CHAPTER 3

NETWORK REPRESENTATION

3.1 INTRODUCTION

This chapter presents an overall model of power system comprising of multi-machine multi-area power system in state space framework. The system representation includes PSS at individual machine and FACTS devices (such as STATCOM, SSSC, and UPFC) in a generalized framework. The model is modular and general enough so as to include any control structure depending on system requirements and numbers of areas connected with transmission network. A Homogeneous model of UPFC has been developed in state space framework. The basic concept of FACTS with LQR control devices has been focused. A new integrated Multi-Stage LQR Power Oscillation Damping (MSLQR-POD) controller and Modified MSLQR controller have been developed for effective oscillation damping. The performance of these controllers (such as LQR controller, POD controller and MSLQR controller), in terms of settling time and overshoot/undershoot has been analysed.

3.2 CONCEPT OF SYSTEM REPRESENTATION

This section presents modeling which includes Power System Stabilizers (PSS) and Flexible AC Transmission System (FACTS) devices in a power network. A homogeneous state space model has been developed in generalized framework which can be extended so as to include number of machines in multi-areas on modular basis.

3.2.1 Machine Dynamics Representation in State Space Framework

A sample two area power system has been used to develop mathematical model of the complete system. In each area, ‘n’ number of generators are connected which are represented by G1, G2….Gn. This can be extended to any number of areas with some modular changes. Multi-area multi-machine system has been shown in Figure 3.1 in which 'n' number of generators are connected in parallel in area 1 as well as in area 2. Armature current and the terminal voltage of area 1 and area 2 are given by i_1 and v_{t1} and i_2 and v_{t2} respectively. Transmission line impedance is shown by Z whereas Y1 and Y2 represent the loading in the respective area [163-164].

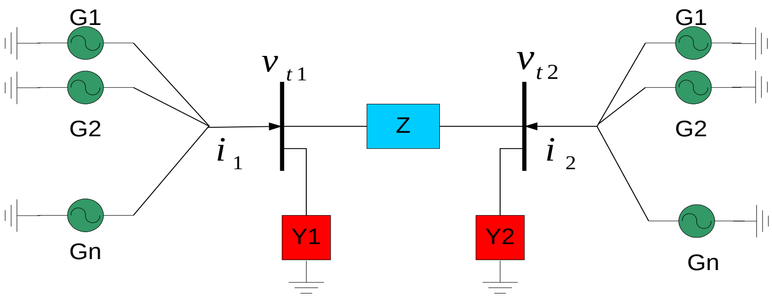


Figure 3.1 Multi-Machine power system

A. Modeling for i^th^ Generator of Area 1

Armature current and terminal voltage can be represented by:

$$i_1 = i_{d1} + ji_{q1}\,; v_{t1} = v_{d1} + jv_{q1} \tag{3.1}$$

Terminal voltage of generators can be given as:

$$v_{di1} + jv_{qi1} - jx_{ti1} * (i_{d1} + ji_{q1}) = v_{d1} + jv_{q1} \tag{3.2}$$

$$\begin{bmatrix} v_{d1} \\ v_{q1} \end{bmatrix} = \begin{bmatrix} v_{di1} \\ v_{qi1} \end{bmatrix} + \begin{bmatrix} 0 & x_{ti1} \\ -x_{ti1} & 0 \end{bmatrix} * \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \tag{3.3}$$

$$v_{ti1} = v_{ti1} * (\sin\delta_{i1} + j\cos\delta_{i1}) \tag{3.4}$$

Where $\delta_{i1} = \angle(E'_{qi1}, v_{ti1})$

From Figure 3.1 we have,

$$i_1 = Y1 * v_{t1} + (v_{t1} - v_{t2}) / Z \tag{3.5}$$

$$Z * i_1 = (1 + Z * Y1) * v_{t1} - v_{t2} \tag{3.6}$$

$$Z = R + jX \;\; \text{(Transmission Line)} \tag{3.7}$$

Equation (3.6) can be written in matrix form:

$$\begin{bmatrix} R & -X \\ X & R \end{bmatrix} * \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = \begin{bmatrix} C11 & -C21 \\ C21 & C11 \end{bmatrix} * \begin{bmatrix} v_{d1} \\ v_{q1} \end{bmatrix} - v_{t2} \qquad (3.8)$$

$$v_{di1} = x_{qi1} * i_{q1} \qquad (3.9)$$

$$v_{qi1} = E'_{qi1} - x'_{di1} * i_{d1} \qquad (3.10)$$

For convenience, the following constants and parameters are introduced:

$$1 + ZY1 = C11 + jC21 \qquad (3.11)$$

Where,

$$C11 = 1 + R*G1 - X*B1;\ C21 = X*G1 + R*B1 \qquad (3.12)$$

$$x'_{qi1} = x_{ti1} + x_{qi1} \qquad (3.13)$$

$$x''_{di1} = x_{ti1} + x'_{di1} \qquad (3.14)$$

$$R'_{i11} = R - C21*x''_{di1}, R'_{i21} = R - C21*x'_{qi1} \qquad (3.15)$$

$$X'_{i11} = X + C11*x'_{qi1};\ X'_{i21} = X + C11*x''_{di1} \qquad (3.16)$$

$$Z_{ei1}^2 = R'_{i11}*R'_{i21} + X'_{i11}*X'_{i21} \qquad (3.17)$$

$$Y_{di1} = (C11*X'_{i11} - C21*R'_{i21}) / Z_{ei1}^2 \qquad (3.18)$$

$$Y_{qi1} = (C11*R'_{i11} + C21*X'_{i21}) / Z_{ei1}^2 \qquad (3.19)$$

Where Y=G+jB (Load in each area),

Equation (3.8) can be written as:

$$\begin{bmatrix} R & -X \\ X & R \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = \begin{bmatrix} C11 & -C21 \\ C21 & C11 \end{bmatrix} \begin{bmatrix} v_{d1} \\ v_{q1} \end{bmatrix} - v_{t2} \begin{bmatrix} Sin(\delta_{iM} - \delta_{jN}) \\ Cos(\delta_{iM} - \delta_{jN}) \end{bmatrix} \qquad (3.20)$$

Where, x_{di1}, x'_{di1}, x''_{di1} are the direct axis synchronous, transient and sub-transient reactance of i^{th} generator in area 1. Where M and N are the number of area in which generator i and j are connected respectively.

But

$$\begin{bmatrix} v_{di1} \\ v_{qi1} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} E'_{qi1} - \begin{bmatrix} 0 & -x_{qi1} \\ x'_{di1} & 0 \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \tag{3.21}$$

Solving the Equations (3.3), (3.20) and (3.21), and rearranging in terms of i_{d1} and i_{q1}, we get:

$$\begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = \begin{bmatrix} Y_{di1} \\ Y_{qi1} \end{bmatrix} E'_{qi1} - (v_{t2} / Z^2_{ei1}) \begin{bmatrix} R'_{i21} & X'_{i11} \\ -X'_{i21} & R'_{i11} \end{bmatrix} \begin{bmatrix} Sin(\delta_{iM} - \delta_{jN}) \\ Cos(\delta_{iM} - \delta_{jN}) \end{bmatrix} \tag{3.22}$$

$$\Delta\delta = \delta_{iM} - \delta_{jN} \tag{3.23}$$

$$\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \end{bmatrix} = \begin{bmatrix} Y_{di1} \\ Y_{qi1} \end{bmatrix} \Delta E'_{qi1} + \begin{bmatrix} F_{di1} \\ F_{qi1} \end{bmatrix} \Delta\delta \tag{3.24}$$

Where:

$$\begin{bmatrix} F_{di1} \\ F_{qi1} \end{bmatrix} = -(v_{t2} / Z^2_{ei1}) \begin{bmatrix} R'_{i21} & X'_{i11} \\ -X'_{i21} & R'_{i11} \end{bmatrix} * \begin{bmatrix} Sin(\delta_{iM} - \delta_{jN}) \\ Cos(\delta_{iM} - \delta_{jN}) \end{bmatrix} \tag{3.25}$$

For calculating K_1, K_2, K_3, K_4, K_5 and K_6 constants:

Formulation from electric torque equation:

Electric torque of a synchronous machine near the synchronous speed can be calculated by:

$$T_{ei1} \approx P_{ei1} = i_{d1} * v_{di1} + i_{q1} * v_{qi1} \tag{3.26}$$

Now putting the values of v_{di1} and v_{qi1} from Equation (3.21) into Equation (3.26):

$$T_{ei1} = i_{q1} * E'_{qi1} + (x_{qi1} - x'_{di1}) i_{d1} * i_{q1} \tag{3.27}$$

Now linearizing the above equation and substituting the value of Δi_{d1} and Δi_{q1} from Equation (3.24), the following is obtained:

$$\Delta T_{ei1} = K_{1i1} * \Delta\delta + K_{2i1} * E'_{qi1} \tag{3.28}$$

Where values of K_{1i1} and K_{2i1} are:

$$\begin{bmatrix} K_{1i1} \\ K_{2i1} \end{bmatrix} = \begin{bmatrix} 0 \\ i_{q1} \end{bmatrix} + \begin{bmatrix} F_{di1} & F_{qi1} \\ Y_{di1} & Y_{qi1} \end{bmatrix} \begin{bmatrix} (x_{qi1} - x'_{di1}) * i_{q1} \\ E'_{qi1} + (x_{qi1} - x'_{di1}) * i_{d1} \end{bmatrix} \tag{3.29}$$

From the field voltage equation:

The field winding circuit voltage can be linearized and written as:

$$(1+sT'_{do})*E'_{qi1} = \Delta E_{fdi1} - (x_{di1} - x'_{di1})*\Delta i_{d1} \tag{3.30}$$

Putting the value of Δi_{d1} from Equation (3.24),

$$(1+sT'_{do}*K_{3i1})*E'_{qi1} = K_{3i1}*[\Delta E_{fdi1} - K_{4i1}*\Delta\delta] \tag{3.31}$$

Here values of K_{3i1} and K_{4i1} are:

$$K_{3i1} = 1/[1+(x_{di1} - x'_{di1})*Y_{di1}] \tag{3.32}$$

$$K_{4i1} = (x_{di1} - x'_{di1})*F_{di1} \tag{3.33}$$

From the terminal voltage magnitude equation:

Magnitude of the generator terminal voltage can be expressed in d and q components as:

$$v^2_{ti1} = v^2_{di1} + v^2_{qi1} \tag{3.34}$$

Now calculating the deviation by following equation:

$$\Delta v_{ti1} = (v_{di1}/v_{ti1})*\Delta v_{di1} + (v_{qi1}/v_{ti1})*\Delta v_{qi1} \tag{3.35}$$

Putting the values of Δv_{di1} and Δv_{qi1} in Equation (3.35),

$$\Delta v_{ti1} = K_{5i1}*\Delta\delta + K_{6i1}*\Delta E'_{qi1} \tag{3.36}$$

Where values of K_{5i1} and K_{6i1} are:

$$\begin{bmatrix} K_{5i1} \\ K_{6i1} \end{bmatrix} = \begin{bmatrix} 0 \\ (v_{qi1}/v_{ti1}) \end{bmatrix} + \begin{bmatrix} F_{di1} & F_{qi1} \\ Y_{di1} & Y_{qi1} \end{bmatrix} \begin{bmatrix} -(x'_{di1}*v_{qi1})/v_{ti1} \\ (x_{qi1}*v_{di1})/v_{ti1} \end{bmatrix} \tag{3.37}$$

The equation of real and reactive power:

$$P_{ei1} + jQ_{ei1} = (i_{d1} + ji_{q1})*(v_{di1} + jv_{qi1}) \tag{3.38}$$

Values of v_{di1} and v_{qi1} are calculated from Equation (3.21) and terminal voltage v_{ti1} is written in terms of v_{di1} and v_{qi1}. Now five independent equations are:

$$P_{ei1} = i_{d1}*v_{di1} + i_{q1}*v_{qi1} \tag{3.39}$$

$$Q_{ei1} = i_{d1} * v_{qi1} - i_{q1} * v_{di1} \tag{3.40}$$

$$v_{di1} = x_{qi1} * i_{q1} \tag{3.41}$$

$$v_{qi1} = E'_{qi1} - x'_{di1} * i_{d1} \tag{3.42}$$

$$v^2_{ti1} = v^2_{di1} + v^2_{qi1} \tag{3.43}$$

From the above five equations, the following results:

$$P_{ei1} * v_{qi1} - Q_{ei1} * v_{di1} = i_{q1} * v^2_{ti1} \tag{3.44}$$

Solving i_{q1} and v_{qi1} from Equations (3.41) and (3.43) and putting this in Equation (3.44), following equation obtained:

$$P_{ei1} * (v^2_{ti1} - v^2_{di1})^{1/2} - Q_{ei1} * v_{di1} = v_{di1} * (v^2_{ti1} / x_{qi1}) \tag{3.45}$$

So, v_{di1} can be found, which leads to the following equations;

$$v_{di1} = P_{ei1} * v_{ti1} * [P_{ei1}{}^2 + (Q_{ei1} + v^2_{ti1} / x_{qi1})^2]^{-1/2} \tag{3.46}$$

$$v_{qi1}{}^2 = (v_{ti1}{}^2 - v_{di1}{}^2)\,; \quad i_{q1} = v_{di1} / x_{qi1} \tag{3.47}$$

$$i_{d1} = (P_{ei1} - i_{q1} * v_{qi1}) / v_{di1} \text{ or } (Q_{ei1} + i_{q1} * v_{di1}) / v_{qi1} \tag{3.48}$$

$$E'_{qi1} = v_{qi1} + x'_{di1} * i_{d1} \tag{3.49}$$

$$\delta_{i1} = \tan^{-1}(v_{di1} / v_{qi1}) \tag{3.50}$$

B. Modeling for ith Generator of Area 2

Armature current and terminal voltage can be represented by:

$$i_2 = i_{d2} + ji_{q2}\,; v_{t2} = v_{d2} + jv_{q2} \tag{3.51}$$

Terminal voltage of ith generators can be given as:

$$v_{di2} + jv_{qi2} - jx_{ti2}(i_{d2} + ji_{q2}) = v_{d2} + jv_{q2} \tag{3.52}$$

$$\begin{bmatrix} v_{d2} \\ v_{q2} \end{bmatrix} = \begin{bmatrix} v_{di2} \\ v_{qi2} \end{bmatrix} + \begin{bmatrix} 0 & x_{ti2} \\ -x_{ti2} & 0 \end{bmatrix} * \begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} \tag{3.53}$$

$$v_{ti2} = v_{ti2} * (\sin\delta_{i2} + j\cos\delta_{i2}) \qquad (3.54)$$

From Figure 3.1 we have,

$$i_2 = Y2 * v_{t2} + (v_{t2} - v_{t1}) / Z \qquad (3.55)$$

$$Z * i_2 = (1 + ZY2) * v_{t2} - v_{t1} \qquad (3.56)$$

Equations (3.55) and (3.56) can be written in matrix form

$$\begin{bmatrix} R & -X \\ X & R \end{bmatrix} * \begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} = \begin{bmatrix} C12 & -C22 \\ C22 & C12 \end{bmatrix} * \begin{bmatrix} v_{d2} \\ v_{q2} \end{bmatrix} - v_{t1} \qquad (3.57)$$

$$v_{di2} = x_{qi2} * i_{q2} \qquad (3.58)$$

$$v_{qi2} = E'_{qi2} - x'_{di2} * i_{d2} \qquad (3.59)$$

For convenience, the following constants and parameters are introduced:

$$1 + ZY2 = C12 + jC22 \qquad (3.60)$$

Where:

$$C12 = 1 + R * G2 - X * B2;\ C22 = X * G2 + R * B2 \qquad (3.61)$$

$$x'_{qi2} = x_{ti2} + x_{qi2} \qquad (3.62)$$

$$x''_{di2} = x_{ti2} + x'_{di2} \qquad (3.63)$$

$$R'_{i12} = R - C22 * x''_{di2},\ R'_{i22} = R - C22 * x'_{qi2} \qquad (3.64)$$

$$X'_{i12} = X + C12 * x'_{qi2};\ X'_{i22} = X + C12 * x''_{di2} \qquad (3.65)$$

$$Z_{ei2}^2 = R'_{i12} * R'_{i22} + X'_{i12} * X'_{i22} \qquad (3.66)$$

$$Y_{di2} = (C12 * X'_{i12} - C22 * R'_{i22}) / Z_{ei2}^2 \qquad (3.67)$$

$$Y_{qi2} = (C12 * R'_{i12} + C22 * X'_{i22}) / Z_{ei2}^2 \qquad (3.68)$$

Where Y=G+jB (Load in each area),

Equation (3.57) can be written as:

$$\begin{bmatrix} R & -X \\ X & R \end{bmatrix}\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix}=\begin{bmatrix} C12 & -C22 \\ C22 & C12 \end{bmatrix}\begin{bmatrix} v_{d2} \\ v_{q2} \end{bmatrix}-v_{t1}\begin{bmatrix} Sin(\delta_{jN}-\delta_{iM}) \\ Cos(\delta_{jN}-\delta_{iM}) \end{bmatrix} \tag{3.69}$$

Where, x_{di2}, x'_{di2}, x''_{di2} are the direct axis synchronous, transient and sub-transient reactance of i[th] generator in area 2.

But

$$\begin{bmatrix} v_{di2} \\ v_{qi2} \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}E'_{qi2}-\begin{bmatrix} 0 & -x_{qi2} \\ x'_{di2} & 0 \end{bmatrix}\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} \tag{3.70}$$

Solving Equations (3.53), (3.69) and (3.70), and rearranging in terms of i_{d2} and i_{q2}, this results as:

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix}=\begin{bmatrix} Y_{di2} \\ Y_{qi2} \end{bmatrix}E'_{qi2}-(v_{t1}/Z_{ei2}^{2})\begin{bmatrix} R'_{i22} & X'_{i12} \\ -X'_{i22} & R'_{i12} \end{bmatrix}\begin{bmatrix} Sin(\delta_{jN}-\delta_{iM}) \\ Cos(\delta_{jN}-\delta_{iM}) \end{bmatrix} \tag{3.71}$$

$$\Delta\delta=\delta_{jN}-\delta_{iM} \tag{3.72}$$

$$\begin{bmatrix} \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix}=\begin{bmatrix} Y_{di2} \\ Y_{qi2} \end{bmatrix}\Delta E'_{qi2}+\begin{bmatrix} F_{di2} \\ F_{qi2} \end{bmatrix}\Delta\delta \tag{3.73}$$

Where:

$$\begin{bmatrix} F_{di2} \\ F_{qi2} \end{bmatrix}=-(v_{t1}/Z_{ei2}^{2})\begin{bmatrix} R'_{i22} & X'_{i12} \\ -X'_{i22} & R'_{i12} \end{bmatrix}*\begin{bmatrix} Sin(\delta_{jN}-\delta_{iM}) \\ Cos(\delta_{jN}-\delta_{iM}) \end{bmatrix} \tag{3.74}$$

For calculating these K_1, K_2, K_3, K_4, K_5 and K_6 constants:

From electric torque equation:

Electric torque of a synchronous machine near the synchronous speed can be calculated by:

$$T_{ei2}\approx P_{ei2}=i_{d2}*v_{di2}+i_{q2}*v_{qi2} \tag{3.75}$$

Now putting the values of v_{di1} and v_{qi1} from Equation (3.70) into Equation (3.75):

$$T_{ei2}=i_{q2}*E'_{qi2}+(x_{qi2}-x'_{di2})i_{d2}*i_{q2} \tag{3.76}$$

Now linearizing the above equation and substituting the value of Δi_{d2} and Δi_{q2} from

Equation (3.73), we get:

$$\Delta T_{ei2} = K_{1i2} * \Delta\delta + K_{2i2} * E'_{qi2} \tag{3.77}$$

Where values of K_{1i2} and K_{2i2} are:

$$\begin{bmatrix} K_{1i2} \\ K_{2i2} \end{bmatrix} = \begin{bmatrix} 0 \\ i_{q2} \end{bmatrix} + \begin{bmatrix} F_{di2} & F_{qi2} \\ Y_{di2} & Y_{qi2} \end{bmatrix} \begin{bmatrix} (x_{qi2} - x'_{di2}) * i_{q2} \\ E'_{qi2} + (x_{qi2} - x'_{di2}) * i_{d2} \end{bmatrix} \tag{3.78}$$

From the field voltage equation:

The field winding circuit voltage can be linearized and written as:

$$(1 + sT'_{do}) * E'_{qi2} = \Delta E_{fdi2} - (x_{di2} - x'_{di2}) * \Delta i_{d2} \tag{3.79}$$

Putting the value of Δi_{d2} from Equation (3.71),

$$(1 + sT'_{do} * K_{3i2}) * E'_{qi2} = K_{3i2} * [\Delta E_{fdi2} - K_{4i2} * \Delta\delta] \tag{3.80}$$

Where values of K_{3i2} and K_{4i2} are:

$$K_{3i2} = 1/[1 + (x_{di2} - x'_{di2}) * Y_{di2}] \tag{3.81}$$

$$K_{4i2} = (x_{di2} - x'_{di2}) * F_{di2} \tag{3.82}$$

From the terminal voltage magnitude equation:

Magnitude of the generator terminal voltage can be expressed in d and q components as:

$$v^2_{ti2} = v^2_{di2} + v^2_{qi2} \tag{3.83}$$

Now calculating the deviation by following equation;

$$\Delta v_{ti2} = (v_{di2} / v_{ti2}) * \Delta v_{di2} + (v_{qi2} / v_{ti2}) * \Delta v_{qi2} \tag{3.84}$$

Putting the values of Δv_{di2} and Δv_{qi2} in Equation (3.84),

$$\Delta v_{ti2} = K_{5i2} * \Delta\delta + K_{6i2} * \Delta E'_{qi2} \tag{3.85}$$

Where values of K_{5i2} and K_{6i2} are:

$$\begin{bmatrix} K_{5i2} \\ K_{6i2} \end{bmatrix} = \begin{bmatrix} 0 \\ (v_{qi2} / v_{ti2}) \end{bmatrix} + \begin{bmatrix} F_{di2} & F_{qi2} \\ Y_{di2} & Y_{qi2} \end{bmatrix} \begin{bmatrix} -(x'_{di2} * v_{qi2}) / v_{ti2} \\ (x_{qi2} * v_{di2}) / v_{ti2} \end{bmatrix} \tag{3.86}$$

The equation of real and reactive Power:

$$P_{ei2} + jQ_{ei2} = (i_{d2} + ji_{q2})*(v_{di2} + jv_{qi2}) \tag{3.87}$$

Values of v_{di2} and v_{qi2} can be calculated from Equation (3.70) and terminal voltage v_{ti2} can be written in terms of v_{di2} and v_{qi2}. Now five independent equations are:

$$P_{ei2} = i_{d2} * v_{di2} + i_{q2} * v_{qi2} \tag{3.88}$$

$$Q_{ei2} = i_{d2} * v_{qi2} - i_{q2} * v_{di2} \tag{3.89}$$

$$v_{di2} = x_{qi2} * i_{q2} \tag{3.90}$$

$$v_{qi2} = E'_{qi2} - x'_{di2} * i_{d2} \tag{3.91}$$

$$v^2_{ti2} = v^2_{di2} + v^2_{qi2} \tag{3.92}$$

From the above five equations, the resultant equation is:

$$P_{ei2} * v_{qi2} - Q_{ei2} * v_{di2} = i_{q2} * v^2_{ti2} \tag{3.93}$$

Solving i_{q2} and v_{qi2} from Equations (3.90) and (3.92) and putting this in Equation (3.93), it can be written as:

$$P_{ei2} * (v^2_{ti2} - v^2_{di2})^{1/2} - Q_{ei2} * v_{di2} = v_{di2} * (v^2_{ti2} / x_{qi2}) \tag{3.94}$$

So v_{di2} can be found, which leads to the following equations;

$$v_{di2} = P_{ei2} * v_{ti2} * [P_{ei2}{}^2 + (Q_{ei2} + v^2_{ti2} / x_{qi2})^2]^{-1/2} \tag{3.95}$$

$$v_{qi2}{}^2 = (v_{ti2}{}^2 - v_{di2}{}^2) \tag{3.96}$$

$$i_{q2} = v_{di2} / x_{qi2} \tag{3.97}$$

$$i_{d2} = (P_{ei2} - i_{q2} * v_{qi2}) / v_{di2} \text{ or } (Q_{ei2} + i_{q2} * v_{di2}) / v_{qi2} \tag{3.98}$$

$$E'_{qi2} = v_{qi2} + x'_{di2} * i_{d2} \tag{3.99}$$

$$\delta_{i2} = \tan^{-1}(v_{di2} / v_{qi2}) \tag{3.100}$$

For **First Area**, final states equation can be written as:

$$M_i s\Delta\omega_{i1} = -(K_{1i1}\Delta\delta_{i1} + K_{2i1}\Delta E'_{qi1}) \qquad (3.101)$$

$$s\Delta\delta_{i1} = \omega_{bi}\Delta\omega_{i1} \qquad (3.102)$$

$$(1 + sT'_{doi}K_{3i1})\Delta E'_{qi1} = K_{3i1}(-K_{4i1}\Delta\delta_{i1} + \Delta E_{fdi1}) \qquad (3.103)$$

$$(1 + sT_{Ai1})\Delta E_{fdi1} = K_{Ai}(UE - \Delta v_{ti1}) = K_{Ai1}(UE - K_{5i1}\Delta\delta_{i1} - K_{6i1}\Delta E'_{qi1}) \qquad (3.104)$$

By writing equations in matrix form,

$$\begin{bmatrix} \Delta\dot{\omega}_{i1} \\ \Delta\dot{\delta}_{i1} \\ \Delta\dot{E}'_{qi1} \\ \Delta\dot{E}_{fdi1} \end{bmatrix} = [A1i] \begin{bmatrix} \Delta\omega_{i1} \\ \Delta\delta_{i1} \\ \Delta E'_{qi1} \\ \Delta E_{fdi1} \end{bmatrix} \qquad (3.105)$$

$$[A1i] = \begin{bmatrix} 0 & -K_{1i1}/M_i & -K_{2i1}/M_i & 0 \\ \omega_{bi} & 0 & 0 & 0 \\ 0 & -K_{4i1}/T'_{doi} & -1/(T'_{doi} * K_{3i1}) & 1/T'_{doi} \\ 0 & -K_{Ai} * K_{5i1}/T_{Ai} & -K_{Ai} * K_{6i1}/T_{Ai} & -1/T_{Ai} \end{bmatrix} \qquad (3.106)$$

Complete State matrix for area 1 only can be written as:

$$[A1] = \begin{bmatrix} A11 & 0 & 0 & 0 \\ 0 & A12 & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & A1n \end{bmatrix} \qquad (3.107)$$

Similarly for **Second Area**, state equations can be written as:

$$M_i s\Delta\omega_{i2} = -(K_{1i2}\Delta\delta_{i2} + K_{2i2}\Delta E'_{qi2}) \qquad (3.108)$$

$$s\Delta\delta_{i2} = \omega_{bi}\Delta\omega_{i2} \qquad (3.109)$$

$$(1 + sT'_{doi}K_{3i2})\Delta E'_{qi2} = K_{3i2}(-K_{4i2}\Delta\delta_{i2} + \Delta E_{fdi2}) \qquad (3.110)$$

$$(1 + sT_{Ai2})\Delta E_{fdi2} = K_{Ai}(UE - \Delta v_{ti2}) = K_{Ai2}(UE - K_{5i2}\Delta\delta_{i2} - K_{6i2}\Delta E'_{qi2}) \qquad (3.111)$$

By writing equations in matrix form,

$$\begin{bmatrix} \Delta \dot{\omega}_{i2} \\ \Delta \dot{\delta}_{i2} \\ \Delta \dot{E}'_{qi2} \\ \Delta \dot{E}_{fdi2} \end{bmatrix} = [A2i] \begin{bmatrix} \Delta\omega_{i2} \\ \Delta\delta_{i2} \\ \Delta E'_{qi2} \\ \Delta E_{fdi2} \end{bmatrix} \tag{3.112}$$

$$[A2i] = \begin{bmatrix} 0 & -K_{1i2}/M_i & -K_{2i2}/M_i & 0 \\ \omega_{bi} & 0 & 0 & 0 \\ 0 & -K_{4i2}/T'_{doi} & -1/(T'_{doi} * K_{3i2}) & 1/T'_{doi} \\ 0 & -K_{Ai} * K_{5i2}/T_{Ai} & -K_{Ai} * K_{6i2}/T_{Ai} & -1/T_{Ai} \end{bmatrix} \tag{3.113}$$

Complete state matrix for area 2 can be written as:

$$[A2] = \begin{bmatrix} A21 & 0 & 0 & 0 \\ 0 & A22 & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & A2n \end{bmatrix} \tag{3.114}$$

Hence complete state matrix of two area power system can be written as:

$$[A] = \begin{bmatrix} [A1] & 0 \\ 0 & [A2] \end{bmatrix} \tag{3.115}$$

Where A is the complete system matrix for two area system, the number of generators are represented as i = 1, 2,..., n. For multi-area system the size of matrix will change accordingly. State space equation of the system is obtained as:

$$\Delta \dot{x} = [A]\Delta x \tag{3.116}$$

$$\begin{bmatrix} \Delta \dot{\omega}_{i1} \\ \Delta \dot{\delta}_{i1} \\ \Delta \dot{E}'_{qi1} \\ \Delta \dot{E}_{fdi1} \\ \Delta \dot{\omega}_{i2} \\ \Delta \dot{\delta}_{i2} \\ \Delta \dot{E}'_{qi2} \\ \Delta \dot{E}_{fdi2} \end{bmatrix} = [A] \begin{bmatrix} \Delta \omega_{i1} \\ \Delta \delta_{i1} \\ \Delta E'_{qi1} \\ \Delta E_{fdi1} \\ \Delta \omega_{i2} \\ \Delta \delta_{i2} \\ \Delta E'_{qi2} \\ \Delta E_{fdi2} \end{bmatrix} \tag{3.117}$$

Where, $\Delta x = \left[\Delta\omega_{i1}, \Delta\delta_{i1}, \Delta E'_{qi1}, \Delta E_{fdi1}, \Delta\omega_{i2}, \Delta\delta_{i2}, \Delta E'_{qi2}, \Delta E_{fdi2}\right]^T$

3.2.2 Inclusion of Power System Stabilizers (PSS) Model in the System

Power System Stabilizer (PSS) has been used along with excitation control for low-frequency oscillation (0.2-3 Hz) damping. Since low-frequency oscillations are observed due to the dynamical changes in large power network and reflected at generator terminals if not properly controlled at transmission system. It is known that if these oscillations are not damped at earliest, the magnitude of these oscillations may increase and ultimately can collapse the entire system due to system interaction. In order to reduce the low-frequency oscillation, PSS adds stabilizing signal to AVR that modulates the excitation precisely by modulating electrical torque component in phase with angular speed variation of rotor, thus damping in the generator oscillation quickly.

Figure 3.2 presents block diagram for low-frequency oscillation studies. There are two major loops in Figure 3.2. Top loop represents mechanical system and lower loop represents electrical system. Small perturbation has been reflected with linearized equations as Δ. In mechanical loop, incremental torque is used as input and rotor angle (Δδ) as the output. First block from left to right is based on the torque equilibrium equation and second block shows the relation between angular speed and phase angle. Here, M represents the inertia constant of the synchronous machine, D is the damping coefficient and 2πf is the synchronous speed. In the electrical loop, input is the difference between supplementary control (UE) and terminal voltage Δv_t and output is the incremental internal voltage $\Delta E'_q$. First block from right to left represents exciter and voltage regulator with gain K_A and time constant T_A. Second block represents field winding circuit as affected by armature reaction.

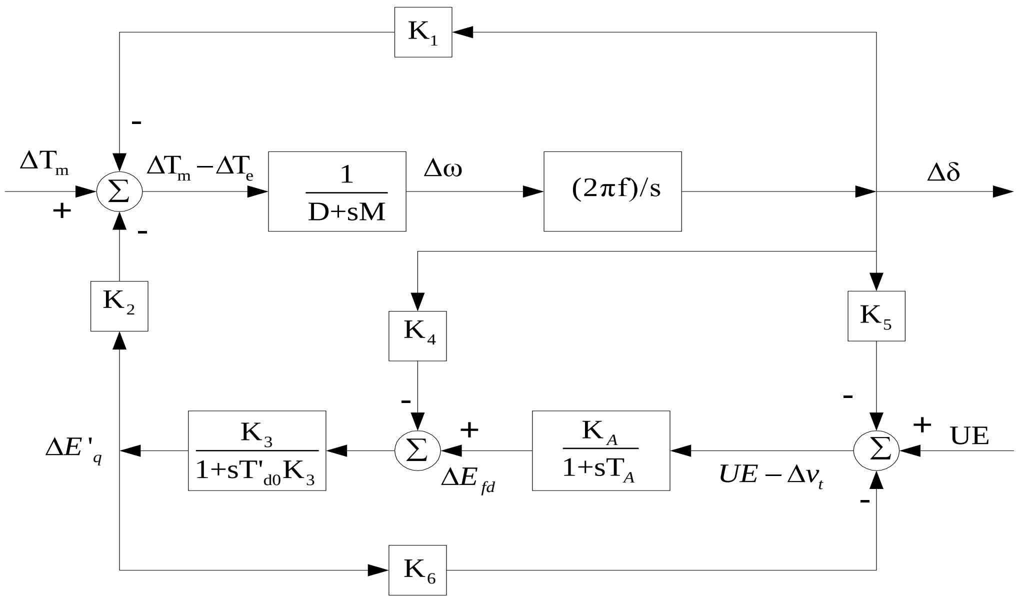


Figure 3.2 Transfer function block diagram for low-frequency oscillation studies

Due to phase lag of excitation system and field circuit, a phase lead circuit is included in the supplementary excitation control in the form of PSS. PSS precisely modulates damping torque ΔT_E in phase with angular speed at the oscillating frequency. An adequate damping is introduced by gain (Kc). Figure 3.3 shows the block diagram of PSS. The first block from left to right is reset block which is used to activate the supplementary excitation only as system oscillation begins and to terminate when the system oscillation ceases, whereas second block provides phase lead/lag compensation depending upon dynamical changes in network.

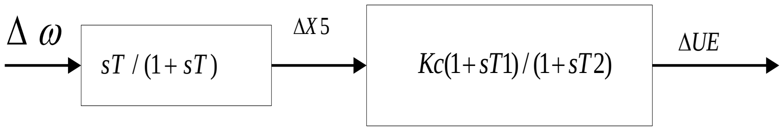


Figure 3.3 Block diagram of PSS model

From Figure 3.3, the following equations can be written:

$$(1+sT)\Delta X5 = sT\Delta\omega \tag{3.118}$$

$$(1+sT2)\Delta UE = Kc(1+sT1)\Delta X5 \tag{3.119}$$

State equations of the generators with PSS connected in area 1 can be formed by taking inverse laplace for Equations (3.118) and (3.119) and rearranging it with Equations (3.101)-(3.104):

$$\Delta\dot{X}5 = -\frac{K_{1i1}}{M_i}\Delta\delta - \frac{K_{2i1}}{M_i}\Delta E'_q - \frac{1}{T_i}X5 \tag{3.120}$$

$$\Delta\dot{UE} = -\frac{K_{1i1}*K_{c1i}*T_{1i}}{M_i*T_{2i}}\Delta\delta - \frac{K_{c1i}*K_{2i1}*T_{1i}}{M_i*T_{2i}}\Delta E'_q + \frac{K_{c1i}}{T_{2i}}(1-T_{1i}/T_{2i})\Delta X5 - \frac{1}{T_{2i}}\Delta UE \tag{3.121}$$

Where, 'i' represents the ith generator related parameters for area 1 and can be generalized modularly. Kc - gain, T1 and T2 are time constant of phase lag-lead block and T is the time constant of washout block of respective generator's PSS.

From Equations (3.120), (3.121) and (3.101)-(3.104), an overall state space model for area 1 can be written as:

$$\Delta\dot{x} = [A]\Delta x + [B]\Delta UE = [Ac]\Delta x \tag{3.122}$$

Where $\Delta x = \left[\Delta\omega_{i1}, \Delta\delta_{i1}, \Delta E'_{qi1}, \Delta E_{fdi1}, X5, UE\right]^T$ (3.123) and

$$Ac1i = \begin{bmatrix} 0 & -K_{1i1}/M_i & -K_{2i1}/M_i & 0 & 0 & 0 \\ \omega_{bi} & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_{4i1}/T'_{doi} & -1/(T'_{doi}*K_{3i1}) & 1/T'_{doi} & 0 & 0 \\ 0 & -K_{Ai}*K_{5i1}/T_{Ai} & -K_{Ai}*K_{6i1}/T_{Ai} & -1/T_{Ai} & 0 & K_{Ai}/T_{Ai} \\ 0 & -K_{1i1}/M_i & -K_{2i1}/M_i & 0 & -1/T_i & 0 \\ 0 & \frac{-K_{c1i}*K_{1i1}*T_{1i}}{M_i*T_{2i}} & \frac{-K_{c1i}*K_{2i1}*T_{1i}}{M_i*T_{2i}} & 0 & \frac{K_{c1i}}{T_{2i}}(1-\frac{T_{1i}}{T_{2i}}) & \frac{-1}{T_{2i}} \end{bmatrix} \quad (3.124)$$

Similarly for second area, modeling of system with PSS can be obtained in state space framework as:

$$Ac2i = \begin{bmatrix} 0 & -K_{1i2}/M_i & -K_{2i2}/M_i & 0 & 0 & 0 \\ \omega_{bi} & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_{4i2}/T'_{doi} & -1/(T'_{doi}*K_{3i2}) & 1/T'_{doi} & 0 & 0 \\ 0 & -K_{Ai}*K_{5i2}/T_{Ai} & -K_{Ai}*K_{6i2}/T_{Ai} & -1/T_{Ai} & 0 & K_{Ai}/T_{Ai} \\ 0 & -K_{1i2}/M_i & -K_{2i2}/M_i & 0 & -1/T_i & 0 \\ 0 & \frac{-K_{c2i}*K_{1i2}*T_{1i}}{M_i*T_{2i}} & \frac{-K_{c2i}*K_{2i2}*T_{1i}}{M_i*T_{2i}} & 0 & \frac{K_{c2i}}{T_{2i}}(1-\frac{T_{1i}}{T_{2i}}) & \frac{-1}{T_{2i}} \end{bmatrix} \quad (3.125)$$

Where Ac11, Ac12, Ac21…etc. can be derived modularly depending upon the areas and the generators connected in each area.

Hence the overall homogeneous state matrix including PSS with each generator for two area system can be written as:

$$[Ac] = \begin{bmatrix} Ac1 & 0 \\ 0 & Ac2 \end{bmatrix} \quad (3.126)$$

3.2.3 Inclusion of Unified Power Flow Controller (UPFC) Model in the System

UPFC has been connected between two areas as shown in Figure 3.4 where multiple generators of the respective area may be grouped, and thus equivalent circuit has been developed. Each area has been represented by Thevenin's equivalent and UPFC is connected at mid-point [145-146]. The concept can be extended to multi-area on modular basis with suitable interface.

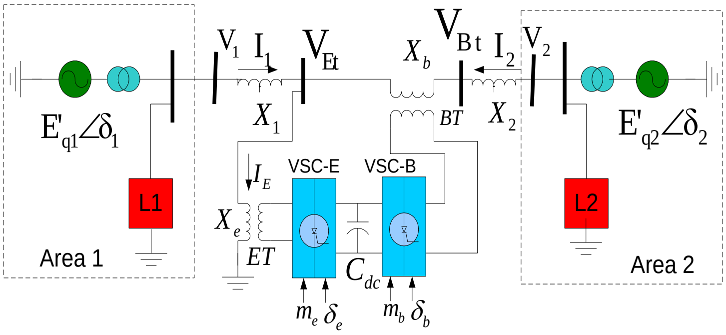


Figure 3.4 Two Area power network with UPFC

Unified Power Flow Controller (UPFC) is composed of an Excitation Transformer (ET), Boosting Transformer (BT) and two three-phase GTO based Voltage Source Converters (VSCs) both connected through DC-link capacitor. VSC-E is connected in shunt while VSC-B is connected in series with the system. Pulse Width Modulation (PWM) technique has been considered for developing the model of UPFC. Where input control signals Δm_e, Δm_b and $\Delta\delta_e$, $\Delta\delta_b$ known as amplitude modulation ratio and phase angle of each voltage source converter respectively.

On neglecting the converter harmonics, the following dynamic equations can be written at each side of the VSC for shunt and series terminal (VSC-E and VSC-B). DC link capacitor charged up to voltage.

$$\begin{aligned} V_E &= V_{Ed} + jV_{Eq} = km_e V_{dc} / 2 * e^{(j\delta_e)} \\ V_B &= V_{Bd} + jV_{Bq} = km_b V_{dc} / 2 * e^{(j\delta_b)} \end{aligned} \qquad (3.127)$$

Where, *k* is the ratio between AC and DC voltage depending on the structure of converter. If converters are assumed to be lossless then the instantaneous power at AC and DC terminals of series and shunt connected converters are equal. Hence power balance equations are:

$$V_{dc}.I_i = V_{Ed}.I_{Ed} + V_{Eq}.I_{Eq}$$
$$V_{dc}.I_o = V_{Bd}.I_{Bd} + V_{Bq}.I_{Bq} \tag{3.128}$$

$$\frac{dV_{dc}}{dt} = \frac{3m_e}{4C_{dc}}\left[\cos\delta_e.I_{1d} + \sin\delta_e.I_{1q}\right] + \frac{3m_b}{4C_{dc}}\left[-\cos\delta_b.I_{2d} - \sin\delta_b.I_{2q}\right] \tag{3.129}$$

Where, I_i and I_o are the currents in VSC for shunt and series terminal. V_{Ed} and V_{Eq} are the direct and quadrature components of voltage connected in shunt side of the UPFC (V_E) and V_{Bd} and V_{Bq} are the direct and quadrature component of voltage connected in series side of the UPFC (V_B). Similarly I_{Ed}, I_{Eq} and I_{Bd}, I_{Bq} are the d-q component of current in shunt and series connected VSC.

Under steady state condition, UPFC neither injects nor absorbs real power, neglecting UPFC losses, i.e., voltage of DC-link capacitor remains constant at specified value. Since net current to capacitor is zero. DC circuit can be written as:

$$C_{dc}\frac{dV_{dc}}{dt} = \left(I_i + I_o\right) \tag{3.130}$$

Where I_i and I_0 are the current at VSC-E and VSC-B end respectively.

$$\overline{V_1} = jX_1.\overline{I_1} + \overline{V_{Et}} \tag{3.131}$$

$$\overline{V_{Et}} = \overline{V_B} + jX_{2b}.\overline{I_B} + \overline{V_2} \tag{3.132}$$

Where,

$$\overline{I_1} = \overline{I_E} + \overline{I_B}\ ;\ \overline{I_2} = -\overline{I_B}\ ;\ X_{2b} = X_b + X_2$$

From above equations, d and q component of current can be obtained as:

$$I_{1d} = \frac{X_{bb}}{X_{dee}}E'_{q1} - m_e\sin\delta_e(V_{dc}/2)\frac{X_{bd}}{X_{dee}} + \frac{X_{de}}{X_{dee}}\left(E'_{q2}\cos\delta - I_{2d}X'_{2d} + m_b\sin\delta_b(V_{dc}/2)\right) \tag{3.133}$$

$$I_{1q} = m_e\cos\delta_e(V_{dc}/2)\frac{X_{bq}}{X_{qee}} - \frac{X_{qe}}{X_{qee}}m_b\cos\delta_b(V_{dc}/2) - \frac{X_{qe}}{X_{qee}}\left(E'_{q2}\sin\delta + I_{2q}X_{2q}\right) \tag{3.134}$$

$$I_{2d} = \frac{X_{dt}}{X_{dee}}\left(E'_{q2}\cos\delta - I_{2d}X'_{2d}\right) + \frac{X_{dt}}{X_{dee}} m_b \sin\delta_b (V_{dc}/2) - m_e \sin\delta_e (V_{dc}/2)\frac{X_{de}}{X_{dee}} - \frac{X_e}{X_{dee}}E'_{q1} \quad (3.135)$$

$$I_{2q} = m_e \cos\delta_e (V_{dc}/2)\frac{X_{qe}}{X_{qee}} - \frac{X_{qt}}{X_{qee}} m_b \cos\delta_b (V_{dc}/2) - \frac{X_{qt}}{X_{qee}}\left(E'_{q2}\sin\delta + I_{2q}X_{2q}\right) \quad (3.136)$$

Where,

$$\delta = \delta_1 - \delta_2\,;\; X_{qe} = X_{1q} + X_1\,;\; X_{de} = X'_{1d} + X_1\,; X_{qt} = X_{qe} + X_e\,;\; X_{dt} = X_{de} + X_e\,;$$

$$X_{qee} = X_{qe}.X_e + X_{qt}.X_{2b}\,;\; X_{dee} = X_{de}.X_e + X_{dt}.X_{2b}$$

Rewriting Equations (3.133)-(3.136):

$$I_{1d} = D_1E'_{q1} + D_2 m_e \sin\delta_e V_{dc} + D_3(E'_{q2}\cos\delta - I_{2d}X'_{2d} + m_b\sin\delta_b V_{dc}/2) \quad (3.137)$$

$$I_{1q} = Q_1 m_e \cos\delta_e V_{dc} + Q_2 m_b \cos\delta_b V_{dc}/2 + Q_2(E'_{q2}\sin\delta + I_{2q}X_{2q}) \quad (3.138)$$

$$I_{2d} = L_1D_4E'_{q2}\cos\delta + L_1D_4 m_b \sin\delta_b V_{dc}/2 + L_1D_5 m_e \sin\delta_e V_{dc}/2 + L_1D_6E'_{q1} \quad (3.139)$$

$$I_{2q} = L_2Q_3 m_e \cos\delta_e V_{dc} + L_2Q_4 m_b \cos\delta_b V_{dc}/2 + L_2Q_4E'_{q2}\sin\delta \quad (3.140)$$

Where

$$D_1 = \frac{X_{bb}}{X_{dee}},\; Q_1 = \frac{X_{bq}}{2X_{qee}},\; D_2 = -\frac{X_{bd}}{2X_{dee}},\; Q_2 = -\frac{X_{qe}}{X_{qee}},\; D_3 = \frac{X_{de}}{X_{dee}},\; Q_3 = \frac{X_{qe}}{2X_{qee}},\; D_4 = \frac{X_{dt}}{X_{dee}}$$

$$Q_4 = -\frac{X_{qt}}{X_{qee}},\; D_5 = -\frac{X_{de}}{X_{dee}},\; D_6 = -\frac{X_e}{X_{dee}},\; L_1 = \frac{1}{1 + D_4X'_{2d}},\; L_2 = \frac{1}{1 - Q_4X_{2q}}$$

Linearizing above Equations, we get

$$\Delta I_{1d} = a_{11}\Delta E'_{q1} + a_{12}\Delta m_e + a_{13}\Delta\delta_e + a_{14}\Delta V_{dc} + a_{15}\Delta E'_{q2} + a_{16}\Delta\delta + a_{17}\Delta m_b + a_{18}\Delta\delta_b \quad (3.141)$$

$$\Delta I_{1q} = a_{21}\Delta m_e + a_{22}\Delta\delta_e + a_{23}\Delta V_{dc} + a_{24}\Delta m_b + a_{25}\Delta\delta_b + a_{26}\Delta E'_{q2} + a_{27}\Delta\delta \quad (3.142)$$

$$\Delta I_{2d} = a_{31}\Delta E'_{q2} + a_{32}\Delta\delta + a_{33}\Delta m_b + a_{34}\Delta\delta_b + a_{351}\Delta V_{dc} + a_{36}\Delta m_e + a_{37}\Delta\delta_e + a_{39}\Delta E'_{q1} \quad (3.143)$$

$$\Delta I_{2q} = a_{41}\Delta m_e + a_{42}\Delta\delta_e + a_{431}\Delta V_{dc} + a_{44}\Delta m_b + a_{45}\Delta\delta_b + a_{47}\Delta E'_{q2} + a_{48}\Delta\delta \quad (3.144)$$

Now

$\Delta I_{td} = \Delta I_{1d} - \Delta I_{2d}$ and $\Delta I_{tq} = \Delta I_{1q} - \Delta I_{2q}$

Substituting the values of ΔI_{1d}, ΔI_{2d}, ΔI_{1q} and ΔI_{2q}, we get:

$$\Delta I_{td} = a_{51}\Delta E'_{q1} + a_{52}\Delta m_e + a_{53}\Delta\delta_e + a_{54}\Delta V_{dc} + a_{55}\Delta E'_{q2} + a_{56}\Delta\delta + a_{57}\Delta m_b + a_{58}\Delta\delta_b \quad (3.145)$$

$$\Delta I_{tq} = a_{61}\Delta m_e + a_{62}\Delta\delta_e + a_{63}\Delta V_{dc} + a_{64}\Delta m_b + a_{65}\Delta\delta_b + a_{66}\Delta E'_{q2} + a_{67}\Delta\delta \quad (3.146)$$

Equation for DC-Link Capacitor:

Let $a_e = a_b = \dfrac{3}{4C_{dc}}$ then Equation (3.129) can be written as:

$$\frac{dV_{dc}}{dt} = a_e m_e[\cos\delta_e I_{1d} + \sin\delta_e I_{1q}] + a_b m_b[-\cos\delta_b I_{2d} - \sin\delta_b I_{2q}] \quad (3.147)$$

Linearizing above equation:

$$\Delta\frac{dV_{dc}}{dt} = a_{e1}\Delta\delta_e + a_{e2}\Delta I_{1d} + a_{e3}\Delta I_{1q} + a_{e4}\Delta m_e + a_{b1}\Delta\delta_b + a_{b2}\Delta I_{2d} + a_{b3}\Delta I_{2q} + a_{b4}\Delta m_b \quad (3.148)$$

Substituting values of ΔI_{1d}, ΔI_{2d}, ΔI_{1q} and ΔI_{2q}, we get:

$$\Delta\frac{dV_{dc}}{dt} = a_{v1}\Delta\delta_e + a_{v2}\Delta E'_{q1} + a_{v3}\Delta m_e + a_{v4}\Delta V_{dc} + a_{v5}\Delta E'_{q2} + a_{v6}\Delta\delta + a_{v7}\Delta m_b + a_{v8}\Delta\delta_b \quad (3.149)$$

For first generator:

1st Equation:

$$(D_1 + M_1 s)\Delta\omega_1 = -\Delta T_{e1} = -\Delta[I_{tq}E'_{q1} + (X_{1q} - X'_{1d})I_{td}I_{tq}] \quad (3.150)$$

$$(D_1 + M_1 s)\Delta\omega_1 = -I_{tq}\Delta E'_{q1} - E'_{q1}\Delta I_{tq} - (X_{1q} - X'_{1d})[I_{tq}\Delta I_{td} + I_{td}\Delta I_{tq}]$$

$$(D_1 + M_1 s)\Delta\omega_1 = -I_{tq}\Delta E'_{q1} - [E'_{q1} + (X_{1q} - X'_{1d})\Delta I_{td}]\Delta I_{tq} - [(X_{1q} - X'_{1d})I_{tq}]\Delta I_{td}$$

OR, $(D_1 + M_1 s)\Delta\omega_1 = n_{11}\Delta E'_{q1} + n_{12}\Delta I_{tq} + n_{13}I_{td}$ (3.151)

Now substituting the value of ΔI_{tq} and ΔI_{td} in Equation (3.151), we get:

$$s\Delta\omega_1 = -\frac{D_1}{M_1}\Delta\omega_1 + \frac{n_{14}}{M_1}\Delta E'_{q1} + \frac{n_{15}}{M_1}\Delta m_e + \frac{n_{16}}{M_1}\Delta\delta_e + .. \\ \frac{n_{17}}{M_1}\Delta V_{dc} + \frac{n_{18}}{M_1}\Delta m_b + \frac{n_{19}}{M_1}\Delta\delta_b + \frac{n_{110}}{M_1}\Delta E'_{q2} + \frac{n_{111}}{M_1}\Delta\delta \quad (3.152)$$

2nd Equation:

$$s\Delta\delta_1 = \omega_0 \Delta\omega_1 \qquad (3.153)$$

3rd Equation:

Field winding circuit voltage equation:

$$(1+sT'_{1do})\Delta E'_{q1} = \Delta E_{fd1} - (X_{1d} - X'_{1d})\Delta I_{td} \qquad (3.154)$$

Putting the value of ΔI_{td} and rearranging, we get

$$s\Delta E'_{q1} = \frac{1}{T'_{1do}}\Delta E_{fd1} + \frac{V_1}{T'_{1do}}\Delta E'_{q1} + \frac{n_{23}}{T'_{1d0}}\Delta m_e + \frac{n_{24}}{T'_{1do}}\Delta E'_{q1} + \frac{n_{25}}{T'_{1do}}\Delta\delta_e + .. \\ \frac{n_{26}}{T'_{1do}}\Delta E'_{q2} + \frac{n_{27}}{T'_{1do}}\Delta\delta + \frac{n_{28}}{T'_{1do}}\Delta m_b + \frac{n_{29}}{T'_{1do}}\Delta\delta_b \qquad (3.155)$$

4th Equation:

$$(1+sT_{1A})\Delta E_{fd1} = K_{1A}(-\Delta V_{1t}) \qquad (3.156)$$

As we know,

$$V^2_{1t} = (E'_{q1} - I_{td}X'_{1d})^2 + (I_{tq}X_{1q})^2$$

On linearizing above equation and putting the values of ΔI_{td} and ΔI_{tq}, we get:

$$s\Delta E_{fd1} = -\frac{1}{T_{1A}}\Delta E_{fd1} + V_3F_{14}\Delta E'_{q1} + V_3F_{15}\Delta m_e + V_3F_{16}\Delta\delta_e + \\ +V_3F_{17}\Delta V_{dc} + \Delta E'_{q2} + V_3F_{19}\Delta\delta + V_3F_{110}\Delta m_b + V_3F_{111}\Delta\delta_b \qquad (3.157)$$

For Second Generator:

1st Equation:

$$(D_2 + M_2s)\Delta\omega_2 = -\Delta T_{e2} = -\Delta[I_{2q}E'_{q2} + (X_{2q} - X'_{2d})I_{2d}I_{2q}] \qquad (3.158)$$

$$(D_2 + M_2s)\Delta\omega_2 = -I_{2q}\Delta E'_{q2} - E'_{q2}\Delta I_{2q} - (X_{2q} - X'_{2d})[I_{2q}\Delta I_{2d} + I_{2d}\Delta I_{2q}]$$

$$(D_2 + M_2s)\Delta\omega_2 = -I_{2q}\Delta E'_{q2} - [E'_{q2} + (X_{2q} - X'_{2d})\Delta I_{2d}]\Delta I_{2q} - [(X_{2q} - X'_{2d})I_{2q}]\Delta I_{2d}$$

OR $(D_2 + M_2s)\Delta\omega_2 = m_{11}\Delta I_{2q} + m_{12}\Delta E'_{q2} + m_{13}\Delta I_{2d}$ (3.159)

Now substituting the value of ΔI_{2q} and ΔI_{2d} in Equation (3.159), we get:

$$s\Delta\omega_2 = -\frac{D_2}{M_2}\Delta\omega_2 + \frac{m_{14}}{M_2}\Delta m_e + \frac{m_{15}}{M_2}\Delta\delta_e + \frac{m_{16}}{M_2}\Delta V_{dc} + ..$$
$$\frac{m_{17}}{M_2}\Delta m_b + \frac{m_{18}}{M_2}\Delta\delta_b + \frac{m_{19}}{M_2}\Delta E'_{q2} + \frac{m_{110}}{M_2}\Delta\delta + \frac{m_{111}}{M_2}\Delta E'_{q1} \quad (3.160)$$

2nd Equation:

$$s\Delta\delta_2 = \omega_0\Delta\omega_2 \quad (3.161)$$

3rd Equation:

$$(1 + sT'_{2do})\Delta E'_{q2} = \Delta E_{fd2} - (X_{2d} - X'_{2d})\Delta I_{2d} \quad (3.162)$$

Putting the value of ΔI_{2d} and rearranging, we get:

$$s\Delta E'_{q2} = \frac{1}{T'_{2do}}\Delta E_{fd2} + \frac{V_2}{T'_{2do}}\Delta E'_{q2} + \frac{m_{23}}{T'_{2do}}\Delta\delta + \frac{m_{24}}{T'_{2d0}}\Delta m_b + ..$$
$$\frac{m_{25}}{T'_{2d0}}\Delta\delta_b + + \frac{m_{26}}{T'_{2do}}\Delta V_{dc} + \frac{m_{27}}{T'_{2do}}\Delta m_e + \frac{m_{28}}{T'_{2do}}\Delta\delta_e + \frac{m_{29}}{T'_{2do}}\Delta E'_{q1} \quad (3.163)$$

4th Equation:

$$(1 + sT_{2A})\Delta E_{fd2} = K_{2A}(\Delta V_{2t}) \quad (3.164)$$

As we know,

$$V^2_{2t} = (E'_{q2} - I_{2d}X'_{2d})^2 + (I_{2q}X_{2q})^2$$

On linearizing above equation and putting the values of ΔI_{2d} and ΔI_{2q}, we get:

$$s\Delta E_{fd2} = -\frac{1}{T_{2A}}\Delta E_{fd2} + V_4G_{14}\Delta E'_{q2} + V_4G_{15}\Delta\delta + V_4G_{16}\Delta m_b + ..$$
$$+ V_4G_{17}\Delta\delta_b + V_4G_{18}\Delta V_{dc} + V_4G_{19}\Delta m_e + V_4G_{110}\Delta\delta_e + V_4G_{111}\Delta E'_{q1} \quad (3.165)$$

Now Since, $s\Delta\delta = s(\Delta\delta_1 - \Delta\delta_2)$

Or, $s\Delta\delta = \omega_0\Delta\omega_1 - \omega_0\Delta\omega_2$ (3.166)

By combining all the Equations (3.152), (3.153), (3.155), (3.157), (3.160), (3.161), (3.163), (3.165) and (3.166) and writing in state space form:

$$\Delta\dot{x} = A\Delta x + B\Delta u \quad (3.167)$$

$$\Delta x = \left[\Delta\delta, \Delta\omega_1, \Delta\omega_2, \Delta E'_{q1}, \Delta E_{fd1}, \Delta E'_{q2}, \Delta E_{fd2}, \Delta V_{dc}\right]^T \tag{3.168}$$

$$\Delta u = \left[\Delta m_e, \Delta\delta_e, \Delta m_b, \Delta\delta_b\right]^T \tag{3.169}$$

Where state matrix A and control input matrix B can be given by:

$$A = \begin{bmatrix} 0 & \omega_0 & -\omega_0 & 0 & 0 & 0 & 0 & 0 \\ \frac{n_{111}}{M_1} & -\frac{D_1}{M_1} & 0 & \frac{n_{14}}{M_1} & 0 & \frac{n_{110}}{M_1} & 0 & \frac{n_{17}}{M_1} \\ \frac{m_{110}}{M_2} & 0 & -\frac{D_2}{M_2} & \frac{m_{111}}{M_2} & 0 & \frac{m_{19}}{M_2} & 0 & \frac{m_{16}}{M_2} \\ \frac{n_{27}}{T'_{1d0}} & 0 & 0 & \frac{V_1}{T'_{1d0}} & \frac{1}{T'_{1d0}} & \frac{n_{26}}{T'_{1d0}} & 0 & \frac{n_{25}}{T'_{1d0}} \\ V_3F_{19} & 0 & 0 & V_3F_{14} & -\frac{1}{T_{1A}} & V_3F_{18} & 0 & V_3F_{17} \\ \frac{m_{23}}{T'_{2d0}} & 0 & 0 & \frac{m_{29}}{T'_{2d0}} & 0 & \frac{V_2}{T'_{2d0}} & \frac{1}{T'_{2d0}} & \frac{m_{26}}{T'_{2d0}} \\ V_4G_{15} & 0 & 0 & V_4G_{111} & 0 & V_4G_{14} & -\frac{1}{T_{2A}} & V_4G_{18} \\ a_{v6} & 0 & 0 & a_{v2} & 0 & a_{v5} & 0 & a_{v4} \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{n_{15}}{M_1} & \frac{n_{16}}{M_1} & \frac{n_{18}}{M_1} & \frac{n_{19}}{M_1} \\ \frac{m_{14}}{M_2} & \frac{m_{15}}{M_2} & \frac{m_{17}}{M_2} & \frac{m_{18}}{M_2} \\ \frac{n_{23}}{T'_{1d0}} & \frac{n_{24}}{T'_{1d0}} & \frac{n_{28}}{T'_{1d0}} & \frac{n_{29}}{T'_{1d0}} \\ V_3F_{15} & V_3F_{16} & V_3F_{110} & V_3F_{111} \\ \frac{m_{27}}{T'_{2d0}} & \frac{m_{28}}{T'_{2d0}} & \frac{m_{24}}{T'_{2d0}} & \frac{m_{25}}{T'_{2d0}} \\ V_4G_{19} & V_4G_{110} & V_4G_{16} & V_4G_{17} \\ a_{v3} & a_{v1} & a_{v7} & a_{v8} \end{bmatrix}$$

3.2.4 Generalized Homogeneous Model of UPFC with Multi-Machine System

Homogeneous modeling of UPFC can be extended to multi-machine system. Here the number of generators are represented as ‘i’, which can be written in generalized form as i=1, 2,..., n. for each area.

The Equations (3.133)-(3.136) can be written for ‘i^{th}’ machine as:

$$I_{Edi} = \frac{X_{bbi1}}{X_{deei1}} E'_{qi1} - m_e \sin\delta_e V_{dc} \frac{X_{bdi1}}{2X_{deei1}} + \frac{X_{dei1}}{X_{deei1}} ((V_b \cos\delta_0)_i + m_b \sin\delta_b (V_{dc}/2)) \tag{3.170}$$

$$I_{Eqi} = \frac{X_{bqi1}}{2X_{qeei1}} m_e \cos\delta_e V_{dc} - \frac{X_{qei1}}{X_{qeei1}} (m_b \cos\delta_b (V_{dc}/2) - (V_b \sin\delta_0)_i) \tag{3.171}$$

$$I_{Bdi} = \frac{-X_{dti1}}{X_{deei1}} ((V_b \sin\delta_0)_i + m_b \sin\delta_b (V_{dc}/2)) + \frac{X_{dei1}}{X_{deei1}} m_e \sin\delta_e (V_{dc}/2) + \frac{X_{ei1}}{X_{deei1}} E'_{qi1} \tag{3.172}$$

$$I_{Bqi} = -m_e \cos\delta_e V_{dc} \frac{X_{qei1}}{2X_{qeei1}} + \frac{X_{qti1}}{X_{qeei1}}(m_b \cos\delta_b (V_{dc}/2) + (V_b \sin\delta_0)_i) \tag{3.173}$$

Where:

$$(V_b \cos\delta_o)_i = E'_{qi2}\cos\delta - I_{2d}X'_{i2d} \tag{3.174}$$

$$(V_b \sin\delta_o)_i = E'_{qi2}\sin\delta - I_{2q}X'_{i2q} \tag{3.175}$$

And

$$I_{Ed} = \sum I_{Edi}\ ,\ I_{Bd} = \sum I_{Bdi}$$

For two machine model current notation can be written as:

$$I_{1d} = I_{Ed}\ ,\ I_{1q} = I_{Eq}\ ,\ I_{2d} = -I_{Bd}\ ,\ I_{2q} = -I_{Bq}$$

Putting the values $(V_b Cos\delta_0)_i$ and $(V_b Sin\delta_0)_i$ from Equations (3.174) and (3.175) to Equations (3.171)-(3.173), we get

$$\begin{aligned} I_{1d} &= \sum \frac{X_{bbi1}}{X_{dee}} E'_{qi1} - m_e \sin\delta_e (V_{dc}/2)\sum \frac{X_{bdi1}}{X_{deei1}} + .. \\ &+ m_b \sin\delta_b (V_{dc}/2)\sum \frac{X_{dei1}}{X_{deei1}} + \sum \frac{X_{dei1}}{X_{deei1}}(E'_{qi2}\cos\delta - I_{2d}X'_{i2d}) \end{aligned} \tag{3.176}$$

$$I_{1q} = m_e \cos\delta_e (V_{dc}/2)\sum \frac{X_{bqi1}}{X_{qeei1}} - m_b \cos\delta_b (V_{dc}/2)\sum \frac{X_{qei1}}{X_{qeei1}} + \sum \frac{X_{qei1}}{X_{qeei1}}(E'_{qi2}\sin\delta - I_{2q}X'_{i2q}) \tag{3.177}$$

$$\begin{aligned} I_{2d} &= m_b \sin\delta_b (V_{dc}/2)\sum \frac{X_{dti1}}{X_{deei1}} - m_e \sin\delta_e (V_{dc}/2)\sum \frac{X_{dei1}}{X_{deei1}} - \\ &\sum \frac{X_{ei1}}{X_{deei1}} E'_{qi1} + \sum \frac{X_{dti1}}{X_{deei1}}(E'_{qi2}\cos\delta - I_{2d}X'_{i2d}) \end{aligned} \tag{3.178}$$

$$\begin{aligned} I_{2q} &= m_e \cos\delta_e (V_{dc}/2)\sum \frac{X_{qei1}}{X_{qeei1}} - m_b \cos\delta_b (V_{dc}/2)\sum \frac{X_{qti1}}{X_{qeei1}} - \\ &\sum \frac{X_{qti1}}{X_{qeei1}}(E'_{qi2}\sin\delta - I_{2q}X'_{i2q}) \end{aligned} \tag{3.179}$$

Linearize above equations and find ΔI_{1d}, ΔI_{2d}, ΔI_{1q} and ΔI_{2q}.

Now, $\Delta I_{td} = \Delta I_{1d} - \Delta I_{2d}$ and $\Delta I_{tq} = \Delta I_{1q} - \Delta I_{2q}$ (3.180)

Putting the values of ΔI_{1d}, ΔI_{2d}, ΔI_{1q} and ΔI_{2q} in Equation (3.180) and rearranging.

we can find ΔI_{td} in terms of $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$ and

ΔI_{tq} in terms of $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

Equation for DC-Link Capacitor:

$$\frac{dV_{dc}}{dt} = \frac{3m_e}{4C_{dc}}\left[\cos\delta_e.I_{1d} + \sin\delta_e.I_{1q}\right] + \frac{3m_b}{4C_{dc}}\left[-\cos\delta_b.I_{2d} - \sin\delta_b.I_{2q}\right] \tag{3.181}$$

Let $a_e = a_b = \dfrac{3}{4C_{dc}}$ then Equation (3.181) can be written as:

$$\frac{dV_{dc}}{dt} = a_e m_e[\cos\delta_e I_{1d} + \sin\delta_e I_{1q}] + a_b m_b[-\cos\delta_b I_{2d} - \sin\delta_b I_{2q}] \tag{3.182}$$

Linearizing above equations:

$$\frac{dV_{dc}}{dt} = a_{e1}\Delta\delta_e + a_{e2}\Delta I_{1d} + a_{e3}\Delta I_{1q} + a_{e4}\Delta m_e + a_{b1}\Delta\delta_b + a_{b2}\Delta I_{2d} + a_{b3}\Delta I_{2q} + a_{b4}\Delta m_b \tag{3.183}$$

Putting the values of ΔI_{1d}, ΔI_{2d}, ΔI_{1q} and ΔI_{2q} in Equation (3.183) and rearranging. We can find dV_{dc}/dt in terms of $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

Dynamics of Generators for First Area:

1st Equation:

$$(D_{i1} + M_{i1}s)\Delta\omega_{i1} = -\Delta T_{ei1} = -\Delta[I_{tq}E'_{qi1} + (X_{i1q} - X'_{i1d})I_{td}I_{tq}] \tag{3.184}$$

$$(D_{i1} + M_{i1}s)\Delta\omega_{i1} = -I_{tq}\Delta E'_{qi1} - [E'_{qi1} + (X_{i1q} - X'_{i1d})I_{td}]\Delta I_{tq} - [(X_{i1q} - X'_{i1d})I_{tq}]\Delta I_{td} \tag{3.185}$$

Putting values of ΔI_{tq} and ΔI_{td}, in Equation (3.185) and rearranging. This results as $s\Delta\omega_{i1}$ which is in terms of $\Delta\omega_{i1}$ $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

2nd Equation:

$$s\Delta\delta_{i1} = \omega_0\Delta\omega_{i1} \tag{3.186}$$

3rd Equation:

$$(1 + sT'_{i1do})\Delta E'_{qi1} = \Delta E_{fdi1} - (X_{i1d} - X'_{i1d})\Delta I_{td} \tag{3.187}$$

Putting the values of ΔI_{td}, in Equation (3.187) and rearranging. This results as $s\Delta E'_{qi1}$

which is in terms of ΔE_{fdi1}, $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

4th Equation:

$$(1+sT_{i1A})\Delta E_{fdi1} = K_{i1A}(-\Delta V_{i1t}) \tag{3.188}$$

As we know,

$$V^2_{i1t} = (E'_{qi1} - I_{td}X'_{i1d})^2 + (I_{tq}X_{i1q})^2$$

On linearizing above equation:

$$V_{i1t}\Delta V_{i1t} = (E'_{qi1} - I_{td}X'_{i1d})[\Delta E'_{qi1} - \Delta I_{td}X'_{i1d}] + I_{tq}X^2_{i1q}\Delta I_{tq} \tag{3.189}$$

Putting the values of ΔI_{tq} and ΔI_{td}, in Equation (3.189) and rearranging. This results as $s\Delta E_{fdi1}$, which is in terms of ΔE_{fdi1}, $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

Dynamics of Generators of Second Area:

1st Equation:

$$(D_{i2} + M_{i2}s)\Delta\omega_{i2} = -\Delta T_{ei2} = -\Delta[I_{tq}E'_{qi2} + (X_{i2q} - X'_{i2d})I_{td}I_{tq}] \tag{3.190}$$

$$(D_{i2} + M_{i2}s)\Delta\omega_{i2} = -I_{tq}\Delta E'_{qi2} - [E'_{qi2} + (X_{i2q} - X'_{i2d})I_{td}]\Delta I_{tq} - [(X_{i2q} - X'_{i2d})I_{tq}]\Delta I_{td} \tag{3.191}$$

Putting the values of ΔI_{tq} and ΔI_{td}, in Equation (3.191) and rearranging. This results as $s\Delta\omega_{i2}$, which is in terms of $\Delta\omega_{i2}$, $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

2nd Equation:

$$s\Delta\delta_{i2} = \omega_0\Delta\omega_{i2} \tag{3.192}$$

3rd Equation:

$$(1+sT'_{i2do})\Delta E'_{qi2} = \Delta E_{fdi2} - (X_{i2d} - X'_{i2d})\Delta I_{2d} \tag{3.193}$$

Putting the values of ΔI_{2d}, in Equation (3.193) and rearranging. This results as $s\Delta E'_{qi2}$ which is in terms of ΔE_{fdi2}, $\Delta E'_{qi1}$, $\Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

4th Equation:

$$(1+sT_{i2A})\Delta E_{fdi2} = K_{i2A}(-\Delta V_{i2t}) \tag{3.194}$$

As we know,

$$V^2_{i2t} = (E'_{qi2} - I_{2d}X'_{i2d})^2 + (I_{2q}X_{i2q})^2$$

On linearizing above equation:

$$V_{i2t}\Delta V_{i2t} = (E'_{qi2} - I_{2d}X'_{i2d})[\Delta E'_{qi2} - \Delta I_{2d}X'_{i2d}] + I_{2q}X^2_{i2q}\Delta I_{2q} \tag{3.195}$$

Putting the values of ΔI_{2d} and ΔI_{2q}, in Equation (3.189) and rearranging. This results as $s\Delta E_{fdi2}$ which is in terms of ΔE_{fdi2}, $\Delta E'_{qi1}, \Delta E'_{qi2}$, Δm_e, $\Delta\delta_e$, ΔV_{dc}, $\Delta\delta$, Δm_b, $\Delta\delta_b$.

Thus with the use of above equations, state space form can be written as:

$$\Delta \dot{x} = A\Delta x + B\Delta u \tag{3.196}$$

$$\Delta x = \left[\Delta\delta, \Delta\omega_{i1}, \Delta E'_{qi1}, \Delta E_{fdi1}, \Delta\omega_{i2}, \Delta E'_{qi2}, \Delta E_{fdi2}, \Delta V_{dc}\right]^T \tag{3.197}$$

$$\Delta u = \left[\Delta m_e, \Delta\delta_e, \Delta m_b, \Delta\delta_b\right]^T \tag{3.198}$$

UPFC Interface Model with the System:

From the machine modeling following equations can be written:

$$V_{Edi} = I_{1q}X_{i1qe} + X_e I_{Eq}\text{, where } X_{i1qe} = X_{i1q} + X_{te} \tag{3.199}$$

$$V_{Eqi} = E'_{qi1} - I_{1d}X_{i1de} - X_e I_{Ed}\text{, where } X_{i1de} = X_{i1d} + X_{te} \tag{3.200}$$

Similarly;

$$V_{Bdi} = I_{1q}X_{i1qe} + I_{2q}X_{i2BB} - E'_{qi2}\cos\delta \tag{3.201}$$

$$X_{i2BB} = -(X_{i2q} + X_{Be}) \;\&\; V_{Bqi} = E'_{qi1} - E'_{qi2}\sin\delta - I_{1d}X_{i1de} + I_{2d}X_{i2Be} \tag{3.202}$$

Where $X_{i2Be} = X_{Be} + X'_{i2d}$

For 'n' generator system V_{Ed} and V_{Eq} can be written as:

$$V_{Ed} = \begin{bmatrix} V_{Ed1} \\ V_{Ed2} \\ . \\ . \\ V_{Edn} \end{bmatrix}, V_{Eq} = \begin{bmatrix} V_{Eq1} \\ V_{Eq2} \\ . \\ . \\ V_{Eqn} \end{bmatrix} \qquad (3.203)$$

Similarly:

$$V_{Bd} = \begin{bmatrix} V_{Bd1} \\ V_{Bd2} \\ . \\ . \\ V_{Bdn} \end{bmatrix}, V_{Bq} = \begin{bmatrix} V_{Bq1} \\ V_{Bq2} \\ . \\ . \\ V_{Bqn} \end{bmatrix} \qquad (3.204)$$

These equations can be transformed into a common d-q frame with the help of following equations:

$$\begin{bmatrix} f_d \\ f_q \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} f_d \\ f_q \end{bmatrix}$$

Let $M_i = \begin{bmatrix} \cos\delta_{i1} & \sin\delta_{i1} \\ -\sin\delta_{i1} & \cos\delta_{i1} \end{bmatrix}$, and $H_i = \begin{bmatrix} \cos\delta_{i2} & \sin\delta_{i2} \\ -\sin\delta_{i2} & \cos\delta_{i2} \end{bmatrix}$,

$$V_{Edqi} = \begin{bmatrix} V_{Edi} \\ V_{Eqi} \end{bmatrix}, V_{Bdqi} = \begin{bmatrix} V_{Bdi} \\ V_{Bqi} \end{bmatrix}$$

Thus

$$\begin{bmatrix} V_{Ed} \\ V_{Eq} \end{bmatrix} = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & M_n \end{bmatrix} \begin{bmatrix} V_{Edq1} \\ V_{Edq2} \\ . \\ . \\ V_{Edqn} \end{bmatrix} \text{and} \begin{bmatrix} V_{Bd} \\ V_{Bq} \end{bmatrix} = \begin{bmatrix} H_1 & 0 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & H_n \end{bmatrix} \begin{bmatrix} V_{Bdq1} \\ V_{Bdq2} \\ . \\ . \\ V_{Bdqn} \end{bmatrix} \qquad (3.205)$$

By using above Equation (3.205), V_{Ed}, V_{Eq}, V_{Bd} and V_{Bq} can be found.

Four equations from the dynamics of series and shunt part of UPFC can be written as:

$$m_e = \frac{\sqrt{(V^2{}_{Ed} + V^2{}_{Eq})}}{kV_{dc}} \qquad (3.206)$$

$$\delta_e = \tan^{-1}\left(\frac{V_{Eq}}{V_{Ed}}\right) \tag{3.207}$$

$$m_b = \frac{\sqrt{(V^2{}_{Bd} + V^2{}_{Bq})}}{kV_{dc}} \tag{3.208}$$

$$\delta_b = \tan^{-1}\left(\frac{V_{Bq}}{V_{Bd}}\right) \tag{3.209}$$

By linearizing above Equations (3.206)-(3.209) and putting the values of V_{Ed}, V_{Eq}, V_{Bd} and V_{Bq} from Equation (3.205), final multi-machine equation will be:

$$[Y][\Delta u] = [Z][\Delta x] \tag{3.210}$$

$$[\Delta u] = [\Delta m_e, \Delta\delta_e, \Delta m_b, \Delta\delta_b]^T \tag{3.211}$$

$$\Delta x = \left[\Delta\delta, \Delta\omega_{i1}, \Delta E'_{qi1}, \Delta E_{fdi1}, \Delta\omega_{i2}, \Delta E'_{qi2}, \Delta E_{fdi2}, \Delta V_{dc}\right]^T \tag{3.212}$$

With the help of above equation two machine model can be transformed into multi-machine system:

$$\Delta \dot{x} = A\Delta x + B\Delta u \tag{3.213}$$

$$\Delta \dot{x} = A\Delta x + BY^{-1}Z\Delta x$$

$$\Delta \dot{x} = (A + BY^{-1}Z)\Delta x$$

Or, $\Delta \dot{x} = M_{UPFC}\Delta x$ (3.214)

Where, $M_{UPFC} = (A + BY^{-1}Z)$

3.2.5 Homogeneous Model of UPFC with Two Machine System

Utilizing the above approach, the generalized homogeneous model of UPFC for six area sample power system has been developed. Here in each area single generator is connected. FACTS devices (UPFC, STATCOM or SSSC) are suitably placed between two areas. To develop the complete system model, homogeneous model of UPFC with two machines has been developed in this section.

From Figure 3.4, the following can be written:

$$V_{t1} = jX_{tE}I_1 + V_{Et} \text{ or } V_{Et} = V_{t1} - jX_{tE}I_1 \quad (3.215)$$

$$V_{Et} = V_{Bt} + jX_{Bv}I_B + V_b \quad (3.216)$$

Or, $V_{t1} = jE'_{q1} - jI_{1d}X'_{1d} + I_{1q}X_{1q}$ (3.217)

Now, $V_{Et} = jE'_{q1} - jI_{1d}X'_{1d} + I_{1q}X_{1q} - jX_{tE}I_1$ (3.218)

$$V_{Et} = (I_{1q}X_{1q} + X_{tE}I_{1q}) + j(E'_{q1} - I_{1d}X'_{1d} - X_{tE}I_{1d}) \quad (3.219)$$

And, $V_{Et} = V_E + jX_eI_E$ (3.220)

$$V_{Et} = (V_{Ed} + jV_{Eq}) + jX_e(I_{Ed} + jI_{Eq}) \quad (3.221)$$

Or, $V_{Et} = (V_{Ed} - X_eI_{Eq}) + j(V_{Eq} + X_eI_{Ed})$ (3.222)

Since, $V_{Et} = V_B + jX_BI_B + V_b + jX_{Bv}I_B$ (3.223)

And, $V_b = E'_{q2}\cos\delta + jE'_{q2}\sin\delta - jI_{2d}X'_{2d} + I_{2q}X_{2q}$ (3.224)

Substituting these values in Equation (3.216):

$$V_{Et} = (V_{Bd} + jV_{Bq}) + jI_BX_B + jX_{Bv}I_B + E'_{q2}\cos\delta + .. \\ jE'_{q2}\sin\delta - jI_{2d}X'_{2d} + I_{2q}X_{2q} \quad (3.225)$$

By rearranging and putting $I_{Bd} = -I_{2d}$ and $I_{Bq} = -I_{2q}$, we get:

$$V_{Et} = (V_{Bd} + I_{2q}X_{Be} + E'_{q2}\cos\delta + I_{2q}X_{2q}) + j(V_{Bq} - I_{2d}X_{Be} + E'_{q2}\sin\delta - I_{2d}X'_{2d}) \quad (3.226)$$

Equating real and imaginary parts of Equations (3.219) and (3.222) the following is obtained:

$$V_{Ed} = I_{1q}X_{qe} + X_eI_{Eq} \quad (3.227)$$

& $V_{Eq} = E'_{q1} - I_{1d}X_{de} - X_eI_{Ed}$ (3.228)

Similarly equating real and imaginary parts of Equations (3.222) and (3.226) the following results:

$$V_{Bd} = I_{1q}X_{qe} + I_{2q}X_{BB} - E'_{q2}\cos\delta \tag{3.229}$$

& $V_{Bq} = E'_{q1} - E'_{q2}\sin\delta - I_{1d}X_{de} + I_{2d}X_{BE}$ (3.230)

Since

$$\overline{V}_E = m_e V_{dc}/2(\cos\delta_e + j\sin\delta_e) = V_{Ed} + jV_{Eq} \tag{3.231}$$

$$\overline{V}_B = m_b V_{dc}/2(\cos\delta_b + j\sin\delta_b) = V_{Bd} + jV_{Bq} \tag{3.232}$$

From the above equation, we get;

$$m_e = \frac{\sqrt{(V^2_{Ed} + V^2_{Eq})}}{kV_{dc}} \tag{3.233}$$

$$\delta_e = \tan^{-1}\left(\frac{V_{Eq}}{V_{Ed}}\right) \tag{3.234}$$

$$m_b = \frac{\sqrt{(V^2_{Bd} + V^2_{Bq})}}{kV_{dc}} \tag{3.235}$$

$$\delta_b = \tan^{-1}\left(\frac{V_{Bq}}{V_{Bd}}\right) \tag{3.236}$$

Using shunt part of the UPFC:

$$m_e = \frac{\sqrt{(V^2_{Ed} + V^2_{Eq})}}{kV_{dc}} \text{ or } k^2V_{dc}^2m_e^2 = V^2_{Ed} + V^2_{Eq} \tag{3.237}$$

Now as we know,

$$V^2_{Ed} = (I_{1q}X_{qe} + X_eI_{Eq})^2 \text{ and } V^2_{Eq} = (E'_{q1} - I_{1d}X_{de} - X_eI_{Ed})^2 \tag{3.238}$$

Hence:

$$\begin{aligned} k^2V_{dc}^2m_e^2 &= E'^2_{q1} + I_{1d}^2X_{de}^2 + I_{1q}^2X_{qe}^2 + X_e^2I_{Eq}^2 + .. \\ & X_e^2I_{Ed}^2 + 2I_{1q}I_{Eq}X_{qe}X_e + 2I_{1d}I_{Ed}X_eX_{de} - 2E'_{q1}I_{1d}X_{de} - 2E'_{q1}I_{Ed}X_e \end{aligned} \tag{3.239}$$

Linearizing above equation and rearranging:

$$2k^2 m_e^{\,2}\Delta V_{dc} + 2k^2 V_{dc}^{\,2}\Delta m_e = (2E'_{q1} - 2I_{1d}X_{de} - 2I_{Ed}X_e)\Delta E'_{q1} + (-2E'_{q1}X_{de} + 2I_{1d}X^2_{de} + 2I_{Ed}X_eX_{de})\Delta I_{1d} + (2I_{1q}X_{qe}^{\,2} + 2I_{Eq}X_{qe}X_e)\Delta I_{1q} + (2X_e^{\,2}I_{Ed} + 2I_{1d}X_eX_{de} - 2E'_{q1}X_e)\Delta I_{Ed} + (2X_e^{\,2}I_{Eq} + 2I_{1q}X_{qe}X_e)\Delta I_{Eq} \quad (3.240)$$

Or, $N_1\Delta m_e + N_2\Delta V_{dc} = N_3\Delta E'_{q1} + N_4\Delta I_{1d} + N_5\Delta I_{1q} + N_6\Delta I_{Ed} + N_7\Delta I_{Eq}$ (3.241)

Here

$\Delta I_{1d} = \Delta I_{td}$ & $\Delta I_{1q} = \Delta I_{tq}$

$\Delta I_{Ed} = \Delta I_{1d}$ & $\Delta I_{Eq} = \Delta I_{1q}$

Putting these values in Equation (3.241) and rearranging, this results as:

$$N_8\Delta m_e + N_9\Delta\delta_e + N_{10}\Delta m_b + N_{11}\Delta\delta_b = N_{12}\Delta\delta + N_{13}\Delta E'_{q1} + N_{14}\Delta E'_{q2} + N_{15}\Delta V_{dc} \quad (3.242)$$

From Equation (3.234):

$$\tan\delta_e = \left(\frac{V_{Eq}}{V_{Ed}}\right)$$

Putting the values of V_{Eq} and V_{Ed} in above equation, this results as:

$$(I_{1q}X_{qe} + X_eI_{Eq})\tan\delta_e = E'_{q1} - I_{1d}X_{de} - X_eI_{Ed} \quad (3.243)$$

Linearizing above equation and rearranging:

$$P_1\Delta\delta_e = \Delta E'_{q1} + P_2\Delta I_{1d} + P_3\Delta I_{1q} + P_4\Delta I_{Ed} + P_5\Delta I_{Eq} \quad (3.244)$$

Now substituting the values of ΔI_{1d}, ΔI_{1q}, ΔI_{Ed} and ΔI_{Eq} in above equation and rearranging:

$$P_6\Delta m_e + P_7\Delta\delta_e + P_8\Delta m_b + P_9\Delta\delta_b = P_{10}\Delta\delta + P_{11}\Delta E'_{q1} + P_{12}\Delta E'_{q2} + P_{13}\Delta V_{dc} \quad (3.245)$$

Using series part of the UPFC:

$m_b = \dfrac{\sqrt{(V^2_{Bd} + V^2_{Bq})}}{kV_{dc}}$ or $k^2V_{dc}^{\,2}m_b^{\,2} = V^2_{Bd} + V^2_{Bq}$

$$V_{Bd}^{\,2} = (I_{1q}X_{qe} + I_{2q}X_{BB} - E'_{q2}\cos\delta)^2$$

$$V^2{}_{Bq} = (E'_{q1} - E'_{q2}\sin\delta - I_{1d}X_{de} + I_{2d}X_{BE})^2$$

Hence $k^2V^2{}_{dc}m^2{}_b = V^2{}_{Bd} + V^2{}_{Bq}$

$$\begin{aligned} k^2V_{dc}{}^2m_b{}^2 &= E'_{q1}{}^2 + E'_{q2}{}^2 + I^2{}_{1d}X_{de}{}^2 + I_{1q}{}^2X_{qe}{}^2 + I_{2d}{}^2X_{BE}{}^2 + I_{2q}{}^2X_{BB}{}^2 + 2I_{1q}I_{2q}X_{BB}X_{qe} - \\ & 2I_{1q}E'_{q2}\cos\delta X_{qe} - 2I_{2q}E'_{q2}\cos\delta X_{BB} - 2E'_{q1}E'_{q2}\sin\delta + 2I_{1d}E'_{q2}\sin\delta X_{de} \\ & + 2E'_{q1}I_{2d}X_{BE} - 2I_{2d}E'_{q2}\sin\delta X_{BE} - 2I_{1d}E'_{q1}X_{de} - 2I_{1d}I_{2d}X_{de}X_{BE} \end{aligned} \tag{3.246}$$

Linearizing above equation and rearranging, we get:

$$N_{16}\Delta m_b + N_{17}\Delta V_{dc} = N_{18}\Delta E'_{q1} + N_{19}\Delta E'_{q2} + N_{20}\Delta I_{1d} + N_{21}\Delta I_{1q} + N_{22}\Delta I_{2d} + N_{23}\Delta I_{2q} + N_{24}\Delta\delta \tag{3.247}$$

Now substituting the values of ΔI_{1d}, ΔI_{1q}, ΔI_{2d} and ΔI_{2q} in above equation and rearranging:

$$N_{25}\Delta m_e + N_{26}\Delta\delta_e + N_{27}\Delta m_b + N_{28}\Delta\delta_b = N_{29}\Delta\delta + N_{30}\Delta E'_{q1} + N_{31}\Delta E'_{q2} + N_{32}\Delta V_{dc} \tag{3.248}$$

From Equation (3.236):

$$\delta_b = \tan^{-1}\left(\frac{V_{Bq}}{V_{Bd}}\right) \text{ or } V_{Bd}\tan\delta_b = V_{Bq} \tag{3.249}$$

Putting the values of V_{Bd} and V_{Bq} in above equation, we get:

$$I_{1q}\tan\delta_b X_{qe} + I_{2q}\tan\delta_b X_{BB} - E'_{q2}\cos\delta\tan\delta_b = E'_{q1} - E'_{q2}\sin\delta - I_{1d}X_{de} + I_{2d}X_{BE} \tag{3.250}$$

After linearizing and rearranging:

$$P_{14}\Delta\delta_b = P_{15}\Delta\delta + P_{16}\Delta E'_{q1} + P_{17}\Delta E'_{q2} + P_{18}\Delta I_{1d} + P_{19}\Delta I_{1q} + P_{20}\Delta I_{2d} + P_{21}\Delta I_{2q} \tag{3.251}$$

Now substituting the values of ΔI_{1d}, ΔI_{1q}, ΔI_{2d} and ΔI_{2q} in above equation and rearranging:

$$P_{22}\Delta m_e + P_{23}\Delta\delta_e + P_{24}\Delta m_b + P_{25}\Delta\delta_b = P_{26}\Delta\delta + P_{27}\Delta E'_{q1} + P_{28}\Delta E'_{q2} + P_{29}\Delta V_{dc} \tag{3.252}$$

Now arranging Equations (3.242), (3.245), (3.248) and (3.252) in matrix form:

$$\begin{bmatrix} N_8 & N_9 & N_{10} & N_{11} \\ P_6 & P_7 & P_8 & P_9 \\ N_{25} & N_{26} & N_{27} & N_{28} \\ P_{22} & P_{23} & P_{24} & P_{25} \end{bmatrix} \begin{bmatrix} \Delta m_e \\ \Delta \delta_e \\ \Delta m_b \\ \Delta \delta_b \end{bmatrix} = \begin{bmatrix} N_{12} & 0 & 0 & N_{13} & 0 & N_{14} & 0 & N_{15} \\ P_{10} & 0 & 0 & P_{11} & 0 & P_{12} & 0 & P_{13} \\ N_{29} & 0 & 0 & N_{30} & 0 & N_{31} & 0 & N_{32} \\ P_{26} & 0 & 0 & P_{27} & 0 & P_{28} & 0 & P_{29} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega_1 \\ \Delta \omega_2 \\ \Delta E'_{q1} \\ \Delta E_{fd1} \\ \Delta E'_{q2} \\ \Delta E_{fd2} \\ \Delta V_{dc} \end{bmatrix} \qquad (3.253)$$

Or $[Y][\Delta u] = [Z][\Delta x]$

$$[\Delta u] = [\Delta m_e, \Delta \delta_e, \Delta m_b, \Delta \delta_b]^T \qquad (3.254)$$

$$\Delta x = \left[\Delta \delta, \Delta \omega_1, \Delta \omega_2, \Delta E'_{q1}, \Delta E_{fd1}, \Delta E'_{q2}, \Delta E_{fd2}, \Delta V_{dc} \right]^T \qquad (3.255)$$

$$[Y] = \begin{bmatrix} N_8 & N_9 & N_{10} & N_{11} \\ P_6 & P_7 & P_8 & P_9 \\ N_{25} & N_{26} & N_{27} & N_{28} \\ P_{22} & P_{23} & P_{24} & P_{25} \end{bmatrix}, \; [Z] = \begin{bmatrix} N_{12} & 0 & 0 & N_{13} & 0 & N_{14} & 0 & N_{15} \\ P_{10} & 0 & 0 & P_{11} & 0 & P_{12} & 0 & P_{13} \\ N_{29} & 0 & 0 & N_{30} & 0 & N_{31} & 0 & N_{32} \\ P_{26} & 0 & 0 & P_{27} & 0 & P_{28} & 0 & P_{29} \end{bmatrix}$$

$$\Delta \dot{x} = A\Delta x + B\Delta u \qquad (3.256)$$

$$\Delta \dot{x} = (A + BY^{-1}Z)\Delta x$$

Or $\Delta \dot{x} = M_{UPFC}\Delta x$ (3.257)

Here: $M_{UPFC} = (A + BY^{-1}Z)$

3.3 SAMPLE TEST SYSTEMS

The above developed state space model of power system has been used to study three sample test system in order to comment on the effectiveness of proposed concept in this section.

3.3.1 Two Area Four Machine Test System (2A4M)

A sample two area system has been developed with two machines in each area as shown in Figure 3.5. The excitation system of each machine is equipped with PSS and FACTS device is connected between both the areas. Complete system data has been shown in the Appendix A.

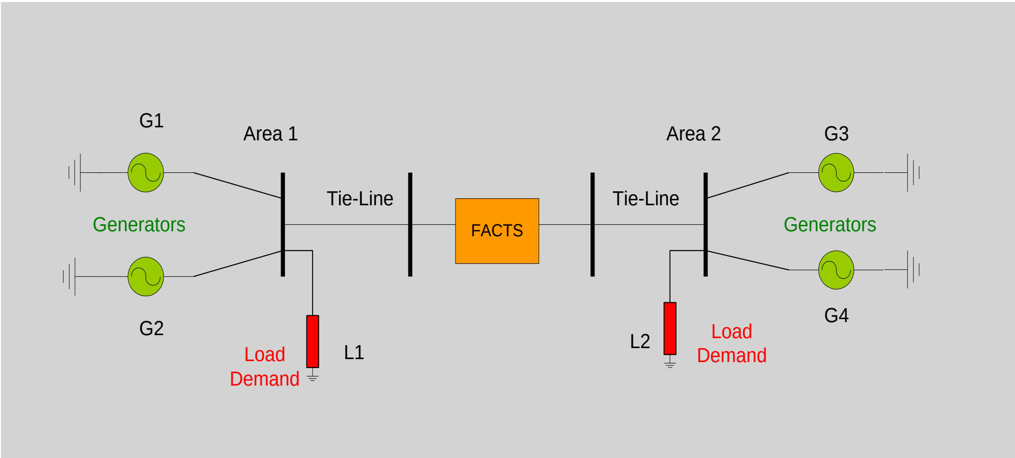


Figure 3.5 Two Area Four Machine power system

3.3.2 Six Area Six Machine Test System (6A6M)

Figure 3.6 shows sample six area system. In each area, generator excitation is equipped with PSS. FACTS device is connected between area 5 and area 6. The criterion for the placement of FACTS is based on power oscillations noticed in the system. The FACTS device has been placed between those areas where the inter-area oscillation takes place frequently with relatively larger magnitude. In order to demonstrate the capability of controller, a sample six area system has been considered where generators in area 5 and area 6 are weak as compared to other generators with weak connecting tie-lines such that oscillations are frequent. Therefore, FACTS device is connected between area 5 and area 6. Placement of the FACTS device may vary depending upon the system specification and operating conditions in general.

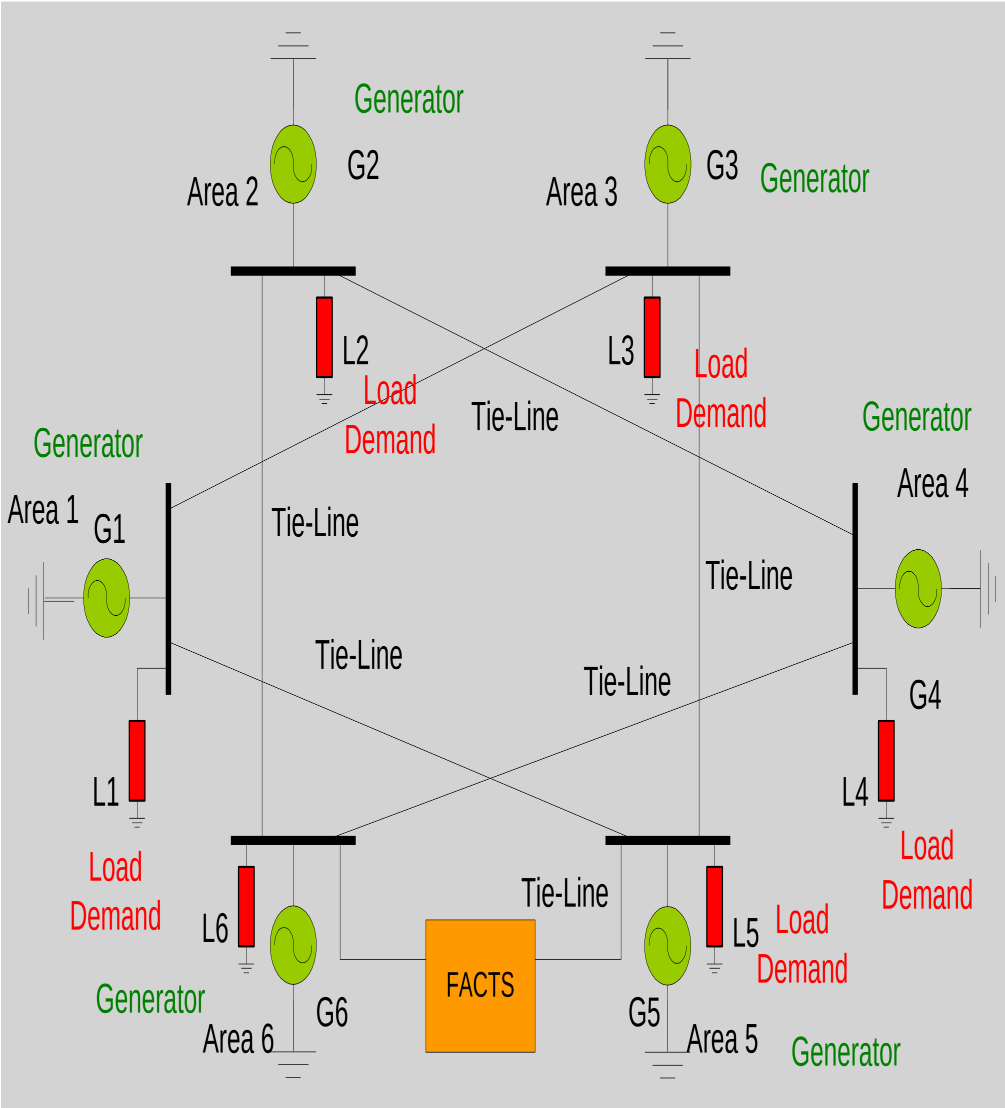


Figure 3.6 Six Area Six Machine power system

3.3.3 Ten Area Fifty Machine Test System (10A50M)

Ten area fifty machine test system has been developed where each generator excitation is equipped with PSS as shown in Figure 3.7. FACTS device is connected between area 1 and area 2 and the detailed specification depends on the system under study. It is to be noted that FACTS controller may be STATCOM, SSSC and UPFC depending upon system requirements which are decided by oscillation under dynamically changing conditions.

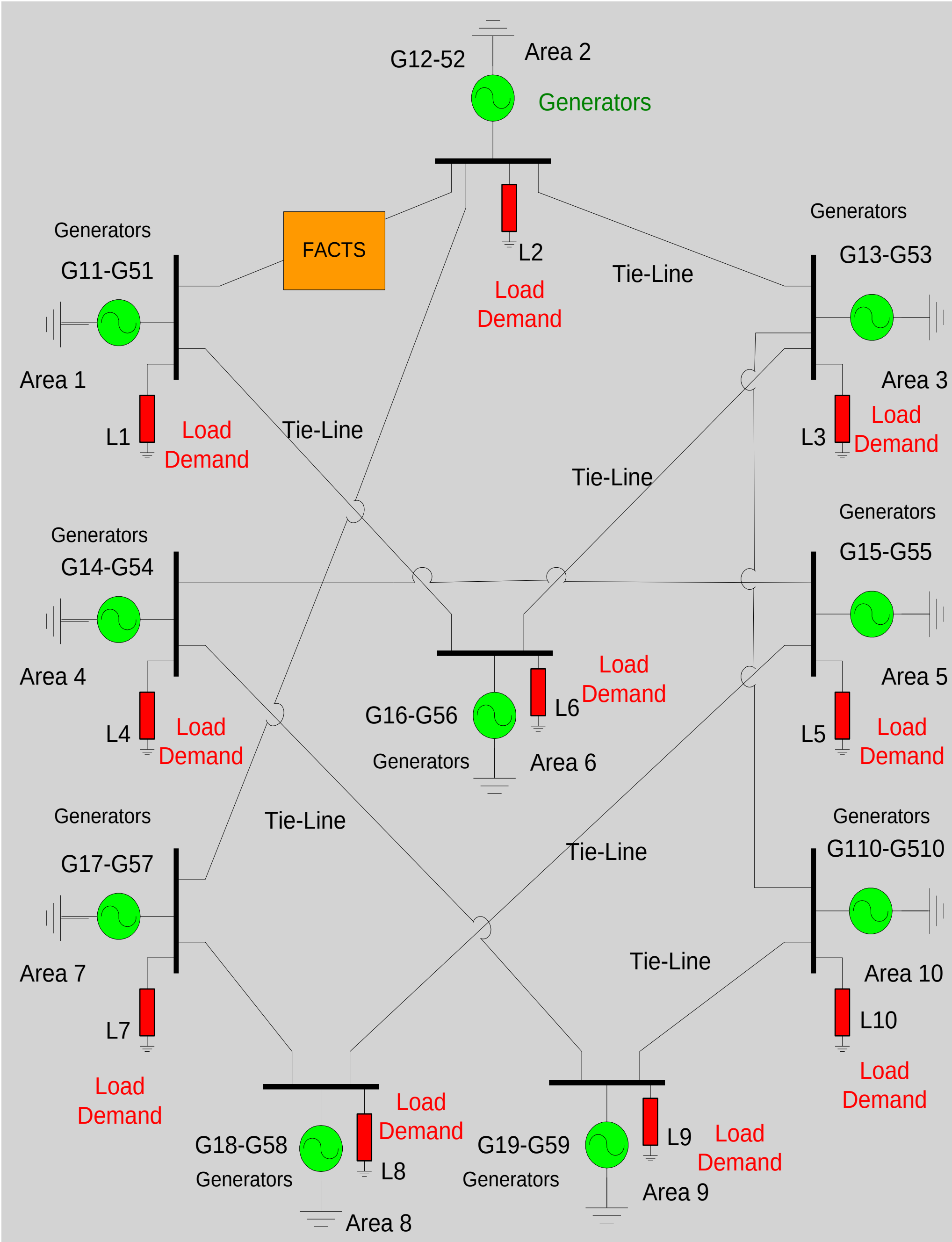


Figure 3.7 Ten Area Fifty Machine power system

3.4 STATE PREDOMINANT CONCEPT (LQR CONTROL) FOR IMPROVED SYSTEM STABILITY

This section presents the state predominant concept for controller design for improved system stability. LQR control has been modified based the proposed concept and the same is modeled in overall system representation.

3.4.1 Linear Quadratic Regulator (LQR) Control Concept

The objectives of the FACTS device have been to maintain the power flow by control injection in the existing transmission corridor. The operation of any FACTS devices is governed by their input control variables. For UPFC, both shunt and series converters control variables (m_e, δ_e and m_b, δ_b) are important to govern the action of the controller. Hence these four control parameters govern complete operation of UPFC. Similarly, for SSSC and STATCOM, respective control inputs govern their operation. The multi-control inputs along with proper coordination decide the system performance. For the satisfactory operation of these devices, multi-variable control concept has been used and coordination of all the control variables simultaneously has been ensured with system dynamics. Multi-Input-Multi-Output (MIMO) design based on the Linear Quadratic Regulator (LQR) control is attempted for multivariable controller design. LQR is an optimal control method to improve the system stability with minimum requirement of control efforts. This method minimizes the cost function J in order to determine the feedback gain matrix with compromise between the control efforts, magnitude and the speed of the response which guarantee stable system with improved system performance.

For a given system-

$$\Delta \dot{x} = A\Delta x + B\Delta u \qquad (3.258)$$

Determine the matrix K of the LQR vector

$$\Delta u(t) = -K\Delta x(t) \qquad (3.259)$$

In order to minimize the performance index:

$$J=\int_{0}^{\infty}\left(x^{T}Qx+U^{T}RU\right)dt \tag{3.260}$$

Where, Q and R are the positive-definite Hermitian or real symmetric matrix. The matrix R and Q determine the relative importance of the error and the expenditure of this energy [146].

From above equations:

$$J=\int_{0}^{\infty}\left(x^{T}Qx+x^{T}K^{T}RKx\right)dt=\int_{0}^{\infty}x^{T}\left(Q+K^{T}RK\right)xdt \tag{3.261}$$

For solving this optimization problem; set

$$x^{T}\left(Q+K^{T}RK\right)x=-d/dt(x^{T}Px) \tag{3.262}$$

$$x^{T}\left(Q+K^{T}RK\right)x=-x^{T}Px-x^{T}Px=-x^{T}\left[\left(A-BK\right)^{T}P+P\left(A-BK\right)\right]x \tag{3.263}$$

Comparing both sides of above equation;

$$\left(A-BK\right)^{T}P+P\left(A-BK\right)=-\left(Q+K^{T}RK\right) \tag{3.264}$$

Since R has been assumed to be positive-definite Hermitian or real symmetric matrix as;

$R=T^{T}T$

where; T is a non-singular matrix; and

$$A^{T}P+PA+\left[TK-\left(T^{T}\right)^{-1}B^{T}P\right]^{T}\left[TK-\left(T^{T}\right)^{-1}B^{T}P\right]-PBR^{-1}B^{T}P+Q=0 \tag{3.265}$$

Minimization of J with respect to K requires the minimization of-

$$x^{T}\left[TK-\left(T^{T}\right)^{-1}B^{T}P\right]^{T}\left[TK-\left(T^{T}\right)^{-1}B^{T}P\right]x \tag{3.266}$$

Which is non-negative, the minimum occurs when it is zero, or when

$$TK = \left(T^{T}\right)^{-1} B^{T} P \tag{3.267}$$

$$K = T^{-1}\left(T^{T}\right)^{-1} B^{T} P = R^{-1} B^{T} P \tag{3.268}$$

Thus a control law comes as;

$$U(t) = -Kx(t) = -R^{-1} B^{T} P x(t) \tag{3.269}$$

In which P must satisfy the reduced Riccati Equation:

$$A^{T} P + PA - PBR^{-1} B^{T} P + Q = 0 \tag{3.270}$$

3.4.1.1 Calculation of Weight Matrices for LQR Control

For the design of optimal LQR controller, quadratic performance index J has to be calculated first. Performance index J can be calculated by obtaining feedback gain matrix K. To calculate K, weighting matrices Q and R must be defined based on the requirements of control. The most tedious part is the selection of state weighting matrix Q. Usually, it is decided based on repetitive trial and error method depending on the physical reasoning in order to get desired performance.

3.4.2 Power Oscillation Damping (POD) Controller

Damping and synchronizing power are related to real and imaginary part of the eigenvalues which corresponds to the variation in rotor speed and rotor angle. The dynamic characteristics of any system can be influenced by the location of eigenvalues. For good dynamic response of a system in terms of settling time and overshoot/undershoot, eigenvalues must be placed in a particular location. So the damping of power oscillation can be improved by shifting the real part of the eigenvalues associated with mode of oscillation to the left side in complex s-plane as desired [146].

For the power system represented in state space form, if d>0 is the required degree of system stability, then close loop system matrix can be calculated as:

$$Ac = A - B(-R^{-1}B^{T}M) = A - B*K \qquad (3.271)$$

This has been obtained through; $u = -Kx$

System matrix given in Equation (3.271) can be represented with its eigenvalue lying on the left side of d- vertical strip in s-plane such that M is the solution of below Riccati equation:

$$(A + dI_n)^T M + M(A + dI_n) - MBR^{-1}B^T M + Q = 0_n \qquad (3.272)$$

Let dl and d2 are two real positive values such that d2 >dl, when lying on negative real axis give a vertical strip of [-d2, -d1].

When the control law is changed to u = -Kx, where K= $R^{-1}B^{T}M$ such that M is the solution of modified Riccati equation.

$$(A + d1I_n)^T M + M(A + d1I_n) - MB(R^{-1}B^T M) = 0_n \qquad (3.273)$$

And $\beta = \frac{1}{2} + \frac{d2 - d1}{trace(BK)}$ (3.274)

This will have eigenvalues lying inside the vertical strip of [-d2, -d1] in complex s-plane which were on the right side of that vertical strip earlier. So for equal weighting of all control inputs matrix R is selected unity matrix and for solving this modified equation no need of Q matrix. This eliminates the main difficulty to determine the weighing matrix through trial and error in order to have satisfactory performance of designed system. So the design of optimal power oscillation damping controller which is based on the eigenvalue assignment technique to improve the system performance is easy and effective.

3.4.2.1 Procedural Steps to Implement POD Controller

1) Find the system matrix (A) and control matrix (B).

2) Calculate eigenvalues of the system.

3) Set the vertical strip [-d2, -d1] to LHS of the complex plane, Q as zero matrix.

4) Calculate feedback gain matrix K

$$K = lqr(A1 + d1I, B, Q, R) \ \text{and} \ \beta = \frac{1}{2} + \frac{d2 - d1}{trace(B*K)}$$

5) Now calculate modified close loop system matrix

($A_c = A - \beta * B * K$) and find system response.

3.4.3 Multi-Stage LQR (MSLQR) Controller

In Multi-Stage LQR controller, selection of diagonal elements of Q matrix are based on the special control concept known as State Predominant technique in which system eigenvalues nearer to the imaginary axis are assigned higher weights in respective diagonal entry of Q matrix. First system matrix A and control matrix B are calculated and depending upon the state predominant concept element of Q matrix is initialized. Feedback gain matrix K is determine using (K=lqr(A,B,Q,R)) with R as identity matrix. Now new control matrix is obtained from A1=A-B*K. This process has to be repeated till all the deviated states are tracked to the desired criteria [149].

3.4.3.1 State Predominant Concept

It is known that the state space representation of a system is essential to design control strategy. States of the system and eigenvalues are interlinked as eigenvalues determine behavior of respective states. Power oscillation damping can be improved if real part of eigenvalue associated with mode of oscillation can be shifted to left side in complex plane. In Multi-Input-Multi-Output (MIMO) system, vulnerable states can be identified by closed loop poles of the system. These vulnerable states may deteriorate performance of the complete system. To maintain all the system states within their limits with faster damping and desired response, proper selection of Q matrix is very crucial. Selection of Q matrix has been evolved in [149] which stated the concept of state predominant approach. In this concept selection of Q matrix has been linked with the position of eigenvalues of system. Some vulnerable states whose eigenvalues are very close to the imaginary axis are termed as predominant states and depending upon that, respective entry in the diagonal elements of Q matrix will have more weightage as compared to other elements. So by identifying predominant states with their respective real part of the eigenvalues, elements of Q matrix has been redefined. Proper regulation of these large deviated states (predominant states) is ensured by assigning high weightage to the respective diagonal elements of Q matrix on the basis of the real part of the eigenvalues. This concept is derived from the very basic fundamentals of states regulation and feedback gain interrelation. With Q matrix, feedback gain matrix K is calculated for ensuring system stabilization. In general, for small perturbations the concept of calculating gain K is mentioned; however, for dynamically changing systems, the control design is based on state predominant approach with multi-stage. Thus, iterative process has to be repeated till all deviated states are tracked to desired criterion. This procedure is very systematic and proves to be effective [149].

3.4.3.2 Procedural Steps for MSLQR Controller

1) Find system matrix (A) and control matrix (B).

2) Obtain the array of gain K with some initialized Q matrix and R as identity matrix.

3) New state matrix A1 is obtained as (A1=A-B*K) and then find eigenvalue of A1 (eig(A1)).

4) Calculate new matrix Q1 with the help of eig(A1) by state predominant approach.

5) New gain matrix K1 is obtained from K1=lqr(A1,B,Q1,R) and new system matrix is formed with combination of plant and controller (as A2=A1-(B*K1)).

6) Set A1=A2 and repeat step 3 and 4.

This process will continue depending upon the required specification for MSLQR controller.

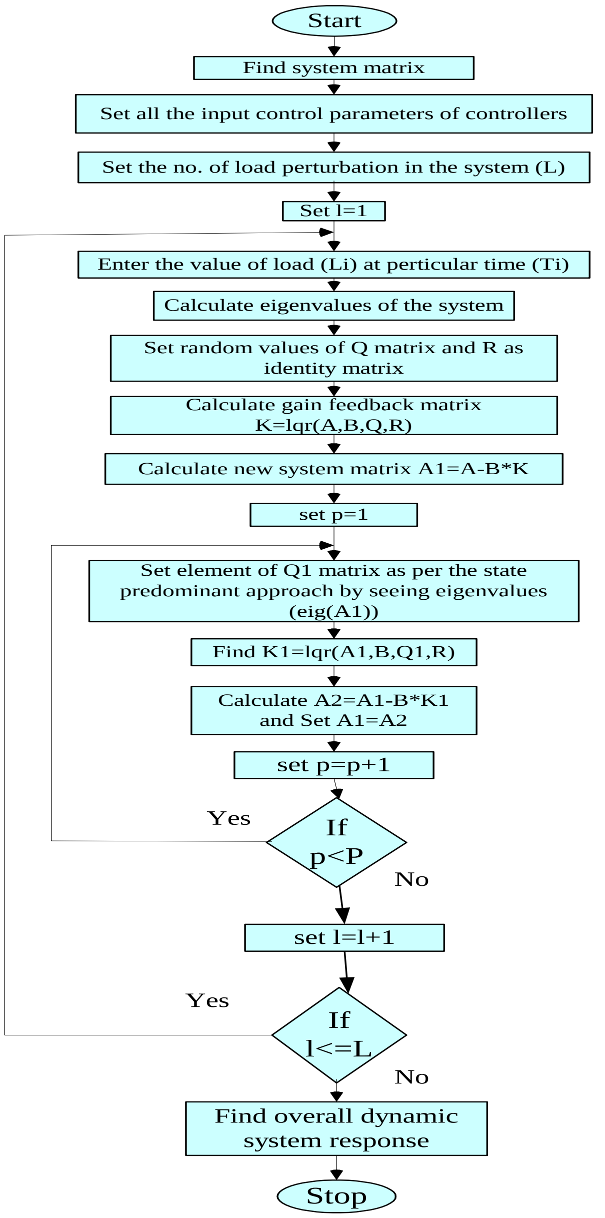


Figure 3.8 Flowchart of MSLQR controller

Flowchart of MSLQR controller has been shown in Figure 3.8, where first step is to find the state matrix of entire system. Set the value of perturbation occurring in the system at particular time and calculate the eigenvalues of the system. Define the elements of Q matrix randomly and calculate feedback gain matrix (K) and new system matrix. Find eigenvalues of new system matrix and set elements of Q by seeing real part of eigenvalue using state predominant approach. Repeat process until all states are tracked to desired response. The process will be same for all other perturbation in the system. In the flowchart, ‘P’ denotes the total number of stages required for desired controller response in multi-stage LQR (MSLQR) controller and p is the initial point of that stage. The multi-stage process is repeated till all the deviated states are tracked to the desired response. For large system with dynamically changing conditions, number of stages may be extended to get the desired system response. As a thumb rule after rigorous analysis for various cases, it can be said that the numbers of stages may be more as complexity of system is increasing. In the proposed work, total number of stage taken for desired system response are 3. The concept show better performance in dynamically changing conditions. In the flowchart ‘l’ shows variation in load and ‘L’ represent the total number of load variation with time.

3.4.4 Proposed Integrated Multi-Stage LQR (MSLQR)-POD Controller

It has been observed in MSLQR controller that even if most of eigenvalues are shifted to left side of the complex plane, but one or two eigenvalues stay very close to the imaginary axis which is not acceptable for the stability of system, as such states may get excited due to operational changes in dynamical mode, therefore a POD controller has been integrated with MSLQR using eigenvalue assignment technique. The proposed controller places eigenvalues corresponding to the mode of oscillations at desired location such that eigenvalues get placed within a vertical strip on LHS in complex plane after applying state predominant approach. The selection of vertical strip can be decided by the desired accuracy of system response. If vertical strip assigned is too far from the imaginary axis then, system will stabilize very quickly but peak overshoot/undershoot of some of the state variables may cross their threshold limits which might affect the entire system in terms of stress in components and also control effort required is more. Therefore while selecting vertical strip; a care has to be taken looking into the stress on the system components.

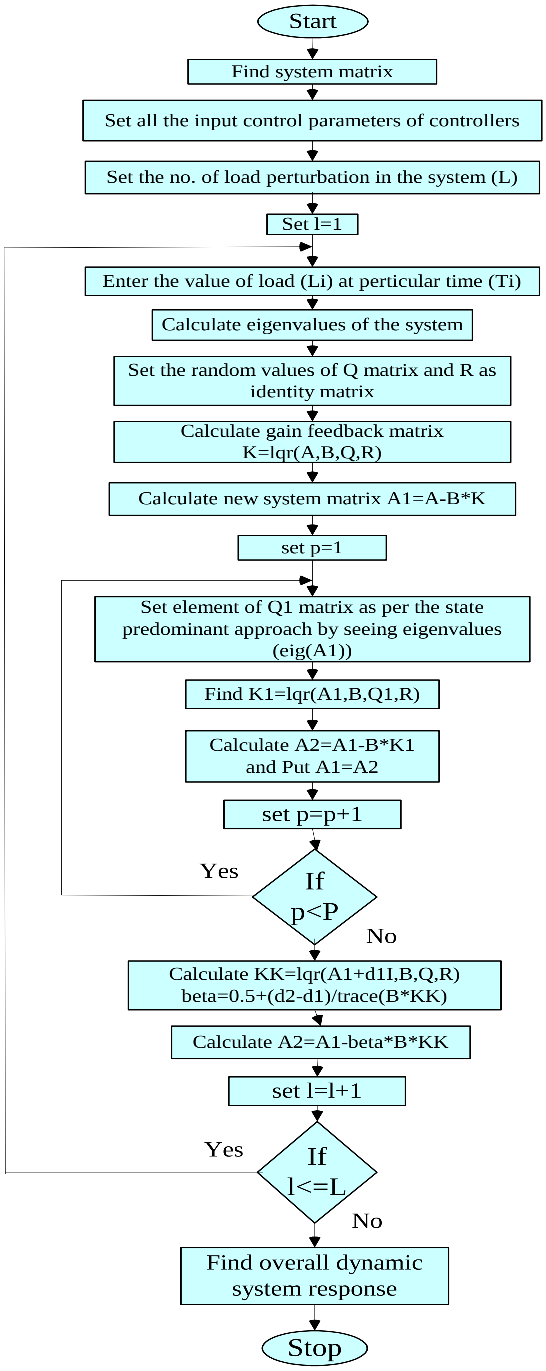


Figure 3.9 Flowchart of Integrated MSLQR-POD controller

Flowchart of proposed control strategy is given in Figure 3.9, where first step is to find the state matrix of entire system. Initialize the input control parameters of PSS and UPFC with any intelligent optimization technique. Set the value of perturbation occurring in the system at particular time and calculate the eigenvalues of the system.

Define the elements of Q matrix randomly and calculate feedback gain matrix (K) and new system matrix. Again find eigenvalues and set the elements of Q by seeing real part of eigenvalue using state predominant approach. Repeat the process until all the states are tracked to desired response. The process will be same for all other perturbation in the system. In the flow chart, 'P' denotes the total number of stages required for desired controller response in Multi-Stage LQR (MSLQR) controller and p is the initial point of that stage. After completing all the stages in MSLQR concept, define vertical strip on the left side of s-plane and calculate feedback gain matrix K and β. Now calculate final state matrix and find the overall dynamic response of the system.

3.4.4.1 Procedural Steps for Integrated Multi Stage LQR (MSLQR)-POD Controller

1) Find system matrix (A) and control matrix (B) and obtain array of gain K with some initialized Q matrix and R as identity matrix.

2) New state matrix A1 is obtained as (A1=A-B*K) and eigenvalue of A1 (eig(A1)).

3) Calculate new matrix Q1 with the help of eig(A1) by state predominant approach.

4) New gain matrix K1 is obtained from K1=lqr(A1,B,Q1,R) and new system matrix is formed with combination of plant and controller (as A2=A1-(B*K1)).

5) Set A1=A2 and repeat step 3 and 4. This process will continue depending upon the required specification for MSLQR controller.

6) For POD controller- Set the vertical strip [-d2, -d1] to the LHS of the complex plane, Q as zero matrix and calculate

$KK = lqr(A1 + d1I, B, Q, R)$ and $\beta = \frac{1}{2} + \frac{d2 - d1}{trace(B * KK)}$

7) Now calculate the modified closed loop system matrix

($A_c = A1 - \beta * B * KK$)

3.5 UTILIZATION OF STATES FOR CONTROL ACTIVATION

States deviation of the system represents complete system behavior which is linked with stability. In general, all states variables are interlinked with eigenvalues and eigenvalues corresponding to the vulnerable states (placed very close to the imaginary axis) need to be taken for priority signal selection. These states are termed as state predominant states and even a small perturbation in the network may aggravate these states which can ultimately collapse the entire system. Control activation is initiated based on eigenvalues corresponding to states. In MSLQR controller concept, the relatively largely perturbed states linked with eigenvalues need to be included for initiating control injection by assigning higher weight in Q matrix for quick stabilization. The identification of predominant states is primarily carried out by respective real part of eigenvalues, elements of Q matrix thus are redefined and linked with perturbation. Proper regulation of these large deviated states (predominant states) are ensured by assigning high weightage to respective diagonal elements of Q matrix on the basis of real part of eigenvalues. The perturbation model has been used for state predominant concept with proper Q assignment, however for relatively changing perturbation with changing system dynamics, process of state predominant concept has to be repeated until all deviated states are tracked to acceptable system response. This concept is very systematic and effective in dealing with operational shift in system dynamics.

3.6 CONTROLLER REALIZATION WITH STATES REGULATION

Integrated MSLQR-POD controller has been used to regulate input control parameters of FACTS devices. Sample six-area system and two-area four-machine test system has been considered to demonstrate the effectiveness of proposed controller in terms of states regulation with improved system stability.

3.6.1 Simulation Results for Two Area Four Machine Test System

UPFC has been connected between both areas as shown in Figure 3.10. Simulation has been carried out for the proposed concept with 10% increase of loading in both the areas.

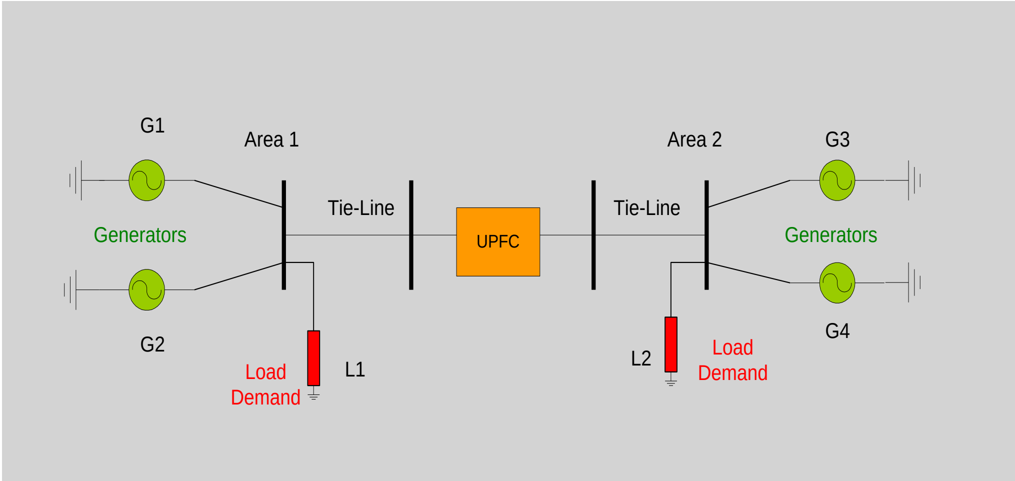
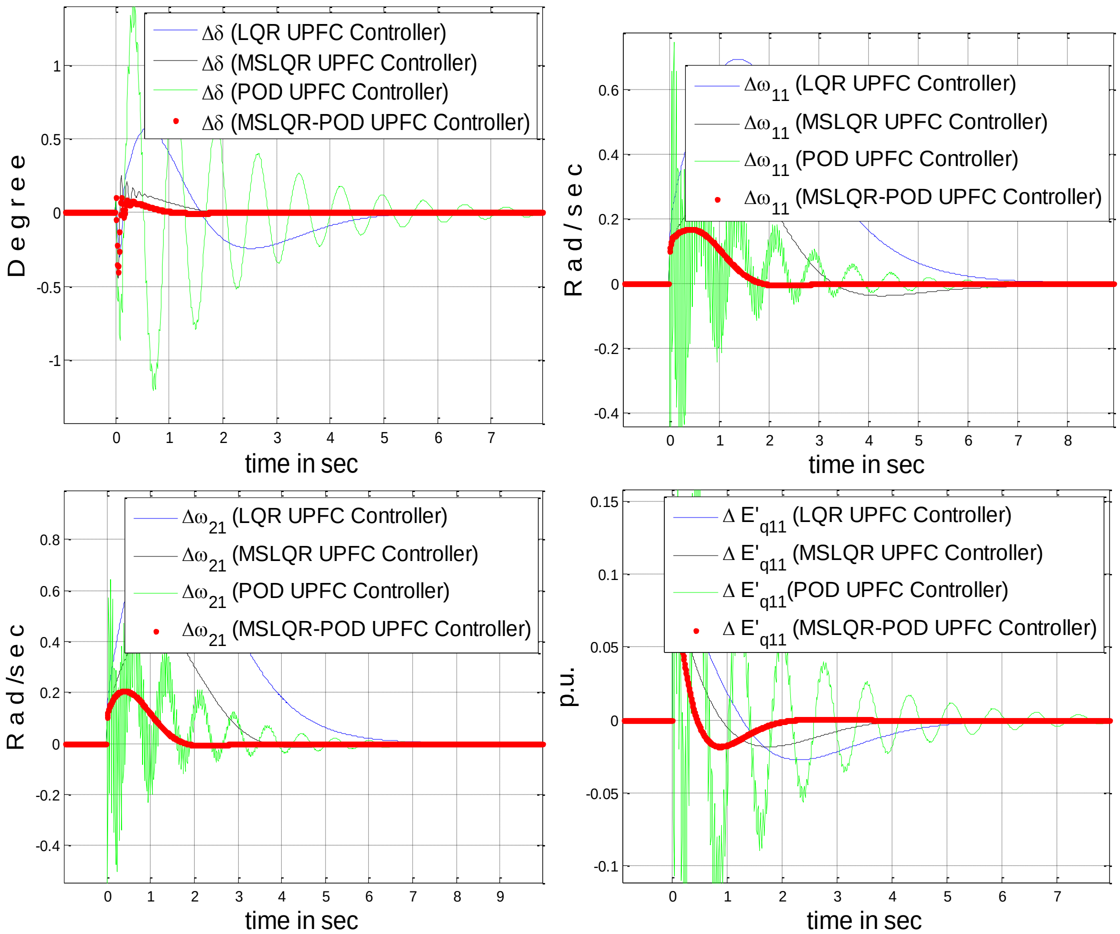
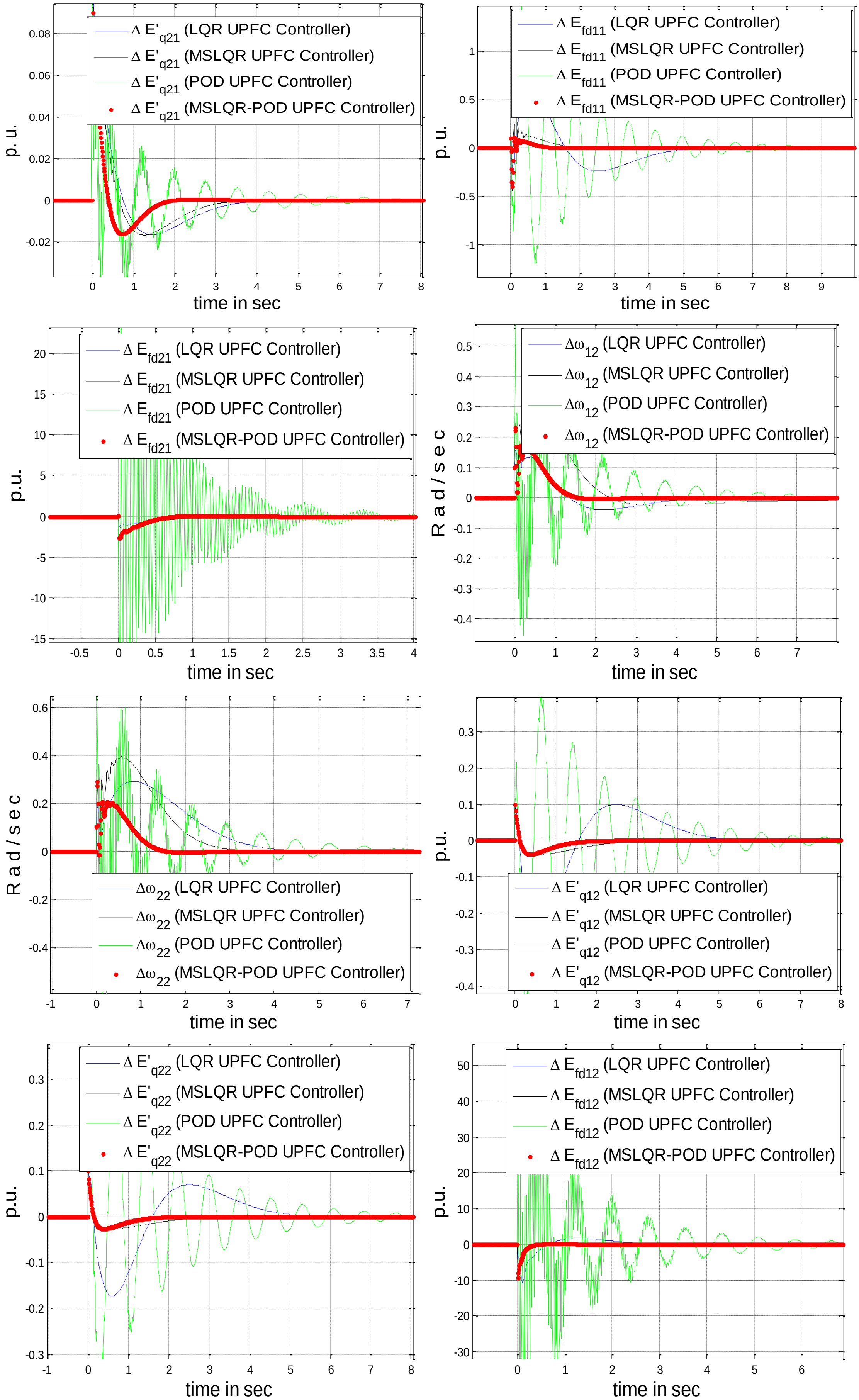


Figure 3.10 Two Area Four Machine power system with UPFC




Δ E'q21 (LQR UPFC Controller)
Δ E'q21 (MSLQR UPFC Controller)
Δ E'q21 (POD UPFC Controller)
Δ E'q21 (MSLQR-POD UPFC Controller)
p. u.
time in sec
Δ Efd11 (LQR UPFC Controller)
Δ Efd11 (MSLQR UPFC Controller)
Δ Efd11 (POD UPFC Controller)
Δ Efd11 (MSLQR-POD UPFC Controller)
p. u.
time in sec
Δ Efd21 (LQR UPFC Controller)
Δ Efd21 (MSLQR UPFC Controller)
Δ Efd21 (POD UPFC Controller)
Δ Efd21 (MSLQR-POD UPFC Controller)
p.u.
time in sec
Δω12 (LQR UPFC Controller)
Δω12 (MSLQR UPFC Controller)
Δω12 (POD UPFC Controller)
Δω12 (MSLQR-POD UPFC Controller)
R a d / s e c
time in sec
Δω22 (LQR UPFC Controller)
Δω22 (MSLQR UPFC Controller)
Δω22 (POD UPFC Controller)
Δω22 (MSLQR-POD UPFC Controller)
R a d / s e c
time in sec
Δ E'q12 (LQR UPFC Controller)
Δ E'q12 (MSLQR UPFC Controller)
Δ E'q12 (POD UPFC Controller)
Δ E'q12 (MSLQR-POD UPFC Controller)
p.u.
time in sec
Δ E'q22 (LQR UPFC Controller)
Δ E'q22 (MSLQR UPFC Controller)
Δ E'q22 (POD UPFC Controller)
Δ E'q22 (MSLQR-POD UPFC Controller)
p.u.
time in sec
Δ Efd12 (LQR UPFC Controller)
Δ Efd12 (MSLQR UPFC Controller)
Δ Efd12 (POD UPFC Controller)
Δ Efd12 (MSLQR-POD UPFC Controller)
p.u.
time in sec

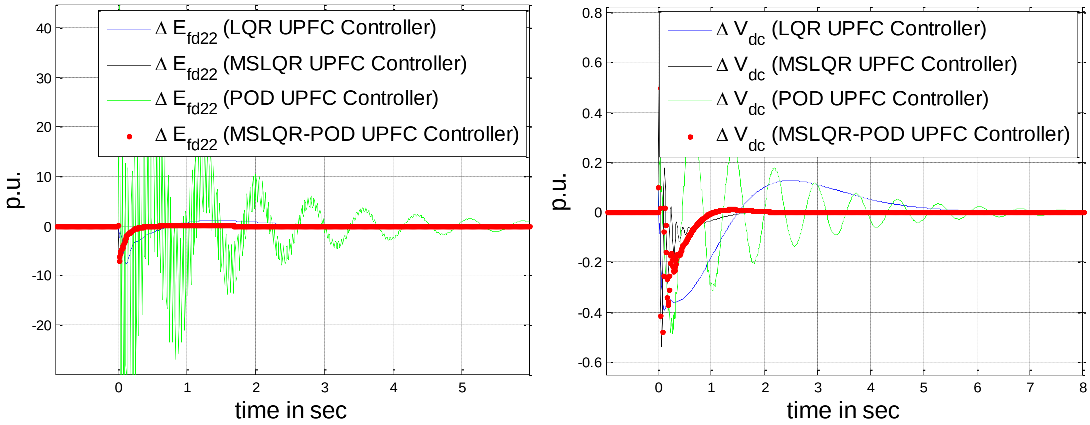


Figure 3.11 Perturbation response for different controllers with UPFC (Two Area Four Machine System)

Table 3.1: Comparison of system response for Two Area Four Machine system

System	States	Over-shoot	Settling time (sec)	Eigenvalues 1.0e+05*
UPFC with LQR controller	$\Delta\delta$(degree)	0.58	4.5	-0.15783
	$\Delta\omega_{11}$(Rad/sec)	0.7	6.9	-0.01835
	$\Delta\omega_{21}$ (Rad/sec)	0.45	6.1	-0.00254
	$\Delta E'_{q11}$ (p.u.)	0.12	5.8	-0.00067
	$\Delta E'_{q21}$ (p.u.)	0.1	3.8	-0.00053
	ΔE_{fd11}(p.u.)	0.58	5.1	-.00015 + 0.00032i
	ΔE_{fd21} (p.u.)	-1.2	2.0	-.00015 - 0.00032i
	$\Delta\omega_{12}$(Rad/sec)	0.14	4.2	-0.00011
	$\Delta\omega_{22}$ (Rad/sec)	0.27	4.1	-0.00003 + .00001i
	$\Delta E'_{q12}$ (p.u.)	-0.25	5.0	-0.00003 - .00001i
	$\Delta E'_{q22}$ (p.u.)	-0.17	5.0	-0.00001 + .00000i
	ΔE_{fd12}(p.u.)	-10.1	2.5	-0.00001 - .00000i
	ΔE_{fd22} (p.u.)	-7.5	3.1	-0.00001 + .00001i
	ΔVdc(p.u.)	-0.39	4.9	-0.00001 - .00001i
UPFC with POD (EAT) controller	$\Delta\delta$(degree)	1.5	7.5	-0.0000123 + .0014183i
	$\Delta\omega_{11}$(Rad/sec)	-0.4	6.1	-0.0000123 - .0014183i
	$\Delta\omega_{21}$ (Rad/sec)	-0.4	6.1	-0.0006094
	$\Delta E'_{q11}$ (p.u.)	-.12	7.5	-0.0005033
	$\Delta E'_{q21}$ (p.u.)	0.08	6.9	-0.0004113
	ΔE_{fd11}(p.u.)	1.2	7.2	-0.0001141 + 0.0001904i
	ΔE_{fd21} (p.u.)	6.0	5.5	-0.0001141 - 0.0001904i
	$\Delta\omega_{12}$(Rad/sec)	0.357	7.0	-0.0002506 + 0.0001093i
	$\Delta\omega_{22}$ (Rad/sec)	-0.45	6.5	-0.0002506 - 0.0001093i
	$\Delta E'_{q12}$ (p.u.)	-0.47	6.8	-0.0000052 + 0.0000813i
	$\Delta E'_{q22}$ (p.u.)	0.3	7.9	-0.0000052 - 0.0000813i
	ΔE_{fd12}(p.u.)	-3.0	6.9	-0.0000050 + 0.0000035i
	ΔE_{fd22} (p.u.)	4.2	9.5	-0.0000050 - 0.0000035i
	ΔVdc(p.u.)	-0.45	6.1	-0.0050
UPFC with MSLQR controller	$\Delta\delta$(degree)	0.21	3.1	-1.0110
	$\Delta\omega_{11}$(Rad/sec)	0.39	6.7	-0.0536
	$\Delta\omega_{21}$ (Rad/sec)	0.48	3.9	-0.0270
	$\Delta E'_{q11}$ (p.u.)	0.1	4.0	-0.0042
	$\Delta E'_{q21}$ (p.u.)	0.1	3.6	-0.0001 + 0.0006i
	ΔE_{fd11}(p.u.)	0.22	2.0	-0.0001 - 0.0006i
	ΔE_{fd21} (p.u.)	-1.4	2.1	-0.0005
	$\Delta\omega_{12}$(Rad/sec)	0.27	8.0	-0.0001

	$\Delta\omega_{22}$ (Rad/sec)	0.4	3.1	-0.0000 + 0.0000i
	$\Delta E'_{q12}$ (p.u.)	-0.04	3.1	-0.0000 - 0.0000i
	$\Delta E'_{q22}$ (p.u.)	-0.025	2.9	-0.0000 + 0.0000i
	ΔE_{fd12}(p.u.)	-7.8	1.9	-0.0000 - 0.0000i
	ΔE_{fd22} (p.u.)	-6.0	1.8	-0.0000
	ΔVdc(p.u.)	-0.55	1.8	-0.0000
Integrated MSLQR-POD UPFC Controller [Proposed]	$\Delta\delta$(degree)	0.21	1.1	-1.0115
	$\Delta\omega_{11}$(Rad/sec)	0.39	2.1	-0.0521
	$\Delta\omega_{21}$ (Rad/sec)	0.48	2.0	-0.0362
	$\Delta E'_{q11}$ (p.u.)	0.1	2.1	-0.0046
	$\Delta E'_{q21}$ (p.u.)	0.1	1.9	-0.0001 + 0.0006i
	ΔE_{fd11}(p.u.)	0.22	1.5	-0.0001 - 0.0006i
	ΔE_{fd21} (p.u.)	-1.4	1.9	-0.0005
	$\Delta\omega_{12}$(Rad/sec)	0.27	1.9	-0.0001
	$\Delta\omega_{22}$ (Rad/sec)	0.4	1.8	-0.0000 + 0.0000i
	$\Delta E'_{q12}$ (p.u.)	-0.04	1.8	-0.0000 - 0.0000i
	$\Delta E'_{q22}$ (p.u.)	-0.025	1.9	-0.0000 + 0.0000i
	ΔE_{fd12}(p.u.)	-7.8	1.8	-0.0000 - 0.0000i
	ΔE_{fd22} (p.u.)	-6.0	1.7	-0.0000 + 0.0000i
	ΔVdc(p.u.)	-0.55	1.6	-0.0000 - 0.0000i

Perturbation response of state variables is shown in Figure 3.11 and Table 3.1 shows the peak overshoot/undershoot, settling time and eigenvalues of all the controllers for the sample two area four machine test system. The proposed controller shows fast damping as compared to other controllers in terms of settling time and peak overshoot. The vertical strip assigned for the eigenvalues by POD controller is [-2.0, -3.5] for UPFC. Assignment of this vertical strip depends on the desired response of system as mentioned earlier. Since system keeps on changing over a period of time, this strategy may prove beneficial for developing a modular controller over the existing control structure.

3.6.2 Simulation Results for Sample Six Area Six Machine Test System

Three cases have been considered to demonstrate the effectiveness of proposed controller.

Case I: Integrated MSLQR-POD STATCOM Controller

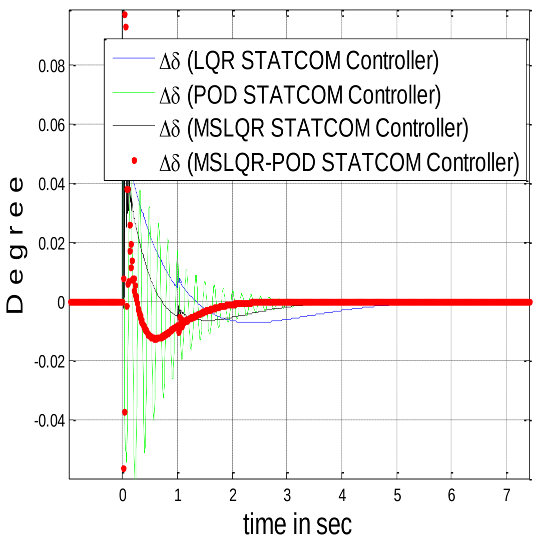
Case II: Integrated MSLQR-POD SSSC Controller

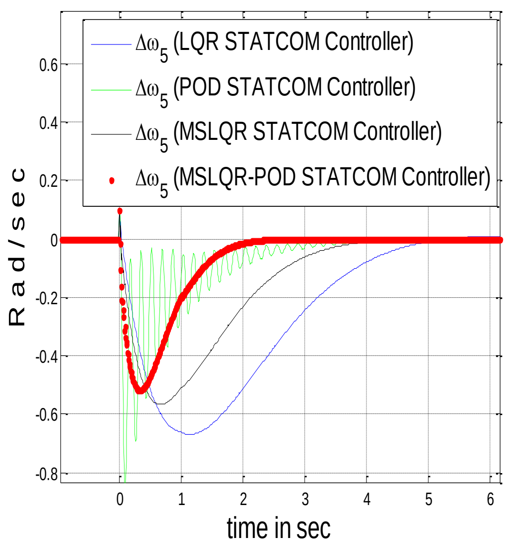
Case III: Integrated MSLQR-POD UPFC Controller

3.6.2.1 Case I: Integrated MSLQR-POD STATCOM Controller

(PSS to all the generators and STATCOM between area 5 and area 6)- at time t_1=0 sec, Load in area 5 and 6 is L=1.0 p.u. with p.f.=0.95 and at time t_2=1.0 sec load increased up to 20% (L=1.2 p.u.) with p.f.=0.85.

Perturbation response of state variables is shown in Figure 3.12 and Table 3.2 shows the peak overshoot/undershoot, settling time and eigenvalues of all the controllers for six area system. The proposed controller shows the fast damping as compared with MSLQR controller in terms of settling time. Peak overshoot for all the state variables are within the acceptable limits. Table 3.3 shows the eigenvalue of the complete system for different controllers. The vertical strip assigned for the eigenvalues by POD controller is [-2.5, -3.5] for STATCOM. The assignment of this vertical strip depends upon the desired response of system. The designer may select the suitable strip depending upon system requirements. Since the system keeps on changing over a period of time, this strategy may prove beneficial for developing a modular controller over the existing control structure.





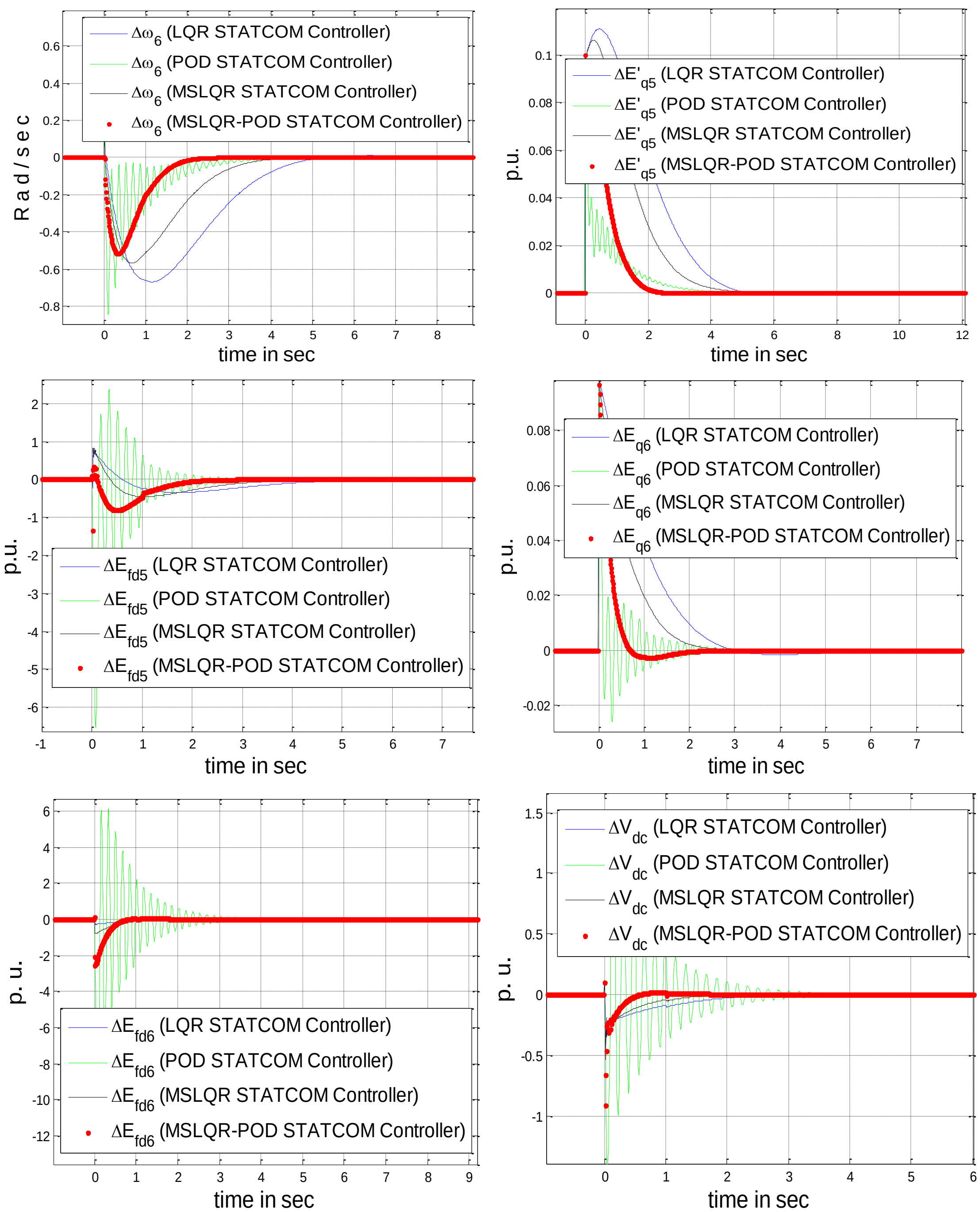


Figure 3.12 Perturbation response for different controllers with STATCOM (Area 5 and Area 6) - Case I

Table 3.2: Comparison of system response for Six Area system with STATCOM-Case I

System	**States**	**Over-shoot**	**Settling time (sec)**	**Eigenvalues 1.0e+03***
STATCOM with LQR controller	$\Delta\delta$ (degree)	0.10	4.50	-1.4172
	$\Delta\omega_5$ (Rad/sec)	-0.67	4.80	-0.6720
	$\Delta\omega_6$ (Rad/sec)	-0.68	4.80	-0.0342 + 0.2296i
	$\Delta E'_{q5}$ (p.u.)	0.115	4.70	-0.0342 - 0.2296i
	ΔE_{fd5} (p.u.)	0.90	4.00	-0.0737
	$\Delta E'_{q6}$(p.u.)	0.10	4.50	-0.0008 + 0.0006i
	ΔE_{fd6}(p.u.)	-0.40	1.90	-0.0008 - 0.0006i
	ΔV_{dc}(p.u.)	-0.41	2.50	-0.0007

STATCOM with POD (EAT) controller	$\Delta\delta$ (degree)	0.08	3.80	-0.03009+0.22779i
	$\Delta\omega_5$ (Rad/sec)	-0.82	3.90	-0.03009 -0.22779i
	$\Delta\omega_6$ (Rad/sec)	-0.81	3.90	-0.00125+0.03731i
	$\Delta E'_{q5}$ (p.u.)	0.10	3.50	-0.00125-0.03731i
	ΔE_{fd5} (p.u.)	-6.50	3.00	-0.01768
	$\Delta E'_{q6}$(p.u.)	-0.018	2.80	-0.00862+0.00625i
	ΔE_{fd6}(p.u.)	-12.0	3.50	-0.00862-0.00625i
	ΔV_{dc}(p.u.)	-1.40	4.00	-0.00100
STATCOM with MSLQR controller	$\Delta\delta$ (degree)	0.09	3.30	-3.9959
	$\Delta\omega_5$ (Rad/sec)	-0.55	3.60	-1.7250
	$\Delta\omega_6$ (Rad/sec)	-0.56	3.60	-0.0182 + 0.2429i
	$\Delta E'_{q5}$ (p.u.)	0.110	4.00	-0.0182 - 0.2429i
	ΔE_{fd5} (p.u.)	0.90	2.90	-0.0958
	$\Delta E'_{q6}$(p.u.)	0.10	2.30	-0.0013 + 0.0006i
	ΔE_{fd6}(p.u.)	-0.80	1.85	-0.0013 - 0.0006i
	ΔV_{dc}(p.u.)	-0.58	1.80	-0.0007
Integrated MSLQR-POD STATCOM Controller [Proposed]	$\Delta\delta$ (degree)	0.04	2.00	-3.9959
	$\Delta\omega_5$ (Rad/sec)	-0.50	2.10	-1.7250
	$\Delta\omega_6$ (Rad/sec)	-0.50	2.10	-0.0182 + 0.2429i
	$\Delta E'_{q5}$ (p.u.)	0.095	2.00	-0.0182 - 0.2429i
	ΔE_{fd5} (p.u.)	-0.90	1.90	-0.0958
	$\Delta E'_{q6}$(p.u.)	0.10	1.90	-0.0028 + 0.0013i
	ΔE_{fd6}(p.u.)	-2.50	1.80	-0.0028 - 0.0013i
	ΔV_{dc}(p.u.)	-0.90	1.50	-0.0029

Table 3.3: Eigenvalues for all the generators in each area - Case I

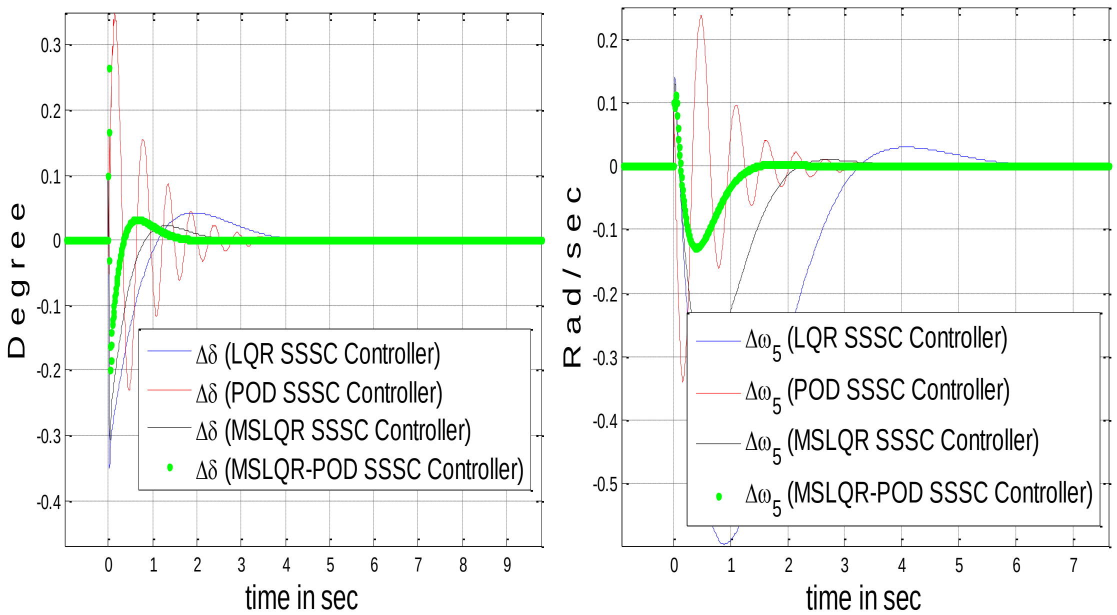
System States		**LQR STATCOM Controller *1.0e+003**	**POD STATCOM Controller *1.0e+002**	**MSLQR STATCOM Controller *1.0e+003**	**Integrated MSLQR-POD STATCOM Controller *1.0e+003**
1	$\Delta\delta$	-1.4172	-0.3009 + 2.2779i	-3.9959	-3.9959
2	$\Delta\omega_5$	-0.6720	-0.3009 - 2.2779i	-1.7250	-1.7250
3	$\Delta\omega_6$	-0.0342 + 0.2296i	-0.0125 + 0.3731i	-0.0182 + 0.2429i	-0.0182 + 0.2429i
4	$\Delta E'_{q5}$	-0.0342 - 0.2296i	-0.0125 - 0.3731i	-0.0182 - 0.2429i	-0.0182 - 0.2429i
5	ΔE_{fd5}	-0.0737	-0.1768	-0.0958	-0.0958
6	$\Delta E'_{q6}$	-0.0008 + 0.0006i	-0.0862 + 0.0625i	-0.0013 + 0.0006i	-0.0028 + 0.0013i
7	ΔE_{fd6}	-0.0008 - 0.0006i	-0.0862 - 0.0625i	-0.0013 - 0.0006i	-0.0028 - 0.0013i
8	ΔV_{dc}	-0.0007	-0.0100	-0.0007	-0.0029
9	$\Delta\omega_1$	-0.0021 + 0.0038i	-0.0213 + 0.0377i	-0.0021 + 0.0038i	-0.0021 + 0.0038i
10	$\Delta\delta_1$	-0.0021 - 0.0038i	-0.0213 - 0.0377i	-0.0021 - 0.0038i	-0.0021 - 0.0038i
11	$\Delta E'_{q1}$	-0.0005 + 0.0031i	-0.0054 + 0.0312i	-0.0005 + 0.0031i	-0.0005 + 0.0031i
12	ΔE_{fd1}	-0.0005 - 0.0031i	-0.0054 - 0.0312i	-0.0005 - 0.0031i	-0.0005 - 0.0031i
13	$\Delta X5_1$	-0.0001	-0.0013	-0.0001	-0.0001
14	ΔUE_1	-0.0003	-0.0033	-0.0003	-0.0003
15	$\Delta\omega_2$	-0.0006 + 0.0046i	-0.0056 + 0.0456i	-0.0006 + 0.0046i	-0.0006 + 0.0046i
16	$\Delta\delta_2$	-0.0006 - 0.0046i	-0.0056 - 0.0456i	-0.0006 - 0.0046i	-0.0006 - 0.0046i
17	$\Delta E'_{q2}$	-0.0021 + 0.0039i	-0.0214 + 0.0388i	-0.0021 + 0.0039i	-0.0021 + 0.0039i
18	ΔE_{fd2}	-0.0021 - 0.0039i	-0.0214 - 0.0388i	-0.0021 - 0.0039i	-0.0021 - 0.0039i
19	$\Delta X5_2$	-0.0004	-0.0037	-0.0004	-0.0004
20	ΔUE_2	-0.0001	-0.0013	-0.0001	-0.0001
21	$\Delta\omega_3$	-0.0066 + 0.0090i	-0.0655 + 0.0901i	-0.0066 + 0.0090i	-0.0066 + 0.0090i
22	$\Delta\delta_3$	-0.0066 - 0.0090i	-0.0655 - 0.0901i	-0.0066 - 0.0090i	-0.0066 - 0.0090i
23	$\Delta E'_{q3}$	-0.0019 + 0.0014i	-0.0189 + 0.0141i	-0.0019 + 0.0014i	-0.0019 + 0.0014i
24	ΔE_{fd3}	-0.0019 - 0.0014i	-0.0189 - 0.0141i	-0.0019 - 0.0014i	-0.0019 - 0.0014i
25	$\Delta X5_3$	-0.0007	-0.0071	-0.0007	-0.0007
26	ΔUE_3	-0.0001	-0.0013	-0.0001	-0.0001

27	$\Delta\omega_4$	-0.0053	-0.0533	-0.0053	-0.0053
28	$\Delta\delta_4$	-0.0004 + 0.0050i	-0.0038 + 0.0498i	-0.0004 + 0.0050i	-0.0004 + 0.0050i
29	$\Delta E'_{q4}$	-0.0004 - 0.0050i	-0.0038 - 0.0498i	-0.0004 - 0.0050i	-0.0004 - 0.0050i
30	ΔE_{fd4}	-0.0006 + 0.0015i	-0.0060 + 0.0153i	-0.0006 + 0.0015i	-0.0006 + 0.0015i
31	$\Delta X5_4$	-0.0006 - 0.0015i	-0.0060 - 0.0153i	-0.0006 - 0.0015i	-0.0006 - 0.0015i
32	ΔUE_4	-0.0003	-0.0031	-0.0003	-0.0003

3.6.2.2 Case II: Integrated MSLQR-POD SSSC Controller

(PSS to all the generators and SSSC between area 5 and area 6)- at time t_1=0 sec, Load in area 5 and 6 is L=1.0 p.u. with p.f.=0.90 and at time t_2=1.0 sec load increased up to 30% (L=1.3 p.u.) with p.f.=0.90.

Perturbation response of state variables with proposed controller concept is shown in Figure 3.13 and Table 3.4 shows the peak overshoot/undershoot, settling time and eigenvalues of all the controllers for six area system. The proposed controller shows the fast damping as compared with MSLQR controller in terms of settling time and peak overshoot. Table 3.5 shows the eigenvalue of the complete system for different controllers. The vertical strip assigned for the eigenvalues by POD controller is [-2.5, -3.5] for SSSC.



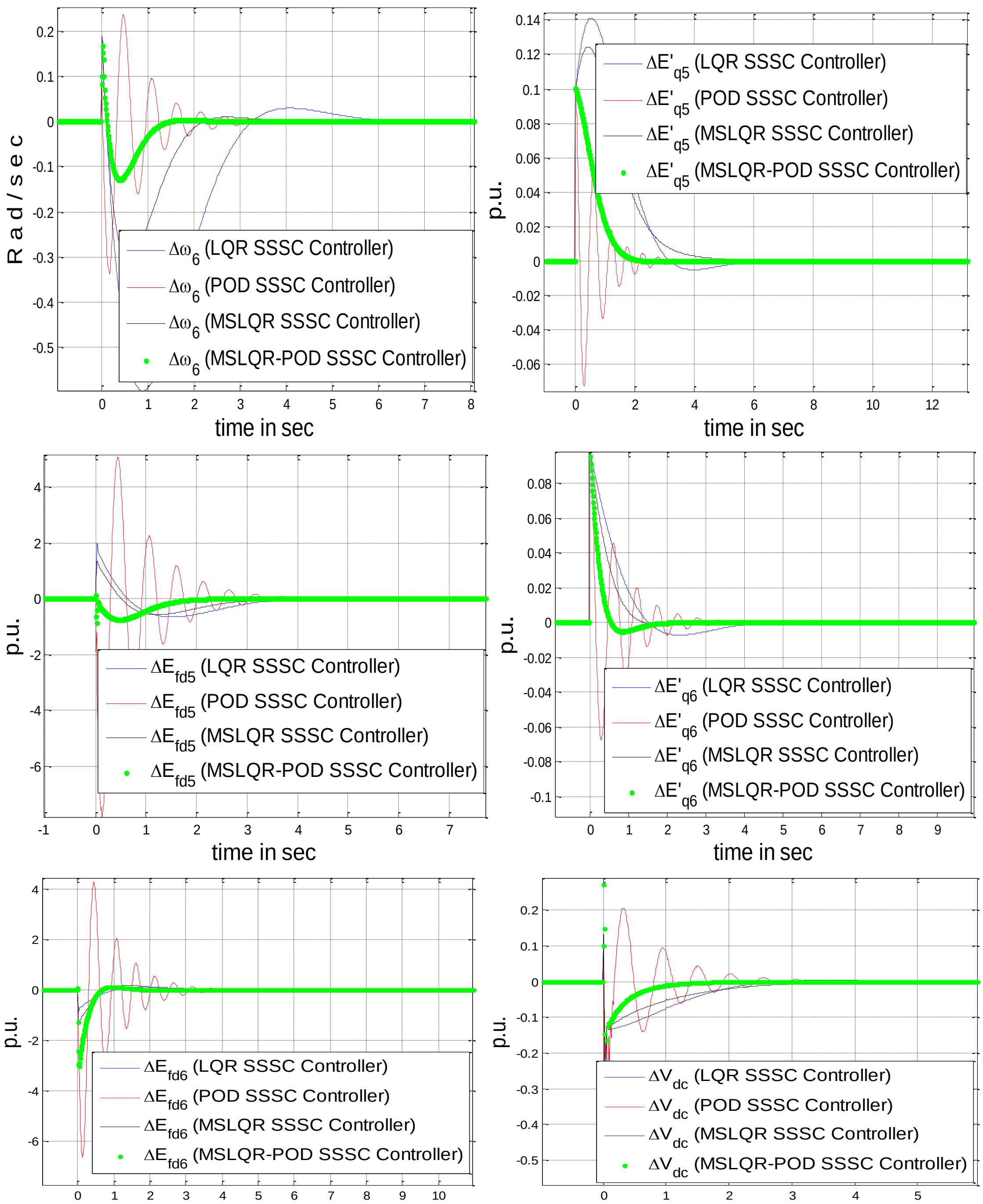


Figure 3.13 Perturbation response of Six Area system for all the controllers with SSSC - Case II

Table 3.4: Comparison of system response for Six Area system with SSSC - Case II

System	States	Over-shoot	Settling time (sec)	Eigenvalues 1.0e+03*
SSSC with LQR Controller	$\Delta\delta$ (degree)	-0.35	3.9	-5.9897
	$\Delta\omega_5$ (Rad/sec)	-0.6	5.5	-1.0207
	$\Delta\omega_6$ (Rad/sec)	-0.6	5.5	-0.0731 + 0.0896i
	$\Delta E'_{q5}$ (p.u.)	0.14	5.1	-0.0731 - 0.0896i
	ΔE_{fd5} (p.u.)	2.0	3.2	-0.1157
	$\Delta E'_{q6}$(p.u.)	0.1	3.9	-0.0009 + 0.0010i

	ΔE_{fd6}(p.u.)	-1.0	1.9	-0.0009 - 0.0010i
	ΔV_{dc}(p.u.)	-0.25	3.0	-0.0013
SSSC with POD (EAT) Controller	$\Delta\delta$ (degree)	0.35	3.5	-.01696 + .21006i
	$\Delta\omega_5$ (Rad/sec)	-0.33	3.1	-.01696 - .21006i
	$\Delta\omega_6$ (Rad/sec)	-0.34	3.1	-.00125 + .01206i
	$\Delta E'_{q5}$ (p.u.)	-0.07	3.0	-.00125 - .01206i
	ΔE_{fd5} (p.u.)	-7.8	3.3	-.01451
	$\Delta E'_{q6}$(p.u.)	-0.07	2.6	-.00867 + .00842i
	ΔE_{fd6}(p.u.)	-6.5	3.5	-.00867 - .00842i
	ΔV_{dc}(p.u.)	-0.59	2.9	-.00160
SSSC with MSLQR Controller	$\Delta\delta$ (degree)	-0.3	2.1	-18.966
	$\Delta\omega_5$ (Rad/sec)	-0.31	2.5	-03.857
	$\Delta\omega_6$ (Rad/sec)	-0.32	2.5	-00.064 + 00.087i
	$\Delta E'_{q5}$ (p.u.)	0.12	4.5	-00.064 – 00.087i
	ΔE_{fd5} (p.u.)	1.9	2.9	-00.115
	$\Delta E'_{q6}$(p.u.)	0.1	2.0	-00.002 + 00.001i
	ΔE_{fd6}(p.u.)	-1.4	1.9	-00.002 – 00.001i
	ΔV_{dc}(p.u.)	-0.25	3.0	-00.001
Integrated MSLQR-POD SSSC Controller [Proposed]	$\Delta\delta$ (degree)	-0.2	1.8	-18.966
	$\Delta\omega_5$ (Rad/sec)	-0.12	1.5	-03.857
	$\Delta\omega_6$ (Rad/sec)	-0.12	1.5	-00.064 + 00.087i
	$\Delta E'_{q5}$ (p.u.)	0.1	2.0	-00.064 – 00.087i
	ΔE_{fd5} (p.u.)	-0.5	1.5	-00.115
	$\Delta E'_{q6}$(p.u.)	0.1	1.9	-00.003 + 00.002i
	ΔE_{fd6}(p.u.)	-3.0	1.4	-00.003 – 00.002i
	ΔV_{dc}(p.u.)	-0.25	1.2	-00.003

Table 3.5: Eigenvalues for all the generators in each area - Case II

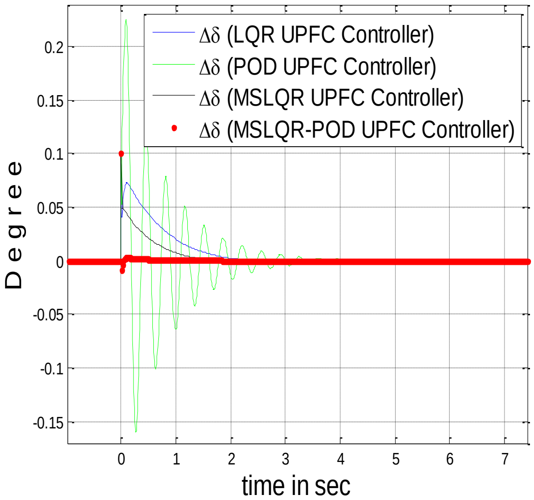
System States		**LQR SSSC Controller *1.0e+003**	**POD SSSC Controller *1.0e+002**	**MSLQR SSSC Controller *1.0e+004**	**MSLQR-POD SSSC Controller *1.0e+004**
1	$\Delta\delta$	-5.9897	-0.1696 + 2.1006i	-1.8966	-1.8966
2	$\Delta\omega_5$	-1.0207	-0.1696 - 2.1006i	-0.3857	-0.3857
3	$\Delta\omega_6$	-0.0731 + 0.0896i	-0.0125 + 0.1206i	-0.0064 + 0.0087i	-0.0064 + 0.0087i
4	$\Delta E'_{q5}$	-0.0731 - 0.0896i	-0.0125 - 0.1206i	-0.0064 - 0.0087i	-0.0064 - 0.0087i
5	ΔE_{fd5}	-0.1157	-0.1451	-0.0115	-0.0115
6	$\Delta E'_{q6}$	-0.0009 + 0.0010i	-0.0867 + 0.0842i	-0.0002 + 0.0001i	-0.0003 + 0.0002i
7	ΔE_{fd6}	-0.0009 - 0.0010i	-0.0867 - 0.0842i	-0.0002 - 0.0001i	-0.0003 - 0.0002i
8	ΔV_{dc}	-0.0013	-0.0160	-0.0002 + 0.0004i	-0.0003
9	$\Delta\omega_1$	-0.0020 + 0.0037i	-0.0205 + 0.0372i	-0.0002 - 0.0004i	-0.0002 + 0.0004i
10	$\Delta\delta_1$	-0.0020 - 0.0037i	-0.0205 - 0.0372i	-0.0001 + 0.0003i	-0.0002 - 0.0004i
11	$\Delta E'_{q1}$	-0.0006 + 0.0031i	-0.0061 + 0.0314i	-0.0001 - 0.0003i	-0.0001 + 0.0003i
12	ΔE_{fd1}	-0.0006 - 0.0031i	-0.0061 - 0.0314i	-0.0000	-0.0001 - 0.0003i
13	$\Delta X5_1$	-0.0001	-0.0013	-0.0001	-0.0000
14	ΔUE_1	-0.0006	-0.0056	-0.0001 + 0.0005i	-0.0001
15	$\Delta\omega_2$	-0.0006 + 0.0047i	-0.0057 + 0.0470i	-0.0001 - 0.0005i	-0.0001 + 0.0005i
16	$\Delta\delta_2$	-0.0006 - 0.0047i	-0.0057 - 0.0470i	-0.0002 + 0.0004i	-0.0001 - 0.0005i
17	$\Delta E'_{q2}$	-0.0021 + 0.0037i	-0.0212 + 0.0371i	-0.0002 - 0.0004i	-0.0002 + 0.0004i
18	ΔE_{fd2}	-0.0021 - 0.0037i	-0.0212 - 0.0371i	-0.0001	-0.0002 - 0.0004i
19	$\Delta X5_2$	-0.0006	-0.0058	-0.0000	-0.0001
20	ΔUE_2	-0.0001	-0.0013	-0.0007 + 0.0009i	-0.0000
21	$\Delta\omega_3$	-0.0067 + 0.0091i	-0.0669 + 0.0906i	-0.0007 - 0.0009i	-0.0007 + 0.0009i
22	$\Delta\delta_3$	-0.0067 - 0.0091i	-0.0669 - 0.0906i	-0.0002 + 0.0002i	-0.0007 - 0.0009i

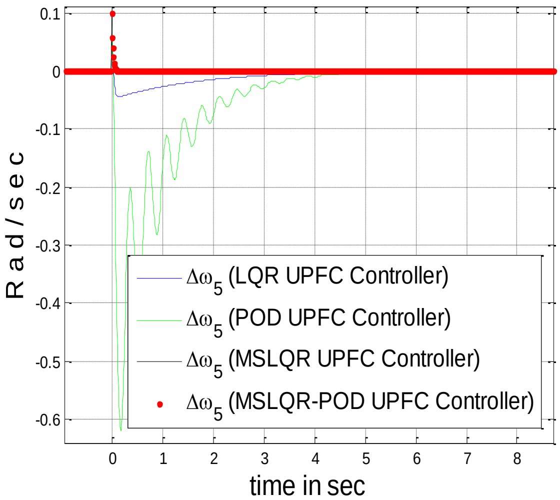
23	$\Delta E'_{q3}$	-0.0018 + 0.0015i	-0.0175 + 0.0153i	-0.0002 - 0.0002i	-0.0002 + 0.0002i
24	ΔE_{fd3}	-0.0018 - 0.0015i	-0.0175 - 0.0153i	-0.0001	-0.0002 - 0.0002i
25	$\Delta X5_3$	-0.0007	-0.0066	-0.0000	-0.0001
26	ΔUE_3	-0.0001	-0.0013	-0.0000 + 0.0004i	-0.0000
27	$\Delta\omega_4$	-0.0002 + 0.0039i	-0.0020 + 0.0388i	-0.0000 - 0.0004i	-0.0000 + 0.0004i
28	$\Delta\delta_4$	-0.0002 - 0.0039i	-0.0020 - 0.0388i	-0.0003 + 0.0002i	-0.0000 - 0.0004i
29	$\Delta E'_{q4}$	-0.0025 + 0.0024i	-0.0252 + 0.0235i	-0.0003 - 0.0002i	-0.0003 + 0.0002i
30	ΔE_{fd4}	-0.0025 - 0.0024i	-0.0252 - 0.0235i	-0.0000	-0.0003 - 0.0002i
31	$\Delta X5_4$	-0.0003	-0.0034	-0.0001	-0.0000
32	ΔUE_4	-0.0006	-0.0060	-0.0002 + 0.0004i	-0.0001

3.6.2.3 Case III: Integrated MSLQR-POD UPFC Controller

(PSS to all the generators and UPFC between area 5 and area 6)- at time t_1=0 sec, Load in area 5 and 6 is L1=1.0 p.u. with p.f.=0.85 and at time t_2=1.0 sec load increased up to 30% (L=1.3 p.u.) with p.f.=0.85.

State variable's perturbation response with proposed controller concept is shown in Figure 3.14 and Table 3.6 shows the peak overshoot/undershoot, settling time and eigenvalues of all the controllers for six area system. Table 3.7 shows the eigenvalue of the complete system for different controllers. The vertical strip assigned for the eigenvalues by POD controller is [-1, -1.5] for UPFC. The designer may select the suitable strip depending upon system requirements. The proposed concept shows the effectiveness by improving the system stability.





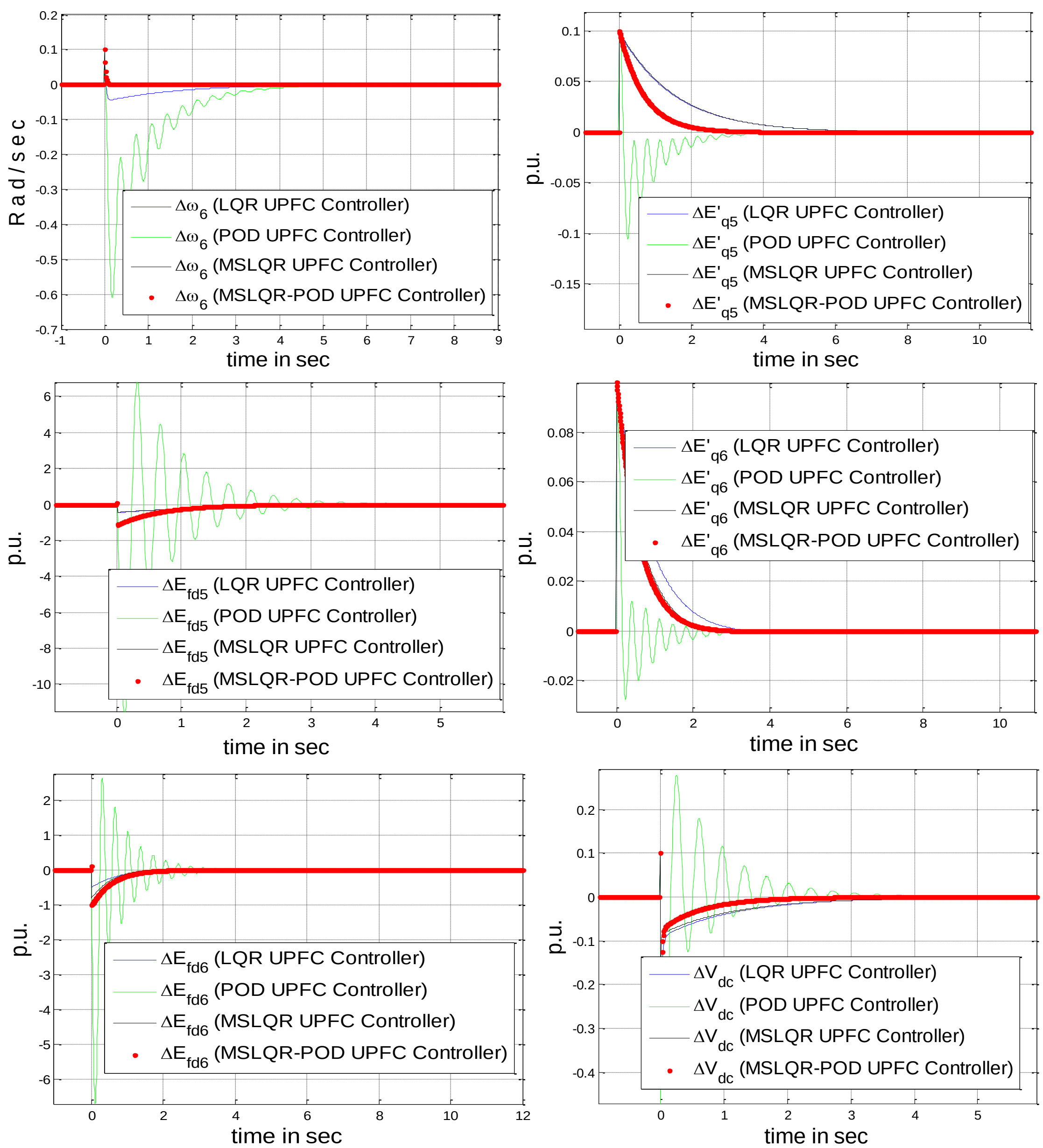


Figure 3.14 Perturbation response of Six Area system for different controllers with UPFC (Area 5 and Area 6) - Case III

TABLE 3.6: Comparison of system response for Six Area system with UPFC - Case III

System	States	Over-shoot	Settling time (sec)	Eigenvalues 1.0e+03*
UPFC with LQR Controller	$\Delta\delta$ (degree)	0.07	2.00	-.42413
	$\Delta\omega_5$ (Rad/sec)	-0.045	2.50	-.40308
	$\Delta\omega_6$ (Rad/sec)	-0.045	2.50	-.02347
	$\Delta E'_{q5}$ (p.u.)	0.10	4.30	-.01052 + .00725i
	ΔE_{fd5} (p.u.)	-1.10	3.50	-.01052 - .00725i
	$\Delta E'_{q6}$(p.u.)	0.10	3.10	-.00404
	ΔE_{fd6}(p.u.)	-0.50	3.90	-.00010
	ΔV_{dc}(p.u.)	-0.28	3.50	-.00007
UPFC with POD	$\Delta\delta$ (degree)	0.22	2.70	-.007461+.030585i
	$\Delta\omega_5$ (Rad/sec)	-0.61	4.10	-.007461-.030585i
	$\Delta\omega_6$ (Rad/sec)	-0.61	4.10	-.000125+.001804i
	$\Delta E'_{q5}$ (p.u.)	-0.11	3.90	-.000125-.001804i

(EAT) Controller	ΔE_{fd5} (p.u.)	-11.3	1.20	-.001630
	$\Delta E'_{q6}$(p.u.)	0.10	2.60	-.000855+.000832i
	ΔE_{fd6}(p.u.)	-6.50	3.50	-.000855-.000832i
	ΔV_{dc}(p.u.)	-0.50	3.00	-.000100
UPFC with MSLQR Controller	$\Delta\delta$ (degree)	0.050	1.20	-.65700
	$\Delta\omega_5$ (Rad/sec)	0.054	1.50	-.37625
	$\Delta\omega_6$ (Rad/sec)	0.054	1.50	-.06205
	$\Delta E'_{q5}$ (p.u.)	0.10	4.30	-.02509 + .01730i
	ΔE_{fd5} (p.u.)	-0.41	1.20	-.02509 - .01730i
	$\Delta E'_{q6}$(p.u.)	0.10	2.30	-.02317
	ΔE_{fd6}(p.u.)	-0.80	3.80	-.00014
	ΔV_{dc}(p.u.)	-0.24	3.00	-.00007
Integrated MSLQR-POD UPFC Controller [Proposed]	$\Delta\delta$ (degree)	0.003	0.80	-5.0554
	$\Delta\omega_5$ (Rad/sec)	0.06	0.01	-0.6716
	$\Delta\omega_6$ (Rad/sec)	0.06	0.01	-0.0666
	$\Delta E'_{q5}$ (p.u.)	0.10	2.10	-0.0553
	ΔE_{fd5} (p.u.)	-0.41	1.20	-0.0304
	$\Delta E'_{q6}$(p.u.)	0.10	2.30	-0.0056
	ΔE_{fd6}(p.u.)	-0.10	2.20	-0.0002
	ΔV_{dc}(p.u.)	-0.24	1.90	-0.0001

Table 3.7: Eigenvalues for all the generators in each area (UPFC) - Case III

System States		**LQR UPFC Controller *1.0e+003**	**POD UPFC Controller *1.0e+002**	**MSLQR UPFC Controller *1.0e+003**	**MSLQR-POD UPFC Controller *1.0e+004**
1	$\Delta\delta$	-4.2413	-0.7461 + 3.0585i	-6.5700	-5.0554
2	$\Delta\omega_5$	-4.0308	-0.7461 - 3.0585i	-3.7625	-0.6716
3	$\Delta\omega_6$	-0.2347	-0.0125 + 0.1804i	-0.6205	-0.0666
4	$\Delta E'_{q5}$	-0.1052 + 0.0725i	-0.0125 - 0.1804i	-0.2509 + 0.1730i	-0.0553
5	ΔE_{fd5}	-0.1052 - 0.0725i	-0.1630	-0.2509 - 0.1730i	-0.0304
6	$\Delta E'_{q6}$	-0.0404	-0.0855 + 0.0832i	-0.2317	-0.0056
7	ΔE_{fd6}	-0.0010	-0.0855 - 0.0832i	-0.0014	-0.0002
8	ΔV_{dc}	-0.0007	-0.0100	-0.0007	-0.0001
9	$\Delta\omega_1$	-0.0021 + 0.0038i	-0.0213 + 0.0377i	-0.0021 + 0.0038i	-0.0002 + 0.0004i
10	$\Delta\delta_1$	-0.0021 - 0.0038i	-0.0213 - 0.0377i	-0.0021 - 0.0038i	-0.0002 - 0.0004i
11	$\Delta E'_{q1}$	-0.0005 + 0.0031i	-0.0054 + 0.0312i	-0.0005 + 0.0031i	-0.0001 + 0.0003i
12	ΔE_{fd1}	-0.0005 - 0.0031i	-0.0054 - 0.0312i	-0.0005 - 0.0031i	-0.0001 - 0.0003i
13	$\Delta X5_1$	-0.0001	-0.0013	-0.0001	-0.0000
14	ΔUE_1	-0.0003	-0.0033	-0.0003	-0.0000
15	$\Delta\omega_2$	-0.0006 + 0.0046i	-0.0056 + 0.0456i	-0.0006 + 0.0046i	-0.0001 + 0.0005i
16	$\Delta\delta_2$	-0.0006 - 0.0046i	-0.0056 - 0.0456i	-0.0006 - 0.0046i	-0.0001 - 0.0005i
17	$\Delta E'_{q2}$	-0.0021 + 0.0039i	-0.0214 + 0.0388i	-0.0021 + 0.0039i	-0.0002 + 0.0004i
18	ΔE_{fd2}	-0.0021 - 0.0039i	-0.0214 - 0.0388i	-0.0021 - 0.0039i	-0.0002 - 0.0004i
19	$\Delta X5_2$	-0.0004	-0.0037	-0.0004	-0.0000
20	ΔUE_2	-0.0001	-0.0013	-0.0001	-0.0000
21	$\Delta\omega_3$	-0.0066 + 0.0090i	-0.0655 + 0.0901i	-0.0066 + 0.0090i	-0.0007 + 0.0009i
22	$\Delta\delta_3$	-0.0066 - 0.0090i	-0.0655 - 0.0901i	-0.0066 - 0.0090i	-0.0007 - 0.0009i
23	$\Delta E'_{q3}$	-0.0019 + 0.0014i	-0.0189 + 0.0141i	-0.0019 + 0.0014i	-0.0002 + 0.0001i
24	ΔE_{fd3}	-0.0019 - 0.0014i	-0.0189 - 0.0141i	-0.0019 - 0.0014i	-0.0002 - 0.0001i
25	$\Delta X5_3$	-0.0007	-0.0071	-0.0007	-0.0001
26	ΔUE_3	-0.0001	-0.0013	-0.0001	-0.0000
27	$\Delta\omega_4$	-0.0053	-0.0533	-0.0053	-0.0005
28	$\Delta\delta_4$	-0.0004 + 0.0050i	-0.0038 + 0.0498i	-0.0004 + 0.0050i	-0.0000 + 0.0005i
29	$\Delta E'_{q4}$	-0.0004 - 0.0050i	-0.0038 - 0.0498i	-0.0004 - 0.0050i	-0.0000 - 0.0005i
30	ΔE_{fd4}	-0.0006 + 0.0015i	-0.0060 + 0.0153i	-0.0006 + 0.0015i	-0.0001 + 0.0002i
31	$\Delta X5_4$	-0.0006 - 0.0015i	-0.0060 - 0.0153i	-0.0006 - 0.0015i	-0.0001 - 0.0002i
32	ΔUE_4	-0.0003	-0.0031	-0.0003	-0.0000

The proposed controller has been tested with changing load conditions for sample six area power system. The investigations reveal that the overshoot and settling time of the proposed controller are much less as compared to other controllers reported earlier and well within the acceptable limits. The controller can readily be extended to include large numbers of multi-area system with some augmentation which may be derived on a modular basis and including interface dynamics of large systems. The simulation results show the capability of the proposed controller in terms of better stability and quick response to system operation. Since the system keeps on changing over a period of time, this strategy may prove beneficial for developing a modular controller over the existing control structure.

3.6.3 Proposed Modified-Multi Stage LQR Controller (M-MSLQR)

All the state variables responses are influenced by the location of each eigenvalues. Particular locations of eigenvalues are desired depending upon the system operating conditions. Power oscillation damping can be improved if real part of eigenvalue associated with mode of oscillation can be shifted to left side in complex plane. In Multi-Input-Multi-Output (MIMO) systems vulnerable states are identified by closed loop poles of the system, i.e., system eigenvalues and states which are vulnerable are interlinked. These states may affect the performance of the system and may introduce instability. State predominant approach is a novel control concept by which vulnerable states can be regulated with some adequate control decisions by assessing the most dominant eigenvalues and generating new control by changing the elements of Q matrix diagonal entry and calculating feedback gain afresh for ensuring system stabilization. In general, the concept of calculating the gain K is for small perturbations; however, for large and dynamic changing systems, design based on state predominant approach may be extended with multiple iterations along with analysis of eigenvalues for system stability. Multi-Stage LQR (MSLQR) controller takes a state predominant concept only looking the real part of eigenvalues and taking the eigenvalues which is very close to the imaginary plane and redefining the Q matrix diagonal entry of concerned states. Such identified states are predominant states and the regulation of such large deviated states can be ensured by assigning high weight in respective diagonal entries of Q matrix by proposed concept.

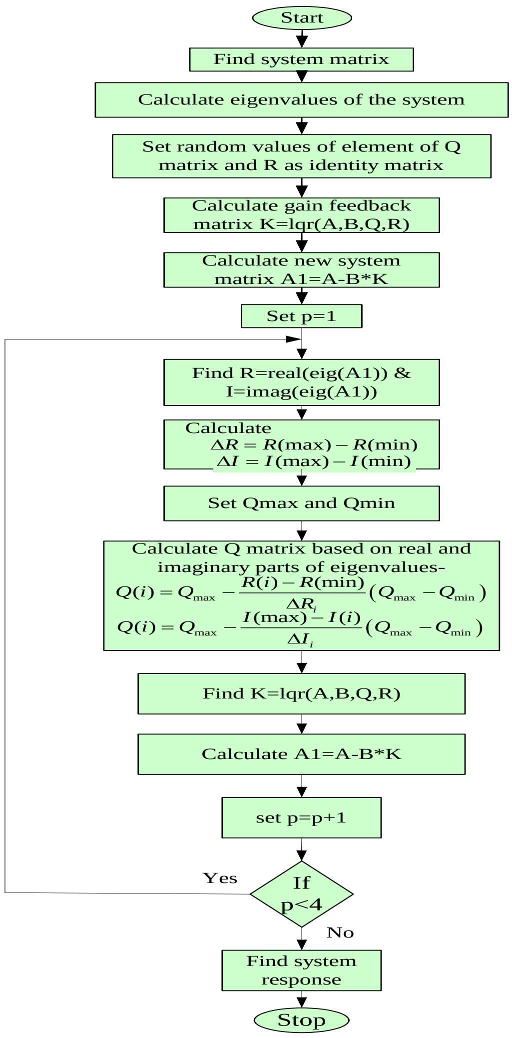


Figure 3.15 Flowchart for Modified-Multi Stage LQR controller

In proposed Modified-Multi Stage LQR (M-MSLQR) controller, assigning diagonal entries of Q matrix is based on the value of real part as well as the imaginary part of the eigenvalues. It’s an iterative process where at each step elements of Q matrix are assigned based on the values of complex eigenvalues. The iterative process has to be repeated till all deviated states are tracked to desired criterion. In general, R matrix is maintained as identity matrix. The design of such M-MSLQR controller results to the quick response regulation for a given system perturbation. Figure 3.15 shows the flowchart for design of Modified-Multi Stage LQR controller.

3.6.3.1 Design of Modified-Multi Stage LQR Controller

1) Obtain the array of gain K with some initialized Q matrix and R as identity matrix.
2) New state matrix A1 is obtained as (A1=A-B*K) and find the eigenvalue of A1 (eig(A1)).
3) Calculate new matrix Q1 with the help of eig(A1) by seeing real part of eigenvalues.

$$Q(i) = Q_{max} - \frac{R(i) - R(\min)}{\Delta R_i}\left(Q_{max} - Q_{min}\right)$$

Then update the element of Q according to the imaginary part of the eigenvalues

$$Q(i) = Q_{max} - \frac{I(\max) - I(i)}{\Delta I_i}\left(Q_{max} - Q_{min}\right)$$

4) New gain matrix K1 is obtained from K1=lqr(A1,B,Q1,R) and new system matrix is formed with combination of plant and controller (as A2=A1-(B*K1)) and eigenvalues (eig(A2)) which create oscillations are identified. Set Q2 matrix by using step 3.
5) The gain matrix K2 is obtained from K2=lqr(A2,B,Q2,R) then A2 is transformed into A3 and again eigenvalues have identified. This process will continue depending on the required specification.

Case Study IV: Two area sample power system (refer Figure 3.4) with UPFC has been analyzed with multi-stage LQR controller and Modified-multi stage LQR controller. Table 3.8 and 3.9 shows the overshoot/undershoot, settling time and eigenvalues of the system with 1.1 p.u. and 0.8 p.f. loading conditions. Perturbed response of state variables of the system for both the controllers have been shown in Figure 3.16.

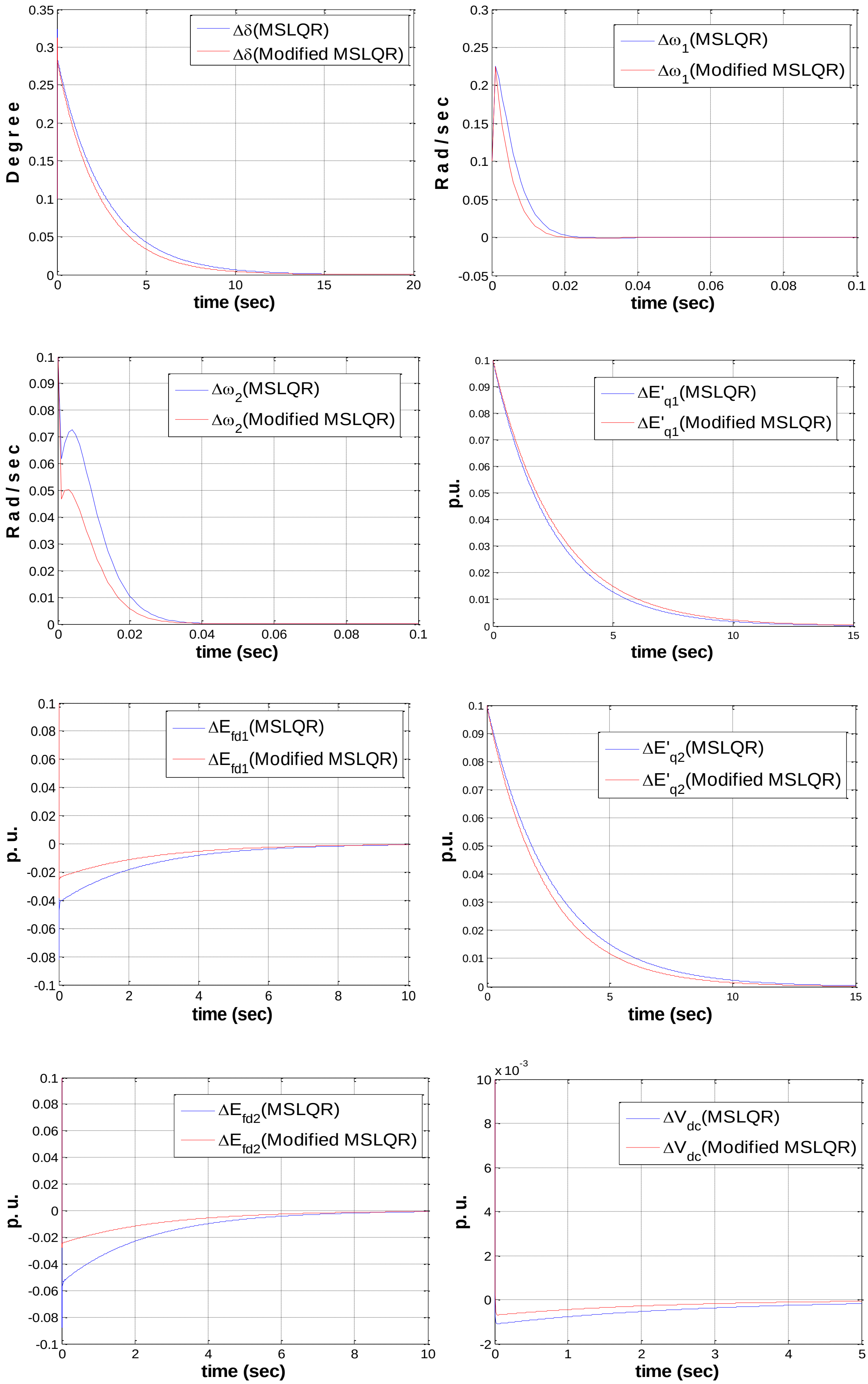


Figure 3.16 Variation in all state variables for both controllers with UPFC - Case IV

Table 3.8: Comparison in system responses for MSLQR and M-MSLQR controllers - Case IV

UPFC Control Parameters: m_e=0.85; δ_e=0.0157; m_b=0.85; δ_b=0.2863				
States	**Multi Stage LQR**		**Modified Multi Stage LQR**	
Load 1.1 p.u 0.8 pf (lag)	**Overshoot**	**Settling time**	**Overshoot**	**Settling time**
$\Delta\delta$	0.32	10.6	0.31	9.5
$\Delta\omega_1$	0.23	0.019	0.23	.015
$\Delta\omega_2$	0.074	0.032	0.05	0.027
$\Delta E'_{q1}$	0.1	10.5	0.1	10.7
ΔE_{1fd}	-0.058	5.2	-0.04	4.5
$\Delta E'_{q2}$	0.1	12.5	0.1	12
ΔE_{2fd}	-0.058	5.6	-0.22	5.1
ΔV_{dc}	0.0005	4.2	0.001	3.1

Table 3.9: Eigenvalues for both controllers - Case IV

	Multi Stage LQR 1.0e+004*	**Modified Multi Stage LQR** 1.0e+005*
$\Delta\delta$	-8.8800	-1.1034
$\Delta\omega_1$	-0.7552	-0.1972
$\Delta\omega_2$	-0.1537	-0.0681
$\Delta E'_{q1}$	-0.0328	-0.0161
ΔE_{fd1}	-0.0197 + 0.0048i	-0.0019 + 0.0006i
$\Delta E'_{q2}$	-0.0197 - 0.0048i	-0.0019 - 0.0006i
ΔE_{fd2}	-0.0000	-0.0000
ΔV_{dc}	-0.0000	-0.0000

Modified-MSLQR controller considers state predominant approach with real part as well as the imaginary part of the eigenvalues of the system for the selection of Q matrix. An improved controller is proposed as Modified-Multi Stage LQR controller over MSLQR controller in terms of peak overshoot/undershoot and settling time of state variables of sample system. The proposed controller driving FACTS controller, can readily be extended for multi-area system with some augmentation related with interface dynamical changes suitably incorporated. The simulation results show the performance of proposed controller in terms of quick stabilization. The effectiveness of proposed controller can be seen as an enhancement in the oscillation damping of two area sample power system which may be applied for even larger area network modularly.

3.7 CONCLUSION

This chapter presents the complete modeling of generators with Power System Stabilizers (PSS) and FACTS devices in power network. The development of concept is modular where any control structure can be embedded with detailed representation in respective blocks. It includes the stepwise procedure for all the components connected in the system with differential equations and finally all the equations are converted in state space framework. With the help of state space model, three sample test systems (Two Area Four Machine System, Six Area Six Machine Test System and Ten Area Fifty Machine Test System) have been developed to demonstrate the effectiveness of the proposed concept and has been used in next chapters for controller design. Also, the basic concept of LQR controller and its integration with FACTS devices has been included. A new integrated MSLQR-POD controller and Modified MSLQR controller has been developed and comparisons show the better performance of the controller in terms of settling time and overshoot/undershoot over other controllers (such as LQR controller, POD controller and MSLQR controller).