PREFACE

This thesis is concerned with the construction and solution of various models of reaction-diffusion problems by operational matrix approaches with collocation scheme. This thesis contains six chapters. In chapter 1, an overview of the field's literature and some fundamental terminology are included. Since diffusion models are taken into consideration in porous media, therefore first chapter starts with the brief discussion of porous media. Additionally, a brief explanation of the advection-reaction diffusion equation is provided, because the solution of this kind of diffusion equation is the main goal of the work.

In second chapter, the author provides the numerical solution for a non-linear reaction-advection diffusion equation with fractional-order space-time derivatives in a finite domain. In the proposed scheme, time fractional derivative in Caputo sense is approximated by using the non-standard finite difference method and the fractional space derivative is specifically approximated by using Vieta-Fibonacci polynomials. These approximations generate a system of ordinary differential equations which is converted into an equivalent system of algebraic equations by using collocation method. Finally, the obtained system of algebraic equations is solved to find the dependent variables (unknowns) of the considered problem. The stability and convergence related to the time discreatization of this approach are also discussed. In this chapter, the effectiveness and precision of the proposed scheme are analyzed with the help of examples, and it is observed that the proposed scheme is sufficiently accurate and efficient technique. Also, the effects of fractional-order derivatives on concentration profiles are discussed.

In the chapter 3, the author presents a novel numerical algorithm based on the operational matrix and collocation approach for solving the two-dimensional non-linear fractional reaction-advection diffusion equations of variable order that is related to the groundwater pollution problem. In this chapter, the operational matrices of variable-order derivatives are derived with the aid of Vieta-Lucas polynomials for two-dimension problem which is used in the proposed algorithm. To find a solution to the considered problem, the terms are approximated by a series of triple-shifted Vieta-Lucas polynomials to construct the residual of the reaction-advection diffusion model. Then collocation technique is applied to transform the problem into algebraic equations, which is solved by Newton's method. Moreover, the convergency and upper bound of the derived error formula for the approximate solution are discussed. Finally, some examples are presented to show the efficiency of the considered scheme, and the results are shown by using graphical presentation and tabular representations.

In chapter 4, the author presents a reaction-advection-diffusion model which describes many physical phenomena, such as the transportation of particles, groundwater pollution, viscoelasticity, and many others. In this chapter, a well-known fractional operator of variable order is used to present the space-time variable-order reaction-advection-diffusion equation. The operational matrix of the variable order derivative is developed with the aid of shifted Vieta-Fibonacci polynomials. This operational matrix is used in the approximation of derivatives of variable order to construct residual associated with the considered problem, and then it is collocated at some points in the domain, which generates a system of non-linear algebraic equations. Newton's method is applied to solve the obtained system of non-algebraic equations. To validate the precision of the proposed scheme, some problems are solved by the proposed scheme, and its comparisons are made with the existing analytical solution, which clearly indicates the improved accuracy of the proposed method. The convergence of the scheme and error analysis are also discussed in this chapter.

In the chapter 5, the one and two-dimensional nonlinear fractional fourth-order sub-diffusion models with neumann boundary are discussed. The numerical solution of the problems are obtained by using a novel approach which depend on shifted Airfoil polynomials of second kind and collocation technique. An operational matrix of differentiation for shifted Airfoil polynomials is derived and applied to deal with the time and space fractional derivative of variable-order. The convergency of the scheme for presented numerical solution and error analysis are also discussed. In this chapter, the author also analyze the order of convergence for the solution of considered problem. For validation, this numerical approach has been discussed with some illustrative problems. The results of the error analysis are also presented through tables, which clearly indicate the improved accuracy of the proposed method.

In the Chapter 6, overall work has been concluded and also the future scope of the related works has been furnished.