Appendix C

Chapter 4

C-I Partitions

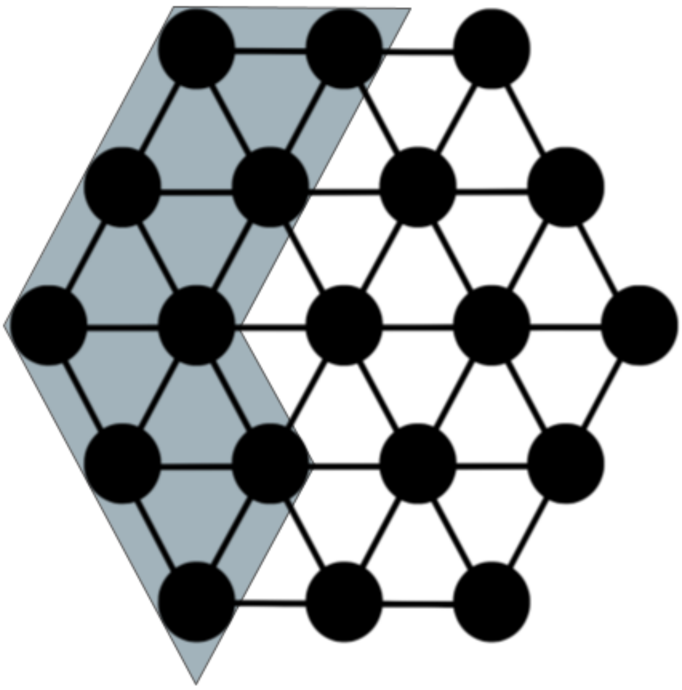

Fig. C.1 The partitioning of the system for computing the entanglement entropy. Shaded region visually represents the subsystem A considered for calculating $\mathscr{S}_A$.

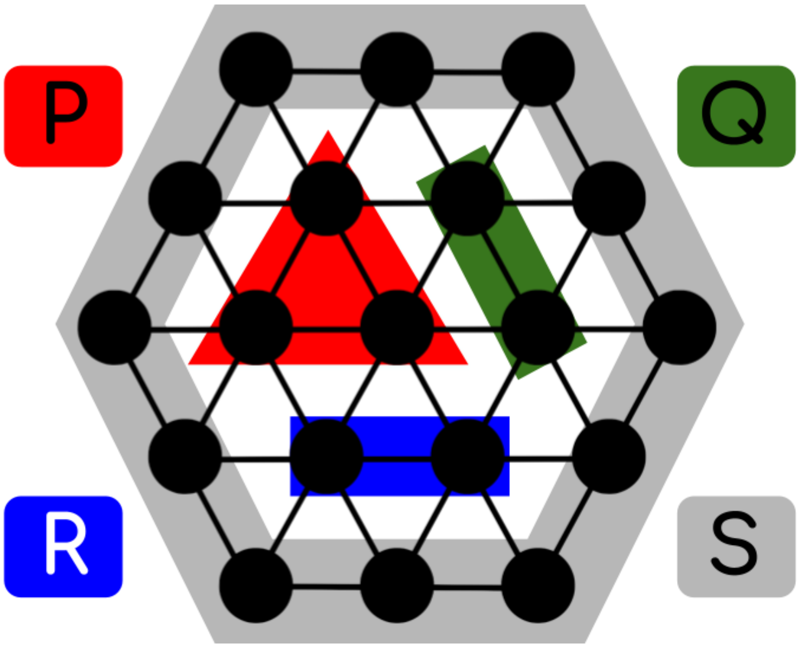

Fig. C.2 The partitioning of the system for computing the topological entanglement entropy (TEE) is depicted in the figure. Each subsystem, identified by different colors, corresponds to *P*, *Q*, *R*, and *S*.

C-II Rate of change of entropy compared with rate function for another model undergoing DQPT

We compared the rate function and rate of change of entropy for another helical model, which demonstrates DQPT. Consider a square lattice of interacting quantum spin $1/2$s coupled to classical control fields at its boundary similar to [C237]. This model is inspired by the work by Spethmann *et al.* [C294], where they modify the film edge of the skyrmion formed by atomic Pd/Fe bilayer on Ir(111) with ferromagnetic Co/Fe patches. This ferromagnetic rim which is the boundary of the skyrmion is referred to as the classical control field. The Hamiltonian of the system is as follows:

$$\begin{aligned}\hat{H} = & -J\sum_{\langle i,j\rangle}(\hat{S}_i^x\hat{S}_j^x+\hat{S}_i^y\hat{S}_j^y)-\Delta\sum_{\langle i,j\rangle}\hat{S}_i^z\hat{S}_j^z \\ & -D\sum_{\langle i,j\rangle}(\mathbf{e_{ij}}\times\hat{z}).\left(\hat{\mathbf{S}}_i\times\hat{\mathbf{S}}_j\right).\end{aligned} \tag{C.1}$$

With a ferromagnetic coupling constant $J=1$, the axial Heisenberg anisotropy $\Delta>0$ and Dzyaloshinskii Moria interaction (DMI) strength D, where $\vec{e}_{ij}$ is the unit vector pointing from $\vec{\hat{S}}_i$ to $\vec{\hat{S}}_j$. The $\vec{\hat{S}}_i=\frac{\hbar}{2}\vec{\hat{\sigma}}_i$ corresponds to a vector of spin operators (vector of Pauli matrices) $\vec{\hat{\sigma}}_i=\left(\sigma_i^x,\sigma_i^y,\sigma_i^z\right)$. The summation runs over all the unique pairs of nearest neighbors. The last term in the Hamiltonian Eq.C.1: $D\sum_{\langle i,j\rangle}(\mathbf{e_{ij}}\times\hat{z}).\left(\hat{\mathbf{S}}_i\times\hat{\mathbf{S}}_j\right)$ describes coupling between electric field E and ferroelectric polarization $\hat{P}=\sum_{\langle i,j\rangle}(\mathbf{e_{ij}}\times\hat{z}).\left(\hat{\mathbf{S}}_i\times\hat{\mathbf{S}}_j\right)$, where $D=E\gamma_{ME}$ is the effective DM constant and γ_{ME} is the magneto electric coupling.

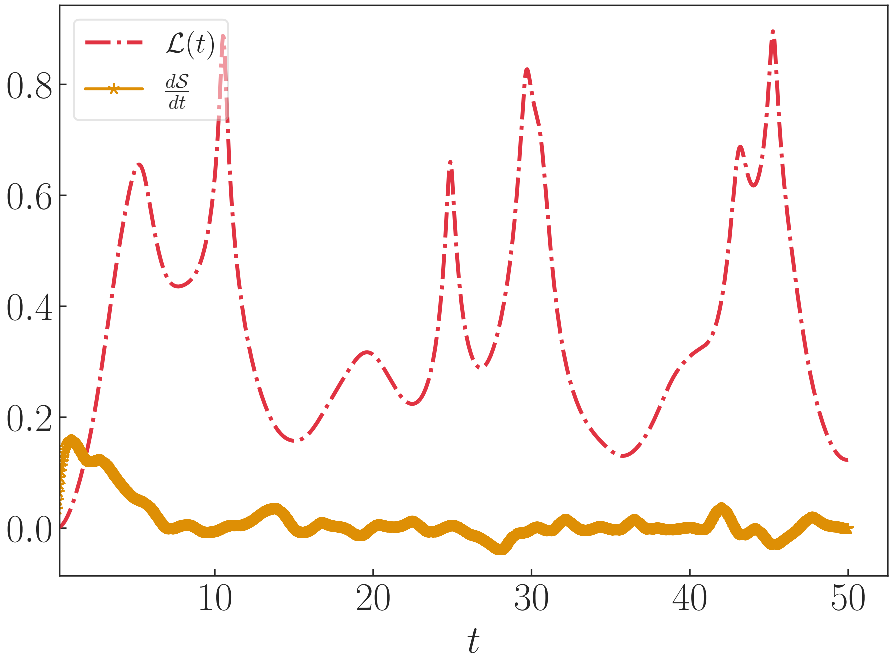

Fig. C.3 EE production rate and rate function compared for system size 3×3.

For a 3×3 lattice when $\Delta=0.5J$ and $D=2J$ we identified a quantum skyrmion phase. We note that parameters $\Delta=0.5J$ and $D=0$ give a quantum ferromagnet. Fig. C.3 shows the comparison between the rate function and rate of change of entanglement entropy for a quench from the skyrmion state to ferromagnetic state. Similar observations were made with 4×4 lattice case, where parameters $\Delta = 0.5J$ and $D = 2J$ form a skyrmion state. $\Delta = 0.5J$ and $D = 0$ form a ferromagnetic ground state. Fig. C.4 shows the comparison between the rate function and rate of change of entanglement entropy for a quench from the skyrmion state to ferromagnetic state. For each of these system sizes, we initiate the system at the skyrmion state at $t = 0$, subsequently quench the system to ferromagetic state by setting $D = 0$.

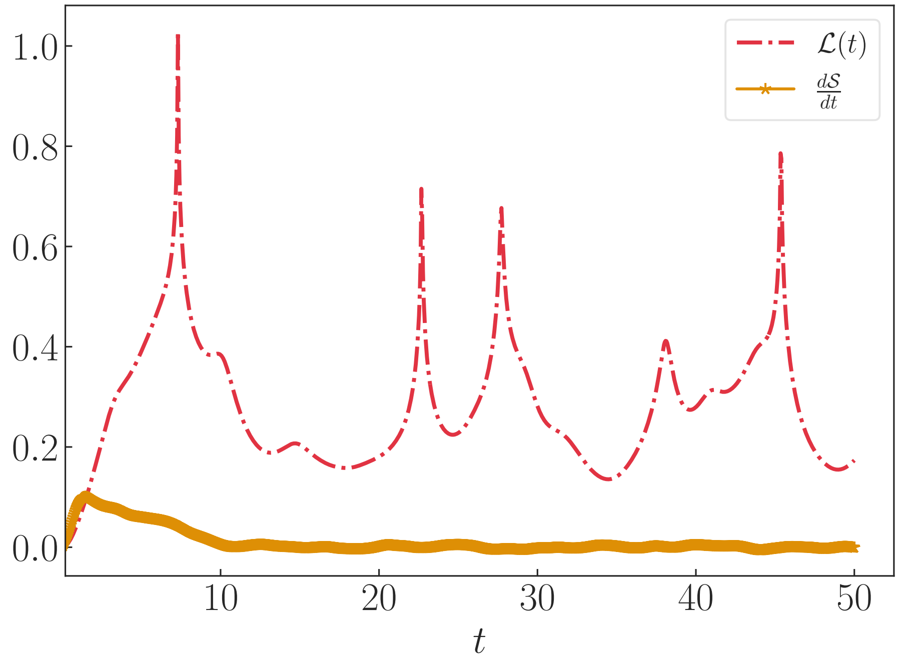

Fig. C.4 EE production rate and rate function compared for a quantum spin system formed in a 4×4 square lattice.