (t, z(t))$ starts its motion from the initial point $Q_{t_0} = (t_0, z_0)$ and move along the solution $\{(t,z); t \geq t_0, z = z(t)\}$ until it reaches the time $t = t_1$ at which the first impulsive effect is activated and transfer the point $Q_{t_1} = (t_1, z(t_1))$ to $Q_{t_1}^+ = (t_1, z(t_1^+))$, where $z(t_1^+) = z(t_1) + I_1(z)$. Further, the point Q_t moves continuously along the trajectory $z(t) = z(t, t_1^+, z(t_1^+))$ until it reaches the time $t_2 > t_1$ at which the second impulse effect is activated and transfer the point $Q_{t_2} = (t_2, z(t_2))$ to $Q_{t_2}^+ = (t_2, z(t_2^+))$, where $z(t_2^+) = z(t_2) + I_2(z)$. Therefore, as long as the solution of (1.1.1) exists, the evolution process will continue in the same manner.

1.1.2 System with Impulses at Variable-Times

In this case, the system is of the form

$$\begin{cases} \dot{z}(t) = g(t,z), & t \neq \tau_l(z), \\ \Delta z(t_l) = I_l(z(t)), & t = \tau_l(z),\ l \in \mathbb{N}, \end{cases} \tag{1.1.2}$$

which involves a sequence S_l of surface given by $S_l : t = \tau_l(z)$ such that $\tau_l(z(t)) < \tau_{l+1}(z(t))$ and $\lim_{l\to\infty} \tau_l(z) = \infty$. The impulsive system with variable time (1.1.2) clearly exhibits a more difficult problem than the impulsive systems with fixed moments. For example, it is clear from the system (1.1.2) that the impulsive moment depends on the solution of $t_l = \tau_l(z(t_l))$, for each l. Thus, the trajectory starts from different initial points and will have finite discontinuity at different points. Also, the solutions may hit the same surface $t = \tau_l(z)$ many numbers of times, such type of behaviour is called "beating or pulse phenomenon". Furthermore, in this case, the phenomenon known as "confluence", occurs when after some time, different solutions coincide and thereafter behave as a single solution.

In this thesis, we study the impulsive dynamical systems with impulses at fixed time. The impulsive phenomena was first considered by Lakshmikantham et al. [3]. He has studied the stability of impulsive systems when there are abrupt changes in the system states during impulsive moments. Subsequently, several researchers and scientists have paid their attention to the stability of impulsive systems [2, 5, 6]. Due to the influence of impulses within the system, the study of the stability of impulsive systems can be categorized into three distinct classes as described below. Let us consider an impulsive system with fixed-time impulses

$$\begin{cases} \dot{z}(t) = g(t, z), & t \neq t_l, \\ \Delta z(t_l) = \mu_l z(t_l^+), & t = t_l, l \in \mathbb{N}, \end{cases} \tag{1.1.3}$$

where μ_l is an impulsive strength or impulsive gain at fixed time t_l. The impulsive gain is characterized into three categories: (i) stabilizing impulses ($|\mu_l| < 1$), these impulsive effects can enhance the stability of the impulsive dynamical system, (ii) destabilizing impulses ($|\mu_l| > 1$), these impulsive effects can be taken as disturbance, which may suppress the stability, (iii) inactive impulses ($|\mu_l| = 1$), these type of impulsive effects neither destroy nor promote the stability of impulsive dynamical systems. The first two categories of impulsive effects are evidently of major concern. This thesis is concerned with the detailed investigation of some stability problems of impulsive dynamical systems for the stabilizing impulses and destabilizing impulses.

1.2 Delay Differential Equation

The stability analysis of nonlinear systems can create various challenges due to time delay. The process of mathematical modeling, analysis and synthesis for delay differential equations deviates from those used for ordinary equations. The primary difference between ordinary equations and delay differential equations emerges from the fact that ordinary equations are typically represented by ordinary differential equations, whereas delay differential equations are characterized by functional differential equations. Time-delay phenomenon was initially observed in biological systems and since then, it has been identified in numerous engineering systems, including fluid and mechanical, metallurgical processes, signal processing problems and network control systems. They are often a source of instability and poor control performances. Delay differential equations are a form of differential equations in mathematics where the derivative of the unknown function at a given time relies on both its prior and current values. A general delay differential equation is defined as follows:

$$\dot{z}(t) = g(t, z(t), z_t), \;\; t \geq 0,$$

where $z(t) \in \mathbb{R}^n$ and $z_t = \{z(\tau) : \tau \leq t\}$ represent the trajectory of the solution in the current and past time respectively. Let the vector-valued function $g(t, z(t), z_t)$ be defined for $t \geq 0$, $z(t) \in \mathbb{R}^n$ and $z_t \in \mathbb{R}$ and assume that this function is continuous. It is possible that the time delay may exist in continuous and discontinuous dynamical systems, therefore, systems with impulses and time delay are divided into impulsive delayed systems and delayed impulsive systems.

1.2.1 Impulsive Delayed Systems

The impulsive delayed systems are mathematical models used to study the dynamical systems that involve both time delay in continuous-time dynamics and abrupt impulsive effects in discrete-time dynamics. According to impulsive delayed systems, it is shown that the evolution of the continuous dynamics is dependent not only on their present state but also on the history of the system's states. For instances, the insulin therapy for both type 1 and type 2 diabetes mellitus with time delay in insulin production as an impulsive delayed system model. It is shown that impulsive exogenous insulin infusions can mimic natural pancreatic insulin production. In addition, the impulsive delayed system finds extensive application in numerous domains, including orbit transfer of satellites, dosage supply in pharmacokinetics, ecosystems management, and synchronization in chaotic secure communications systems. Let's look at the impulsive delayed system that is outlined as follows:

$$\begin{cases} \dot{z}(t) = g(t, z(t), z_t), & t \neq t_l, t \geq t_0, \\ \Delta z(t_l) = I_l(z(t)), & t = t_l, l \in \mathbb{N}, \end{cases} \tag{1.2.1}$$

where $g : \mathbb{R}_+ \times \mathbb{R}^n \times D \to \mathbb{R}^n$, D being an open set in $PC([-\tau, 0], \mathbb{R}^n)$, where $\tau > 0$ and $PC([-\tau, 0], \mathbb{R}^n) = \{\psi : [-\tau, 0] \to \mathbb{R}^n, \psi(t)$ is continuous everywhere except at finite number of points at which it is right continuous and left limit exists$\}$ and $I_l : \mathbb{R}^n \to \mathbb{R}^n$. For $t \geq t_0$, $z_t \in D$ is defined by $z_t(m) = z(t + m)$, $-\tau \leq m \leq 0$. For $l \in \mathbb{N}$, the impulsive sequence $\{t_l\}$ satisfies $0 \leq t_0 < t_1 \cdots < t_l < \cdots$, $\lim_{l \to \infty} t_l = +\infty$. The function $\phi \in D$ denotes the initial value of the system state (1.2.1).

1.2.2 Delayed Impulsive Systems

Delayed impulsive systems describe a phenomenon in which impulsive transients are influenced not only by the current state but also by its historical state. For example, in population dynamics such as in the fishing industry, the effective impulsive controller measures, such as harvesting and re-leasing, are determined based on not only the current fish population but also the populations recent history. This is because immature fish requires time to grow, and the quantity of each impulsive action depends on the history of fish population data. Another example can be found in impulsive synchronization-based communication security systems. In these systems, there are transmission and sampling delays that occur during the information transmission process. The sampling delays result from the impulses being sampled at a discrete instant, which causes the impulsive transients to depend on their past states. Let us consider the delayed impulsive systems as follows:

$$\begin{cases} \dot{z}(t) = g(t, z(t), z_t), & t \neq t_l, t \geq t_0, \\ \Delta z(t_l) = I_l(z(t - \tau_l)^-), & t = t_l, l \in \mathbb{N}, \end{cases} \tag{1.2.2}$$

where $z \in \mathbb{R}^n$ and $g : \mathbb{R}_+ \times \mathbb{R}^n \times D \to \mathbb{R}^n$, where D is an open set in $PC([-\tau, 0], \mathbb{R}^n)$. Also $\tau > 0$, $PC([-\tau, 0], \mathbb{R}^n) = \{\psi : [-\tau, 0] \to \mathbb{R}^n$, $\psi(t)$ is continuous everywhere except at finite number of points at which it is right continuous and left limit exists$\}$ and $I_l : D \to \mathbb{R}^n$. For $t \geq t_0$, $z_t \in D$ is defined by $z_t(m) = z(t + m)$, $-\tau \leq m \leq 0$. $\tau_l \geq 0$ is the time delay in impulses, and sequence $\{\tau_l\}$ is said to be impulsive delay sequence. For $l \in \mathbb{N}$, the impulsive sequence $\{t_l\}$ satisfies $0 \leq t_0 < t_1 \cdots < t_l < \cdots$, $\lim_{l \to \infty} t_l = +\infty$. The function $\phi \in PC([-\varrho, 0], \mathbb{R}^n)$ deontes the initial value of the system state (1.2.2), where $\varrho = \sup_{l \in \mathbb{N}}\{\tau_l - t_l, \tau\}$, the norm of ϕ is defined by $|\phi| = \sup\{||\phi|| : -\varrho \leq m < 0\}$, and $||.||$ is a norm in $\mathbb{R}^n$.

1.3 Stability Analysis

Stability represents the most crucial qualitative attribute of dynamical systems, as unstable dynamical systems have no practical significance. By the concept of system's stability, it is understood that when subjected to small variations in input signals, initial conditions, or system parameters, the state of the system should not exhibit substantial deviations. In other words, these are the minimal requirements that the system must be satisfied. This theory considers the system's behaviour over a long period, up to the time approaches infinity. It is worth noting that linear systems have only a single equilibrium state, whereas nonlinear systems typically exhibit multiple equilibrium states and dynamical behaviour. In this present thesis, the main focus is on the stability analysis of nonlinear systems.

Consider the system of differential equations

$$\dot{z}(t) = g(t, z), \ \ z(t_0) = z_0, \tag{1.3.1}$$

where $g : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$. Let $z(t) = z(t, t_0, z_0)$ be any solution of the system (1.3.1) with the initial value $z(t_0) = z_0$. Now, we recall the following definitions (see [7]):

Definition 1.3.1. A vector $z \in \mathbb{R}^n$ is said to be an equilibrium state of the nonlinear system (1.3.1) if and only if $g(t, z) = 0$.

Definition 1.3.2. The equilibrium z_e of (1.3.1) is called **stable** if, for given any t_0 and for each $\epsilon > 0$ there exist a $\delta = \delta(\epsilon, t_0) > 0$ such that, for any solution $z(t) = z(t, t_0, z_0)$ of (1.3.1), the inequality $||z(t_0) - z_e|| < \delta$ implies that $||z(t) - z_e|| < \epsilon$ for all $t \geq t_0$.

Definition 1.3.3. The equilibrium z_e of (1.3.1) is called **uniformly stable** if, for each $\epsilon > 0$, there exist a $\delta = \delta(\epsilon) > 0$, independent of t_0, such that for any solution $z(t) = z(t, t_0, z_0)$ of (1.3.1), the inequality $||z(t_0) - z_e|| < \delta$ implies that $||z(t) - z_e|| < \epsilon$ for all $t \geq t_0$.

Definition 1.3.4. The equilibrium z_e of (1.3.1) is called **asymptotically stable** if it is stable, for each $\epsilon > 0$, there exist a $\delta = \delta(t_0, \epsilon) > 0$, such that for any solution $z(t) = z(t, t_0, z_0)$ of (1.3.1), the inequality $||z(t_0) - z_e|| < \delta$ implies that $||z(t) - z_e|| \to 0$

as $t \to \infty$.

Definition 1.3.5. The equilibrium z_e of (1.3.1) is said to be unstable if it is not stable.

Definition 1.3.6. The equilibrium z_e of (1.3.1) is called **uniformly asymptotically stable** if it is uniformly stable and there exist a $\delta = \delta(\epsilon) > 0$, independent of t_0, such that for any solution $z(t) = z(t, t_0, z_0)$ of (1.3.1), the inequality $||z(t_0) - z_e|| < \delta$, $||z(t) - z_e|| \to 0$ as $t \to \infty$, uniformly in t_0; that is, for each $\eta > 0$, there is $T = T(\eta) > 0$ such that $||z(t) - z_e|| < \eta$, $\forall t \geq t_0 + T(\eta)$, $\forall ||z(t_0) - z_e|| < \delta$.

Definition 1.3.7. The equilibrium z_e of (1.3.1) is said to be **exponentially stable** if for any initial value $z(t_0) = z_0$, there exist $\alpha > 0$, $M \geq 1$ such that for any solution $z(t) = z(t, t_0, z_0)$ of (1.3.1), the inequality $||z(t) - z_e|| < M||z(t_0) - z_e||e^{-\alpha(t-t_0)}$, for all $t \geq t_0$.

1.4 Lyapunov's Direct Method of Stability

A Russian mathematician Alexander M. Lyapunov introduced the second method in 1982 which is known as the "Lyapunov second method" for determining the stability of solution in linear and nonlinear systems [8]. This method offers a significant advantage as it allows stability analysis without requiring prior knowledge of the system's solutions. Over the years, this fundamental concept has been extensively explored and effectively applied in stability theory. Today, it is widely recognized as a valuable tool not only for analyzing differential equations but also for studying control systems, dynamical systems, systems with time delays, and more. As this method provides stability information directly, without the need to solve the differential equations, we call it as the "Lyapunov direct method". The core of this approach revolves around the construction of a scalar function denoted as W and its derivative W'. If certain properties are satisfied by W and its derivative W', then several conclusions about the stability behaviour of the system can be drawn. This concept is similar to considering an energy function in a dynamical system, where each state of the system is associated with a certain amount of energy, often composed of potential and kinetic energy. If there is some form of friction or dissipation in a system, then this energy decreases slowly with time. Eventually, the system will take the state of lowest energy and become stable.

Now, here are some fundamental theorems (without proof) on the stability of solutions of differential systems without impulses using the concept of Lyapunov functions. These theorems can also be extended to impulsive differential systems. Detailed proofs for these theorems can be found in the book given by Lyapunov [8] and Yoshizawa [9].

Consider the following differential equation defined as:

$$\dot{z}(t) = g(t, z), \tag{1.4.1}$$

where $g : I \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous n-vector function. Assume that $g(t, 0) = 0$ for all t, then the following theorem holds true.

Theorem 1.4.1. (Lyapunov theorem) When the differential equations governing undisturbed motion (representing the steady state of a system before perturbations) can be expressed in the form where there exists a definite function W, and its derivative $W'$is a function with a fixed sign, which is either opposite to the sign of W or consistently equal to zero, then the undistributed motion is characterized as stable.

A further simplification of Theorem 1.4.1 results in the subsequent theorems.

Theorem 1.4.2. Suppose that there exists a function $W(t, z)$ defined for $t \geq 0$, $|z| < \delta_0$ (δ_0 is a positive constant) that satisfies the following properties:

(i) $W(t, z) = 0$,

(ii) $W(t, z) \geq \alpha(|z|)$, where $\alpha(r)$ is continuous monotonically increasing and $\alpha(0) = 0$,

(iii) $\dot{W}(t, z) \leq 0$,

then the zero solution i.e., $z(t) \equiv 0$ of differential system (1.4.1) is stable.

Theorem 1.4.3. Assume conditions (i) and (ii) of Theorem 1.4.2 hold, and if the condition (iii) is replaced with following condition

(iv) $\alpha(|z|) \leq W(t, z) \leq \beta(|z|)$, where $\alpha(r)$ and $\beta(r)$ being continuous monotonically increasing functions and $\alpha(0) = \beta(0) = 0$,

then the zero solution i.e., $z(t) \equiv 0$ of differential system (1.4.1) is uniformly stable.

Theorem 1.4.4. Under the assumption of the Theorem 1.4.3, if

(v) $\dot{W}(t, z) \leq -\gamma(|z|)$,

where $\gamma(r)$ is continuous on $[0, \delta_0]$ and positive definite, and if $g(t, z)$ is bounded then the zero solution i.e., $z(t) \equiv 0$ of differential system (1.4.1) is asymptotically stable.

Theorem 1.4.5. Under the same assumption of Theorem 1.4.3 with condition (v) of Theorem 1.4.4, the zero solution of differential system (1.4.1) is uniformly asymptotically stable.

Theorem 1.4.6. If $\dot{W}(t, z) \leq -\gamma(|z|)$, where $\gamma > 0$ is a constant under the same assumption as in Theorem 1.4.3, then the zero solution of differential system (1.4.1) is uniformly asymptotically stable.

1.5 Some Basic Definitions and Fundamental Concepts

Throughout the thesis, many definitions and concepts will be repeatedly used (see [7]). Some of such definitions are described below:

Let us consider the differential system

$$\dot{z} = g(t, z). \tag{1.5.1}$$

Let the $z(t;t_0,z_0)$ be the solution of system (1.5.1) along with initial condition $z(t_0) = z_0$.

Definition 1.5.1. A solution $z(t;t_0,z_0)$ of system (1.5.1) is bounded if there exist a $\rho > 0$, such that $|z(t;t_0,z_0)| < \rho$, for all $t \geq t_0$, where ρ may depend on each solution.

Definition 1.5.2. A function $\alpha(r)$ is said to belong to class K if $\alpha \in C[\mathbb{R}^+,\mathbb{R}^+]$, $\alpha(0) = 0$ and $\alpha(r)$ is strictly monotonically increasing in r. i.e., Class K=$\{\alpha : \alpha \in C[\mathbb{R}^+,\mathbb{R}^+] : \alpha(r)$ is strictly monotonically increasing in r and $\alpha(0) = 0\}$.

Definition 1.5.3. A scalar function $W(z)$ is called positive definite on set Γ if $W(0) = 0$ and $W(z) > 0$, for all $z \neq 0$, $z \in \Gamma$, or if $W(0) = 0$ and there exists a function $a(r) \in K$ such that $a(||z||) \leq W(z)$, $z \in \Gamma$.

Definition 1.5.4. A scalar function $W(z)$ is said to be negative definite on set Γ if and only if $-W(z)$ is positive definite on set Γ.

Definition 1.5.5. A function $g : X \to Y$ is said to be Lipschitz continuous if there exists a real constant $K > 0$ such that, for all $z_1, z_2 \in Z$,

$$|f(z_1) - f(z_2)| \leq K|z_1 - z_2|.$$

1.6 Biological and Artificial Networks

The meaning of the phrase "neural networks" is originated from the concept of a network or circuit of biological neurons implies, although nowadays, it usually refers to artificial neural networks, which are made up of artificial nodes or neurons. Biological neural networks are made up of actual human neurons that are functionally associated with one another and connected inside a nervous system. In contrast, artificial neural networks are computational models inspired by the natural neurons found in biological nervous systems. The structure components of both biological and artificial neural networks can be outlined from the following figures:

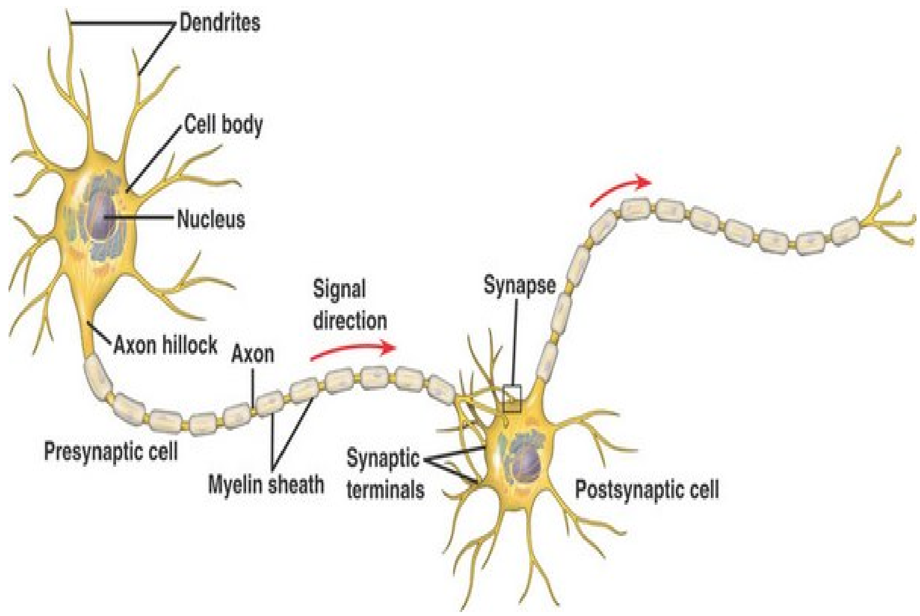

Figure 1.6.1: Schematic structure of a biological network

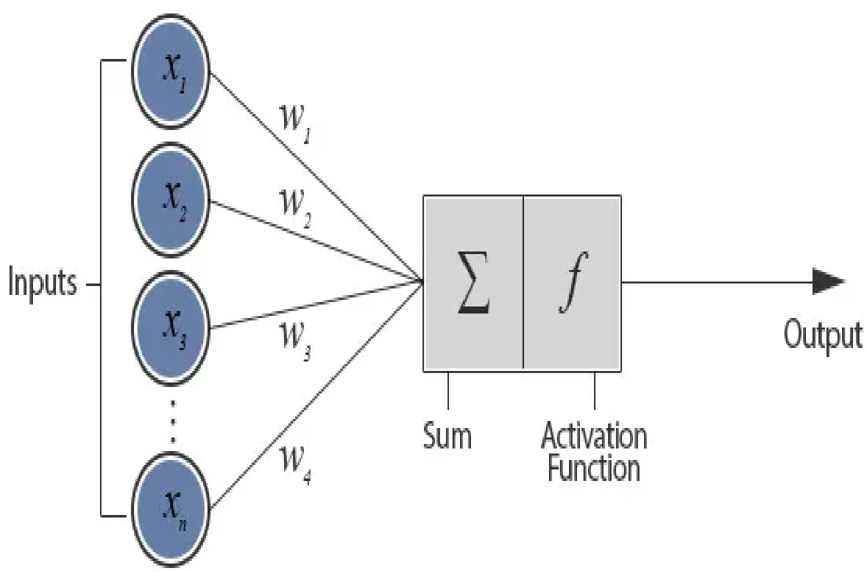

Figure 1.6.2: Schematic structure of an artificial neural network

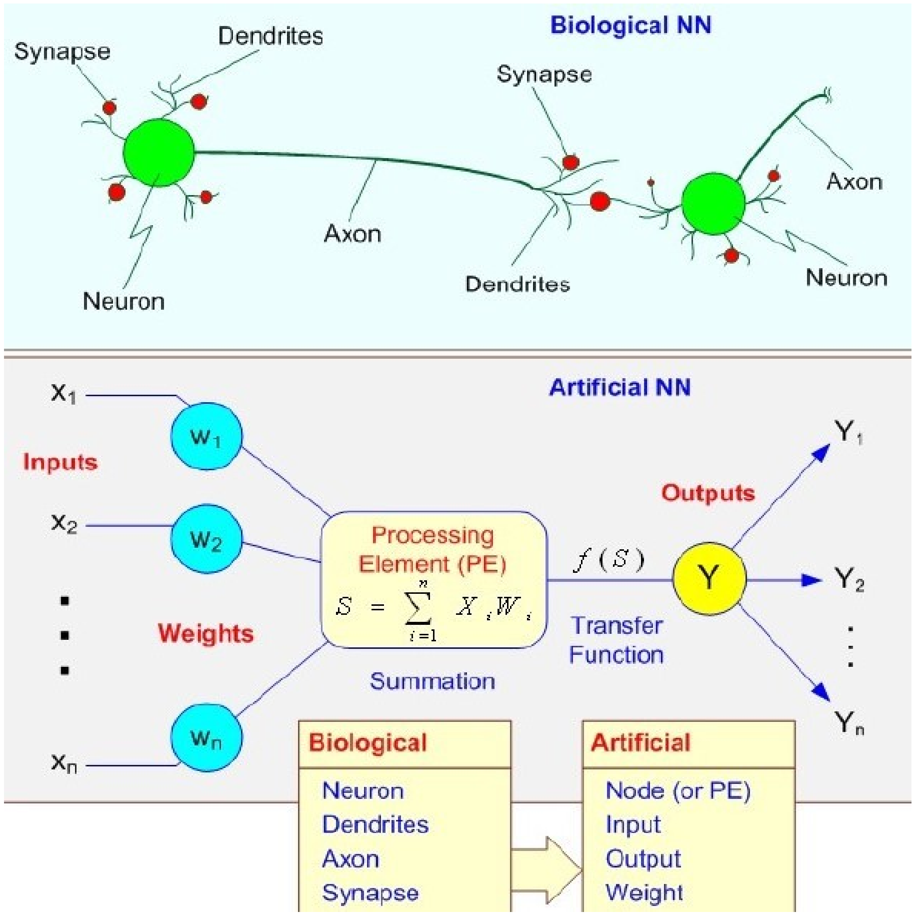

Figure 1.6.3: Comparison of biological and artificial networks

1.7 Neural Network Architecture

The combination of nonlinear functions connected to two or more neurons is represented by a network in which the weighting and combining of the individual functions to produce the total output are determined by the connections between the various units. The network architecture is the arrangement of connections between these neurons. In particular, neurons that analyze identical input variables are organized into layers, and lines of linking units in various layers represent the weights that modify the combination of nonlinear functions.

- **Input layer:** The nodes within the network are termed as input units, and their role is to encode the instances presented to the network for processing. For instance, each input unit may be characterized by an attribute value associated with the attributes possessed by the instance.

- **Hidden layer:** The nodes within the network other than the input nodes are referred to as hidden units, and they are not directly observable, remaining hidden. These hidden units contribute nonlinearities to the network.

- **Output layer:** The nodes within the network are designated as output units which are responsible for encoding potential concepts or values to be assigned to the instance under consideration. For instance, each output unit corresponds to a specific class of objects.

The architecture of various neural network models is determined by the method of the learning principles and the form of the connections between the links. Input units are primarily responsible for distributing information rather than processing it. Neural networks can be classified based on their interconnection scheme into two types: feed-forward networks and recurrent networks. Their descriptions are given below:

1.7.1 Feed-Forward Neural Network

The feed-forward neural network stands as one of the earliest and arguably the simplest forms of artificial neural networks. In this network, data flows in a unidirectional manner, from input to output layer, through hidden layer(s). Nodes from one layer are connected (using interconnection or links) to all nodes in the adjacent layer(s), but no lateral connection between nodes within one layer. In general, there are two primary categories of feed-forward networks:

Single-Layer Feed-Forward Networks

A layered network consists of just two layers at its most basic level: an output layer with target nodes and an input layer with source nodes. The main goal is to map or transform data from the input layer to the output layer, with a focus on the input-to-output flow being unidirectional rather than the other way around.

Multi-Layer Feed-Forward Networks

In this particular neural network architecture, multiple hidden layers are incorporated, with their processing units referred to as hidden neurons. These hidden neurons serve the crucial role of intermediaries, situated between the external input layer and the output layer. The inclusion of these hidden layers enhances the network's ability to capture more complex and higher-order statistical patterns. This becomes particularly advantageous while dealing with a substantial number of input neurons.

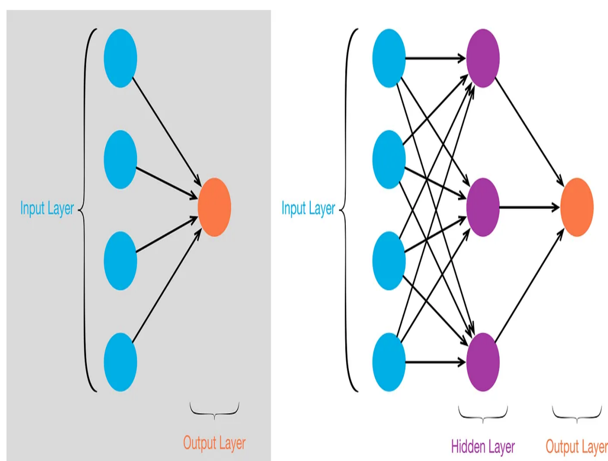

Figure 1.7.1: Structure of single and multi-layered feed-forward neural network

1.7.2 Recurrent Neural Network

One kind of neural network distinguished by connections between units that create a directed cycle is the recurrent neural network. The network's internal state is established by this cyclic structure, which allows it to exhibit dynamic temporal behaviour. The following figure describes its architechture:

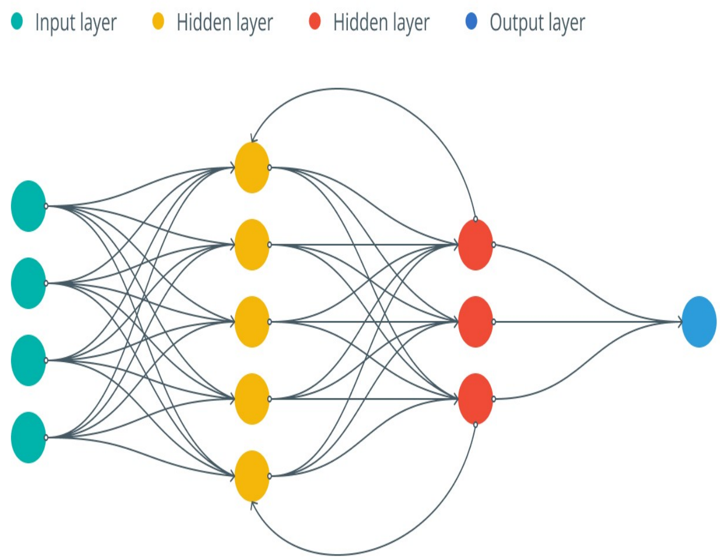

Figure 1.7.2: Structure of recurrent neural network

1.8 Features of Various Types of Neural Networks

Since the early beginning of artificial neural networks, researchers have developed a diverse array of artificial neural network types and their implementations based on the fundamental concept of the simple processing elements. This is a copy of the natural neuron and its extensive interconnections. Despite of their common foundation of neurons and connections, they exhibit notable distinctions in terms of topology, dynamics, feed, and function. Over the last decades, numerous optical neural network architectures have been introduced. In general, the various types of neural networks and their function abilities can be described as follows:

1.8.1 Hopfield Neural Network

In this neural network architecture, every neuron's output is linked through the adjustable connections to the inputs of all other neurons within the network. The only exception is the absence of connections between the output and input of the same neuron. Each neuron executes two consecutive operations: first, it computes the sum of input signals, each weighted by its respective factor, and second, it applies a bounded activation function to the result. This activation function features a linear range in the middle and increasing (or decreasing) portions at the ends. The dynamics of Hopfield neural network was first introduced by John Hopfield in his paper [10] published in 1984. After that, a great deal of attention has been given to its applications in associative memory [11, 12].

1.8.2 Cohen-Grossberg Neural Network

The Cohen-Grossberg neural network is a type of recurrent artificial neural network that was introduced by Cohen and Grossberg in 1983 [13]. These networks are employed to transform and store input patterns through competitive cellular networks. This technique has several uses, from the generation of patterns in developing biology through the firing of morphogenetic gradients to the short-term storage of language or visual patterns in neural networks. It also contributes to the creation of stable, context-sensitive parallel processors and the control of decision behaviour during macromolecular evolution. Furthermore, these systems have the capability to converge towards any of a potentially infinite number of equilibrium points in response to arbitrary input patterns and initial data. Resonance, standing waves, traveling waves, and even chaos are examples of these behaviours.

1.8.3 Bidirectional Associative Memory (BAM) Neural Network

In 1988, Kosko [14] introduced a significant type of artificial neural network, known as the BAM neural network. The BAM neural networks are composed of two layers of processing elements with fully interconnections between these layers. This unit may, or may not, have feedback connections of themselves. When the memory neurons are activated, the network reaches a stable state characterized by the reverberation of two patterns, each pattern at the output of one layer. These dynamics involves interactions between two layers. Because the memory processes information in time and involves bidirectional data flow, these models represent a generalization of the single-layer auto-associative Hebbian correlator to a two-layer pattern-matched hetero-associative model, i.e., content-addressable memory consisting of two layers of processing elements that are fully interconnected between the layers. It uses the forward and backward information flow to perform associative searches for stored stimulus-response associations. Due to such characteristic of the network, BAM neural network has gained increasing research interest due to their potential applications in diverse fields such as classification, optimization, memories, cryptography and so on. Recently, many researchers have investigated the dynamics of BAM neural networks and most of them have focused on the local and global stability analysis [15, 16, 17]. In [18, 19], some sufficient conditions are obtained for the stability of the equilibrium points of BAM neural networks.

1.8.4 Memristive Neural Networks

The concept of the memristor was initially introduced by Leon O. Chua [20] in 1971. Initially defined as components that establish a relationship between charge and magnetic flux, memristors have since been more accurately described as devices with pinched-hysteresis loops, the size of which depends on the frequency of the applied signal. The unique functionality of memristors is not exclusive to two-terminal passive devices but is essentially a memory effect associated with changes in their internal state variables. Their resistances are contingent on the complete history of the signal waveform, encompassing both voltage and current, applied across the memristor. The application of a memristor allows for the realization of circuit functionalities that cannot be achieved using traditional passive components like resistors, capacitors and inductors. This quality makes memristors exceptionally practical. In the field of electronics, they hold significant potential for low-power computation and data storage, as they can retain information without a continuous power supply [21] . Additionally, memristor exhibit properties akin to resistors and share the same unit of measurement (ohm).

In 2008, a team of scientists at Hewlett-Packard Lab created a prototype of the memristor, employing the concept of nanotechnology [22]. The memristor distinguishes itself from the other three fundamental circuit elements by its nonlinearity and its value is not unique. Memristors hold the promise of exceptional and distinctive applications, finding relevance in spintronic devices, high-density data storage, neuromorphic circuits, and programmable electronics. Furthermore, the emergence of the memristor as a novel circuit component has drawn significant attention from researchers, primarily due to its potential application in various physical systems. These encompass nonlinear semiconductor devices, analogue computation, programmable logic and so on. As predicted in [23], the memristors are highly likely to find utility in storage devices, with a potential future role in artificial neural networks. These networks could be used to simulate the functioning of the human brain in tasks like pattern recognition and real-time signal analysis from sensor arrays. A memristor's conductance varies with time or the amount of current flowing through it, working similarly to a biological synapse. Similar to this, the brain modifies the strength of synaptic connections between neurons to configure and learn for itself. Like synapses, memristors can assume any number of values over a range because of their capacity for memory and their ability to function as analogue devices. The brain is composed of memristors, which can self-learn from experience. The memristor neural networks have features of complex brain networks such as node degree, distribution and assortativity-both at the whole-brain scale of human neuro imaging. Taking a physical perspective, a memristive neural network is a hybrid nanomemristor/transistor logic circuit. Neural networks relying on memristors, with their enhanced dynamic behaviours, are poised to play a pivotal role in optimistic computation and associative memory. Therefore, recently the study of stability analysis of memristor neural networks have become very important among researchers [24, 25, 26, 27, 28].

1.9 Transfer Function (Activation Function)

Both the input-output function (transfer function) and the weights that are set for the units affect how an artificial neural network behaves. The following three definitions can be used to classify the most fundamental kinds of transfer functions:

1. Threshold function

The function that maps the total internal activity level of a neuron to a binary state of 0 or 1 can be expressed as follows:

$$f_k = \sum_{j=1}^{p} w_{kj} x_j - \theta_k,$$

where f_k represents the internal activity level of the neuron, w_{kj} is the strength of the neuron and θ_k denotes the threshold value, which has the effect of lowering the net input or increasing the net input depending on being positive or negative. The threshold that is defined as "bias" is utilized when the total input to the activation function increases that is

$$\varphi(f_k) = \begin{cases} 1, & \text{if } f \geq 0, \\ 0, & \text{if } f < 0. \end{cases}$$

2. Piecewise-linear function

Two essential characteristics of the structure of the activation function can be used to approximate a nonlinear amplifier:

1. When the linear region of operation is held without saturation, the output of the linear combiner rises.

2. The activation function essentially becomes a threshold function when the linear region's amplification factor reaches its maximum.

Now, this function can be described with the following equation:

$$\varphi(f) = \begin{cases} 1, & \text{if } f \geq \frac{1}{2}, \\ f, & \text{if } \frac{1}{2} > f > -\frac{1}{2}, \\ 0, & \text{if } f \leq -\frac{1}{2}. \end{cases}$$

3. Sigmoid function

In fact, two of the best examples of sigmoidal activation functions are the logistic function and the hyperbolic tangent function, which have great value for use in the development of artificial neural networks. From a mathematical point of view, the sigmoid activation function is well-behaved, differentiable and strictly increasing function.

$$\varphi(f) = \frac{1}{1 + \exp(-af)},$$

where a denotes the slope parameter. Implementing the hyperbolic tangent function in the activation function enables it to effectively process negative values. The following equation shows this property:

$$\varphi(f) = \tanh(\frac{f}{2}) = \frac{1 - \exp(-f)}{1 + \exp(-f)},$$

and

$$\varphi(f) = \begin{cases} 1, & \text{if } f > 0, \\ 0, & \text{if } f = 0, \\ -1 & \text{if } f < 0. \end{cases}$$

1.10 Complex-Valued Neural Networks

The propagation and interference of electromagnetic waves involve various aspects such as the magnitude of transmission and reflection, phase progression and retardation, and superposition of fields. These phenomena naturally find expression through the use of complex numbers. To address this complexity, neural networks are extended into the complex domain, forming what is known as Complex-Valued Neural Networks (CVNNs). In CVNNs, network parameters are treated as complex numbers, and computations follow the rules of complex algebra. A significant advantage of CVNNs lies in their excellent compatibility with wave phenomena. There are particularly well-suited for processes involving complex amplitudes, as seen in applications like interferometric radar systems. It is worth noting that, different from real-valued cases, the nonlinear function in CVNNs lacks standard complex derivatives. This is due to the fact that the Cauchy-Riemann equations do not hold in this context.

Numerous nonlinear problems such as XOR, parity-n, and symmetry detection problems, are known to be solvable by complex-valued neurons, whereas real-valued neurons are not capable to solve such problems. This provides motivation for an empirical study that uses two innovative activation functions to evaluate the classification performance of single-layered CVNNs on several real-world benchmark classification problems. The outcomes of the experiment show that single-layered CVNNs and multilayered real-valued neural networks have similar classification performances.

1.10.1 Complex-Valued Nonlinear Activation Function

CVNNs require special consideration, emphasizing that simple extensions from real-valued neural networks to CVNNs are insufficient for effective neural processing of complex-valued information. Specifically, the properties of boundedness and differentiability are not satisfied in the case of complex-valued activation function as given in Section 1.9.

For example, consider the log sigmoid given in Section 1.9 and replace $f \in \mathbb{R}$ with $f \in \mathbb{C}$. Then the function takes the form

$$\varphi(f) = \frac{1}{1+\exp(-af)}, f \in \mathbb{C}.$$

When z tends to any value in the set $\{0 \pm i(2n+1)\pi : n \,\text{is any integer and}\ i = \sqrt{i}\}$, $|\varphi(f)| \to \infty$. Therefore, $\varphi(f)$ exhibits singularity at regular intervals along the imaginary axis. This implies that the function is unbounded across the entire complex domain. A similar argument holds for the function $\tanh(\frac{f}{2})$ as $|\tanh(\frac{f}{2})| \to \infty$, whenever z tends to any values in the set $\{0 \pm i(2n+1)\pi\}$.

If a complex function is differentiable at every point in an open domain, it is said to be analytic there. An alternate way to define the requirement for complex differentiability is that a complex function is only considered analytic in an open domain if and only if it fulfils the Cauchy-Riemann equations across the entire domain. The strict limitations imposed by Cauchy-Riemann equations cause some real analytic functions which are analytical in every sense of the real domain to completely lose their analyticity quality when they are extended to the complex domain. It is commonly known that sigmoid functions exhibit singular points when extended into the complex domain. As a result, the sigmoid functions are not entire (analytic throughout the complex domain) functions. Therefore, in the complex domain, it is not differentiable.

When the analytic function is both bounded and entire, the activation function of CVNNs is determined in the same manner as real valued neural networks. However, this optimism such undermined by theorem of Liouville's, which means that a complex function is constant if it is both entire and bounded. A neuron with a constant activation function produces a constant output for every possible input signal. Therefore, there is no meaning of constant activation function in neurons. It is clear that the analytic and bounded properties become evident in CVNNs, as both of the properties cannot be simultaneously attained. This challenge has been identified as a significant concern in the development and application of CVNNs.

1.11 Synchronization

Chaos is a fundamental characteristic of nonlinear dynamical systems, and when the trajectory of such a system displays high sensitivity to initial conditions, then it is termed as chaotic nature. Various investigations, such as those conducted by researchers [29, 30, 31], have demonstrated the chaotic nature of neural networks. Because chaotic systems are extremely sensitive to initial conditions, synchronizing them is a considerable task. Any initial correlation between identical systems, even when starting from very close initial conditions, exponentially decreases to zero with time. Consequently, for practical purposes, any initial synchronization between the systems is practically guaranteed to vanish quickly. Despite these challenges, recent developments have introduced methods to achieve synchronized behaviour in chaotic systems. Pioneering work by Pecora and Carroll [32] introduced the concept of a response system locking onto a drive system. Studying simple systems may offer limited insights into the behaviour of complex systems composed of multiple independent driver systems. Understanding how these systems, which compete with each other to synchronize a common response system, requires a more comprehensive approach. The Pecora-Carroll driving mechanism can be understood as the limit of "strong coupling" in a more general framework of couplings that are oriented in a certain direction inside a network of chaotic elements. Applications for synchronisation of connected chaotic systems may arise in a number of domains, such as information sciences, chemical reactions, biological systems, secure communication, and chaos generator design.

1.12 Literature Review

The study on qualitative analysis of impulsive dynamical systems attracted many researchers in the later part of the nineteenth century after the pioneering work of Milman and Myshkis [1]. They provided fundamental concepts related to systems incorporating impulsive effects and emphasized that the theory of impulsive differential equations offers a broader scope in comparison to the theory of ordinary differential equations. This is because it is more practical and closer to the real-life situation. The main contribution to this theory was acquired when Lakshmikantham et al. [3] authored the book 'Theory of Impulsive Differential Equations' which articulated the effect of impulses on the stability behaviour of the solutions of impulsive systems. Moreover, Bainov and Simeonov [5] in 1989 discussed the mathematical theory of impulsive systems including the existence and continuity theorem, asymptotic properties of solutions, and Lyapunov stability in their book "Systems with Impulse Effect: Stability, Theory and Application". Afterwards, Liu [33] studied the impulsive stabilization of nonlinear systems by employing Lyapunov's direct method for both stabilization and destabilization. Liu [34] also established the stability criteria of impulsive differential equations and applied these results to population growth models. However, they did not consider the delay in the impulsive differential equations. Anokhin et al. [35] established the exponential stability of linear delayed impulsive differential equations with bounded delay and bounded coefficients. In order to control and synchronise chaotic systems, Yang and Chua [36] discovered a number of conclusions based on impulsive stabilisation. They also determined the circumstances in which impulsive control can asymptotically control chaotic systems back to the origin. They have also talked about how secure communication techniques can be implemented via impulsive synchronisation of chaotic systems. Yang et al. [37] extended the impulsive control scheme to the Lorenz system and found the condition under which the impulsively controlled Lorenz system is asymptotically stable. The concept and principle of the impulsive control scheme for asymptotic stability of a nonlinear system have been proposed by Yang [38] and they applied the same to a system in which system states are changeable in a short time. Further, Yang [39] has formulated several concepts of impulsive control such as different types of impulsive events, the existence and continuation of solutions, the solution of impulsive differential equations, beating phenomena and the comparison method in his book "Impulsive Control Theory". Liu and Ballinger [40] have given some sufficient criteria for uniform asymptotic stability for impulsive delay differential equations using the Lyapunov function and Razumikhin technique. Sun et al. [41] studied the asymptotic stability of an impulsive differential system by using a new comparison theorem. Based on the new result, a less conservative condition was derived for the asymptotic stability of impulsive control systems with impulses at fixed-times. Zhang et al. [42] established the mathematical model of impulse time harvest for the famous Logistic equation. Chellaboina et al. [43] developed an invariance principle for a dynamical system possessing left-continuous flow. They have shown that, for hybrid and impulsive dynamical systems, it is sufficient to establish an invariance principle if the system trajectories are guaranteed to be left-continuous over time for each fixed state point and to be continuous in the state for every time within some dense subset of the semi-infinite interval. Jiang and Lu [44] have examined the dynamics of an autonomous system with impulses that include impulsive state feedback control in a predator-prey model. They obtained bifurcation diagrams of periodic solutions using a Poincare map and showed the generation of a chaotic solution via a cascade of period-doubling bifurcation. Liu and Wang [45] generalized the results of exponential stability of impulsive systems with time delay by the implementation of Lyapunov functionals. Benchohara and Slimani [46] obtained a sufficient condition for the existence and uniqueness of a class of initial value problems for impulsive fractional differential equations involving Caputo fractional derivatives. Further, Liu et al. [47] investigated input-to-state stability and integral input-to-state stability of impulsive delayed systems based on the Lyapunov-Krasovski functionals method. They have shown that even if the continuous dynamics of all subsystems without impulses are not input-to-state or integral input-to-state, the system can be effectively stabilized in the input-to-state or integral input-to-state sense through the use of impulses, provided that there are no overly long intervals between impulses. The existence and uniqueness of the solution of interval fuzzy Cohen–Grossberg neural networks with piecewise constant argument was discussed by Bao et al. [48]. Li et al. [49] gave the periodic solution for impulsive systems using Lyapunov's second method and the contraction mapping method. Further, they derived some conditions ensuring the existence and global attractiveness of unique periodic solutions. Some results for nonlinear systems with state-dependent delayed impulses (i.e., impulses involving the delayed state of the systems for which the delay is state-dependent) were also given by Li and Wu [50]. Using the impulsive control theory and a few comparison arguments, they have produced general conclusions for the systems' uniform stability, uniform asymptotic stability, and exponential stability. Furthermore, a new impulsive delay inequality incorporating nondifferentiable and unbounded time-varying delay was introduced by Li and Cao [51]. Using their work as a basis, they were able to deduce adequate conditions that ensured the stability and stabilisation of impulsive systems with unbounded time-varying delay. A thorough and understandable review of impulsive control systems was given by Yang et al. [52]. Six areas of impulsive control systems were covered, including basic theory, Lyapunov stability, and delayed impulsive control systems. Li et al. [53] considered a nonlinear impulsive system subjected to delayed impulses, where the time delays in impulses occur between two consecutive impulsive instants. Utilizing impulsive control theory and employing the concept of average dwell time, they derived a set of Lyapunov-based sufficient conditions ensuring globally exponential stability. Liang and Wanli [54] explored the exponential synchronization problem for inertial Cohen-Grossberg neural networks with time delay. Moreover, the asymptotic stability of Markovian jump inertial Cohen-Grossberg was discussed by Huang and Cao [55]. Li et al. [56] also investigated the Lyapunov stability in impulsive systems based on event-triggered impulsive control. In this framework, the dynamical systems evolve according to continuous-time equations most of the time, but occasionally undergo instantaneous jumps when impulsive events are triggered. They established Lyapunov-based sufficient conditions for both uniform stability and globally asymptotical stability. Jiang et al. [57] studied the exponential stability of nonlinear delayed systems with destabilizing and stabilizing delayed impulses. Using the ideas of average impulsive interval and average impulsive delay, they presented some Lyapunov-based exponential stability criteria for delayed systems with average-delay impulses, where the delays in impulses satisfy the proposed average impulsive delay condition. Liu et al. [58] studied the boundary stabilization for stochastic delayed Cohen–Grossberg neural networks with diffusion terms by the Lyapunov functional method.

Above mentioned literature review is focused on the (Lyapunov) asymptotic or exponential stability of impulsive systems. The main property of asymptotic stability is that the solution approaches the equilibrium state as time approaches infinity. Finite-time stability (FTS) is a specific instance of asymptotic stability in which the system achieves equilibrium within a finite-time. This may lead to rapid transient responses and high-precision performance. Many interesting results have been investigated from theoretical as well as practical point of view by several researchers. It is worth mentioning that Bhat and Bernstein [59] first proposed the FTS of nonlinear systems. It has been demonstrated that the settling-time function and regularity properties of the Lyapunov function are related. Afterwards, Moulay and Perruquetti [60] investigated the sufficient condition for FTS for the continuous systems using the Lyapunov function. Moulay et al. [61] also developed some results on FTS and stabilization of time-delayed systems. Perruquetti et al. [62] discussed finite-time observer problems for the nonlinear system. Hong et al. [63] developed finite-time input-to-state stability and applications to finite-time control design for nonlinear systems. Wang et al. [64] proposed the FTS for a class of delayed neural networks. Peng et al. [65] discussed the finite-time synchronization of Cohen–Grossberg neural networks with both time-varying discrete delay and infinite-time distributed delay. Further, the novel finite-time synchronization of a class of inertial neural networks with time delays based on the integral inequality method was proposed by Zhang and Cao [66]. Kong et al. [67] looked into the finite-time robust synchronisation of fuzzy Cohen-Grossberg neural networks with discontinuous activations. He et al. [68] developed some sufficient conditions to guarantee the FTS for general time-varying systems based on the classical Lyapunov method. Further, the FTS of time-varying nonlinear systems based on a novel differential inequality approach was reported by Wu et al. [69]. Nersesov and Haddad [70] extended the FTS theory to impulsive dynamical systems involving resetting events. Later on, Li et al. [71] studied the FTS for nonlinear impulsive systems. They have established several Lyapunov-based FTS results for stabilizing and destabilizing impulses based on Lyapunov control theory. Xi [72] presented a Lyapunov-based FTS theory for impulsive systems by Lyapunov inequality condition. Wu et al. [73] extended the FTS for time-varying nonlinear impulsive systems. They established several Lyapunov theorems to guarantee the FTS property, which takes into account three distinct categories of impulses: stabilizing, destabilizing, and multiple. Xi et al. [74] presented some problems on uniform FTS, including the estimation of settling-time using time-varying differential inequality in which the derivative of the Lyapunov function derivative was not necessarily negative definite. Wang et al. [75] developed sufficient conditions for the finite-time input-to-state stability of nonlinear impulsive systems, bringing together the analysis of input-to-state stability and the convergence rate within a finite time. The FTS for a general form of nonlinear systems subjected to state-dependent delayed impulsive controller was investigated by Zhang et al. [76]. Yang and Li [77] studied the problem of FTS and finite-time contractive stability for nonlinear systems, where the possibility of time-delay in impulses had been fully considered. The main issue in FTS results is that the settling-time depends on the initial condition, which greatly limits its application in real practices. Therefore, a new definition of fixed-time stability (FxTS) for continuous systems was firstly proposed by Polyakov [78]. Later on, a novel type of Lyapunov functionals called implicit Lyapunov function to study FTS and FxTS of nonlinear systems was proposed by Polyakov et al. [79]. By calculating the inverse function of the Lyapunov function and applying the results to neural networks, Lu et al. [80] developed the FTS/FxTS of nonlinear systems. Hu et al. [81] produced a higher-precision calculation of settling-time, and a FxTS theorem was proposed for reduced absurdity outcomes. Through quantized control, Zhang et al. [82] addressed the finite-time and fixed-time synchronization issues of discontinuous complex networks. The fixed-time consensus problem of complex dynamical networks with fuzzy state-dependent uncertainty and nonlinear coupling effects was studied by Yang et al. [83]. Chen et al. [84] looked into the FxTS of nonlinear systems and put out a unique Lyapunov inequality. The novel FxTS results have been applied to study the FxTS of neural networks and they gave different upper bound estimate formulas for the settling time as compared to the existing literature. Xiao et al. [85] investigated the fixed-time synchronization of discontinuous Cohen–Grossberg neural networks with time-varying delays and matched disturbances based on the sliding-mode control. Udhayakumar et al . [86] investigated the new FxTS for delayed fractional-order systems. After that, Li et al. [87] extended the results to FxTS and stabilization of impulsive dynamical systems. Yang et al. [88] proposed fixed-time synchronization of complex networks with impulsive effects based on Lyapunov function and comparison systems. Li et al. [89] presented the FxTS for impulsive Cohen-Grossberg neural networks via two different controllers. Jao and Li [90] developed fixed-time synchronization of a class of delayed memristor neural networks based on Lyapunov functional, analytical techniques and using discontinuous controller. Using Filippov discontinuity theory and Lyapunov stability, Wei et al. [91] investigated both finite-time synchronization and fixed-time synchronization problems of inertial memristor neural networks with time-varying delay. Further, Chen et al. [92] discussed fixed-time synchronization of inertial memristor neural networks with discrete delay. The fixed-time synchronization problems for stochastic memristors using differential inclusions theory and set-valued maps were proposed by Ren et al. [93].

Wang et al. [94] proposed several unified finite/fixed-time criteria for nonlinear discontinuous impulsive systems under the framework of differential inclusion theory. Wei et al. [95] investigated the fixed-time synchronization for memristive Cohen-Grossberg neural networks with impulses. Zhang et al. [96] developed fixed-time synchronization of the impulsive memristor-based neural networks. Li et al. [97] discussed the fixed-time synchronization of complex dynamical networks with impulses on a proposed novel lemma. Wang et al. [98] elaborated a new two-stage comparison principle method to demonstrate the FxTS of nonlinear impulsive system. They presented a novel idea for the FxTS function set by taking the average impulsive interval into account. Using an indefinite Lyapunov functional method, Udhayakumar et al. [99] investigated the FTS and FxTS theorems of general fractional-order impulsive discontinuous systems. Pang et al. [100] derived fixed/pre-assigned-time synchronization of complex networks with impulses.

1.13 Objective of the Thesis

The main objective of the present thesis is to study the stability of nonlinear dynamical systems under the effects of stabilizing and destabilizing impulses and explore their applications in the synchronization of neural networks. This thesis deals with two types of problems one is impulsive systems without any impulse delay and another is impulsive systems with impulse delay. Moreover, it can also be categorized into two parts based on the concepts of stability: the first part consists of studying the fixed-time stability of nonlinear impulsive systems and extending the proposed results into the synchronization of various types of neural networks. Whereas, the second part is about the analysis of finite-time stability of nonlinear systems with delayed impulses.

In the case of finite-time stability, the upper bound of settling-time depends on the initial conditions, but for fixed-time stability, the upper bound of settling-time is independent of the initial condition. These two theories lead to rapid transient responses and high-precision performance as compared to the asymptotic and exponential stability. In light of these, the fixed-time stability theorem is proposed for impulsive dynamical systems. Two different types of impulsive sequences like, stabilizing and destabilizing impulses are considered to investigate some results on fixed-time stability theory by the concept of Lyapunov function. The estimated settling-time function is derived in this current work based on the comparison principle method and average impulsive interval. The present thesis mainly focuses on the fixed-time stability of impulsive systems in the presence of destabilizing impulses, as it has not been addressed in any existing articles. In this context, several new Lyapunov inequalities are proposed to achieve stability in fixed-time. Based on the proposed Lyapunov inequalities, unified controllers are designed to achieve the fixed-time synchronization of neural networks. Further, the designed controllers are incorporated to synchronize various types of neural networks, such as inertial Cohen-Grossberg neural networks, memristor neural networks, complex-valued neural networks and coupled neural networks in fixed-time.

This thesis also discusses the stability of nonlinear systems under delayed impulses within the finite-time. The delayed impulses mean that the impulses involve time delays. Such kind of impulses describe a phenomena where impulsive transient depend on not only their current but also historical state of the system. In order to analyze the finite-time stability of nonlinear systems for stabilizing and destabilizing impulses, the impulsive control theories and conecpt of Lyapunov stability theory are adopted here. Some Lyapunov-like theorems on finite-time stability along with settling-time estimation are established for two different kinds of delayed impulses. The obtained settling-time function for the nonlinear impulsive systems is proven to be dependent on the impulse time sequence as well as on the initial condition.