

CHAPTER

5

Chapter 5. Modeling, Identification and IMC based PID Control of Single Input Single Output Stable Nonlinear Process

5.1. Introduction

Single Input Single Output (SISO) stable nonlinear processes are encountered at several stages of process industries, such as hoppers for granular materials, storage vessels having conical bottoms for viscous fluids, clarifiers and thickeners, etc. Appropriate IMC based PID control strategy is essential for efficient control of operations under such situations. In view of this, in this section of the present work, an experimental nonlinear conical tank liquid level process has been used to simulate the process and arrive at appropriate control strategy.

5.2 Experimental Annular Flow Conical Tank Liquid Level Nonlinear Process

Conical vessels are extensively used in process industries as their conical shape permits better flow of solid mixtures, slurries and viscous liquids. The process nonlinearity in conical tanks is caused by (a) its constantly varying cross-sectional area and (b) the nonlinear flow resistance. Sufficient published information is available in literature that address the conical tank level control methodologies based on simulation results of selected process models (Pandian & Noel, 2020). While the simulation results may offer satisfactory control, but there are deviations when implemented on practical experimental setups. This may be attributed to the variation in the dynamics associated with the various components of the setup and also to the process noise associated with the instrumentation

(Patranabis, 2001). The present work addresses both to the simulation studies as well as to the experimental validation on laboratory scale physical setup.

5.2.1 Process Description

The experimental setup designed by Apex Innovations Pvt. Ltd. and used in this work is shown in Figure 5.1 and the relevant technical specifications of the setup are shown in Table 5.1. The control objective is to maintain the liquid level (as measured by a Level Transmitter) at the desired steady-state by manipulating the inlet volumetric flowrate to the tank. The setup is interfaced to a computer which records all the desired process variables and implements the PID Control action on the process. The setup consists of a reservoir tank, submersible pump, pneumatic control valve (linear, air to close), I to P converter, annular conical tank and level sensor. The schematic process diagram of the setup is shown in Figure 5.2.

Table 5. 1 Specification of Annular Flow Conical Tank Liquid Level setup

Product	Nonlinear level control trainer
Type of control	PID
Level transmitter	Range 0–250 mm, Output 4–20 mA
I/P converter	Input 4-20mA, Output 3-15 psig.
Control valve	Pneumatic type, Linear Characteristic, Direct Acting, Size 1/4", Input 3–15 psig.
Rotameter	10-100 LPH, Make: Eureka
Pump	Submersible type having fractional horse power.
Process tank	Acrylic cylindrical with cylindrical to linear conical conversion with 0-100% graduated scale.



Figure 5. 1 Experimental setup of Conical Tank Process

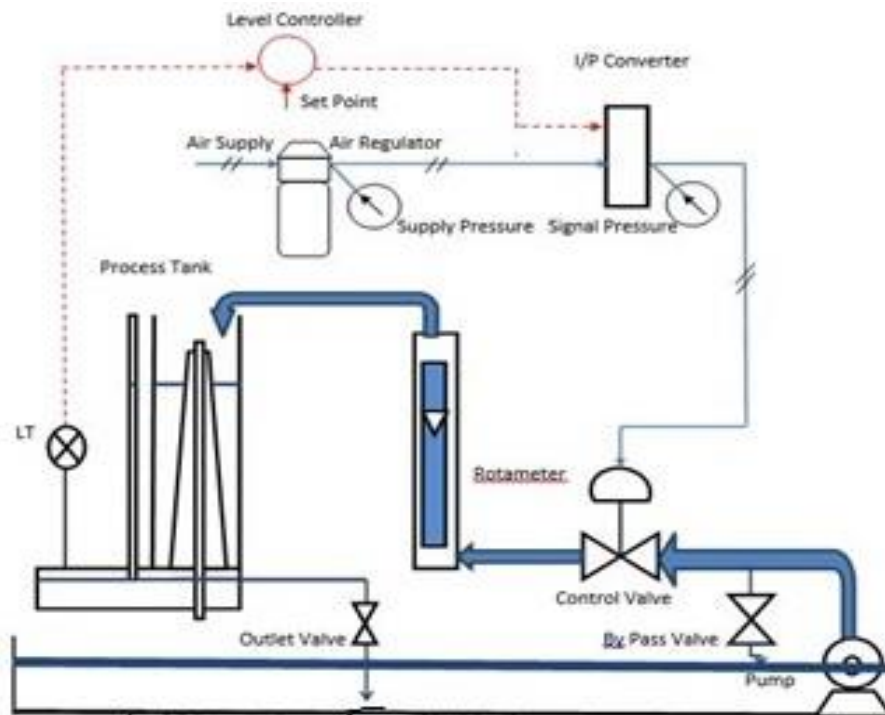


Figure 5. 2 Schematic diagram of Conical Tank Process

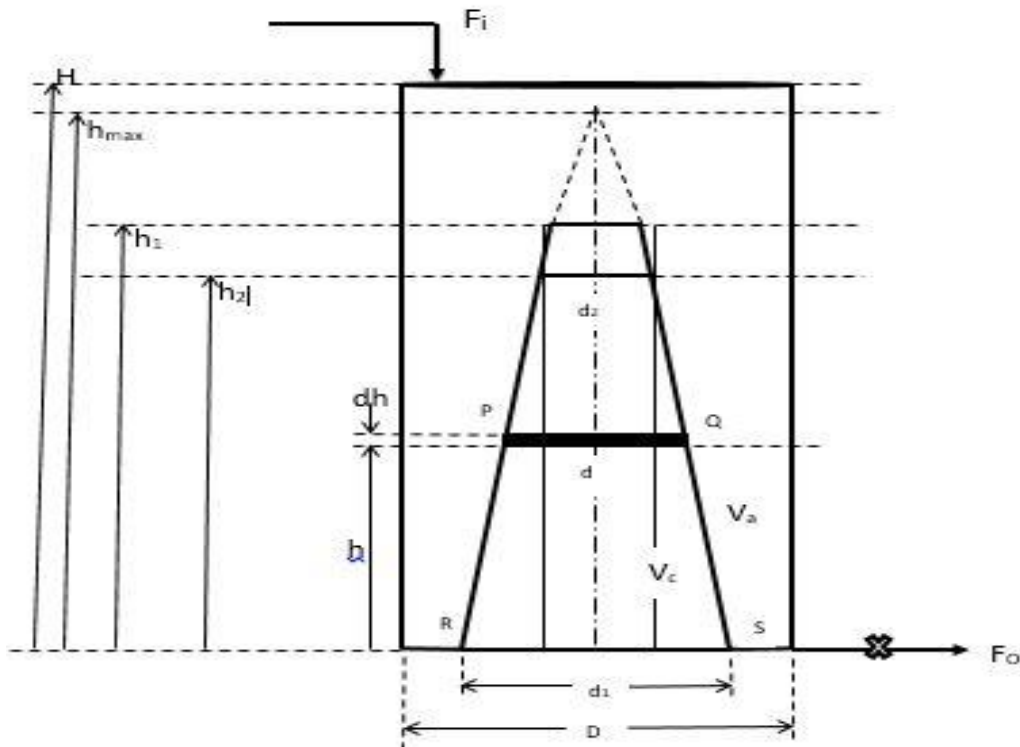


Figure 5.3 Process Model of Conical Tank Liquid Level Process

5.3 Mathematical Modeling of Annular Flow Conical Tank Process

5.3.1 Process Variables and Parameters

Consider the annular flow conical tank process model as shown in Figure 5.3.

The input-output variables and parameters associated with the process model are shown as under:

H = Height of cylindrical tank = 290mm

D = Diameter of the cylindrical tank = 92mm

V_c = Volume of cone

V_a = Volume of annulus

V_T = Volume of cylindrical tank

F_i = Volumetric flowrate of inlet stream (LPH)

F_o = Volumetric flowrate of the outlet stream (LPH)

h_{\max} = Height of the cone

h_1 = 255mm

h_2 = 245mm

d_1 = Bottom diameter of the cone= 88mm

d_2 = Top diameter of the cone= 25mm

β = Nonlinear flow resistance

ρ = Density of fluid

5.3.2 Unsteady State Mass balances

Consider the differential volume of cone at any height:

$$dV_c = \frac{\pi d^2}{4} dh \quad \text{Eq.5. 1}$$

Assuming linear variation of diameter with respect to the height of cone

$$d(h) = a_1 h + a_2 \quad \text{Eq.5. 2}$$

The coefficients a_1 & a_2 are evaluated using the boundary conditions:

Boundary condition 1: at $h = 0$, $d = d_1$ gives:

$$a_2 = d_1 \quad \text{Eq.5. 3}$$

Boundary condition 2: at $h = h_2$, $d = d_2$ gives:

$$a_1 = \frac{d_2 - d_1}{h_2} \quad \text{Eq.5. 4}$$

The maximum height h_{\max} is evaluated using equation 5.2, when $d = 0$:

$$h_{\max} = \frac{d_1 h_2}{d_1 - d_2} \quad \text{Eq.5. 5}$$

Total volume of cone is obtained by integrating equation 5.1

$$\int dV_c = \int \pi(a_1 h + a_2)^2 dh \quad \text{Eq.5. 6}$$

$$V_c = \frac{\pi(a_1 h + a_2)^3}{12a_1} + C_1 \quad \text{Eq.5. 7}$$

The constant of integration, C_1 is evaluated using the Boundary condition:

Boundary condition 3: at $h = 0$; $V_c = 0$

$$C_1 = -\frac{\pi a_2^3}{12a_1} \quad \text{Eq.5. 8}$$

Substituting into equation 5.7:

$$V_c = \frac{\pi(a_1 h + a_2)^3 - \pi a_2^3}{12a_1} \quad \text{Eq.5. 9}$$

The volume of annulus is:

$$V_a = V_T - V_c \quad \text{Eq.5. 10}$$

The volume of cylindrical tank is:

$$V_T = \frac{\pi D^2}{4} h \quad \text{Eq.5. 11}$$

$$V_a = \frac{\pi D^2 h}{4} - \frac{\pi(a_1 h + a_2)^3 - \pi a_2^3}{12a_1} \quad \text{Eq.5. 12}$$

Mass balance around the annulus gives:

$$\frac{d}{dt}(V_a \rho) = F_i \rho - F_o \rho \quad \text{Eq.5. 13}$$

Assuming constant density of fluid:

$$\frac{dV_a}{dt} = F_i - F_o \quad \text{Eq.5. 14}$$

Based on the Bernoulli's principle, a nonlinear square root (hydraulic) relationship between the outlet volumetric flowrate and liquid level is assumed.

$$F_o = \phi(h) = \beta \sqrt{h} \quad \text{Eq.5. 15}$$

Substituting into equation 5.14:

$$\frac{dV_a}{dt} = F_i - \beta \sqrt{h} \quad \text{Eq.5. 16}$$

The LHS of equation 5.16 is written as:

$$\frac{dV_a}{dt} = \left[\frac{dV_T}{dh} - \frac{dV_c}{dh} \right] \frac{dh}{dt} \quad \text{Eq.5. 17}$$

From equation 5.11,

$$\frac{dV_T}{dh} = \frac{\pi D^2}{4} \quad \text{Eq.5. 18}$$

From equation 5.1 and 5.2,

$$\frac{dV_c}{dh} = \frac{\pi(a_1 h + a_2)^2}{4} \quad \text{Eq.5. 19}$$

Substituting equation 5.18 and 5.19 into 5.17,

$$\frac{dV_a}{dt} = \left[\frac{\pi D^2}{4} - \frac{\pi(a_1 h + a_2)^2}{4} \right] \frac{dh}{dt} \quad \text{Eq.5. 20}$$

Substituting equation 5.20 into the mass balance equation 5.16 gives:

$$\left[\frac{\pi D^2}{4} - \frac{\pi(a_1 h + a_2)^2}{4} \right] \frac{dh}{dt} = F_i - \beta \sqrt{h} \quad \text{Eq.5. 21}$$

The (variable) cross sectional area of annulus is represented as:

$$\eta(h) = \left[\frac{\pi D^2}{4} - \frac{\pi(a_1 h + a_2)^2}{4} \right] \quad \text{Eq.5. 22}$$

Equation 5.21 is re-written as:

$$f(h, F_i) = \frac{dh}{dt} = \frac{F_i - \beta \sqrt{h}}{\eta(h)} \quad \text{Eq.5. 23}$$

Equation 5.23 represents the mathematical model of the process

5.3.3 Linearized State Space Model and Transfer Function Model

The equation 5.23 is the nonlinear differential equation that is solved for steady state as well as used for obtaining the dynamic response. From a control perspective, in terms of input-output model, the system is viewed as a Single Input Single Output (SISO) process.

The linearized state space model of a process is elaborately described in section 3.3.1 (equations 3.96).

The state variable and input variable are defined as:

$$x = (h - h_s), \quad u = (F_i - F_{is}) \quad \text{Eq.5. 24}$$

Where, the subscript, 's' denotes steady state condition.

At steady state, equation 5.23 is written as:

$$f(h_s, F_{is}) = \frac{dh_s}{dt} = \frac{F_{is} - \beta\sqrt{h_s}}{\eta(h_s)} = 0 \quad \text{Eq.5. 25}$$

Solution of equation 5.25 yields the steady state liquid level:

$$h_s = \left(\frac{F_{is}}{\beta} \right)^2 \quad \text{Eq.5. 26}$$

For a Single Input Single Output (SISO) process, the matrices **A** and **B** (equation 3.96) in the linearized state space model are scalar and are obtained using equation 5.23.

$$A = \left. \frac{\partial f(h, F_i)}{\partial h} \right|_s = -\frac{1}{\eta(h_s)} \frac{\beta}{2\sqrt{h_s}} \quad \text{Eq.5. 27}$$

$$B = \left. \frac{\partial f(h, F_i)}{\partial F_i} \right|_s = \frac{1}{\eta(h_s)} \quad \text{Eq.5. 28}$$

For a Single Input Single Output (SISO) system, the process transfer function model (equation 3.106) is written as:

$$g_p(s) = \frac{B}{(s - A)} \quad \text{Eq.5. 29}$$

Writing the above transfer function in gain- time constant form,

$$g_p(s) = \frac{K_p}{(\tau_p s + 1)} = \frac{(-B / A)}{[(-1 / A)s + 1]} \quad \text{Eq.5. 30}$$

Where, the Process (steady state) gain is denoted as:

$$K_p = (-B / A) = \frac{2\sqrt{h_s}}{\beta} \quad \text{Eq.5. 31}$$

And the process time constant is denoted as:

$$\tau_p = (-1/A) = \eta(h_s) \frac{2\sqrt{h_s}}{\beta} \quad \text{Eq.5. 32}$$

The process has a variable gain and variable time constant and since the Eigen value of A matrix (as described by equation 5.27) is negative, the system is stable. Equation 5.30 suggests that the experimental annular conical tank is first order system, characterized by its capacitance C and resistance R , as defined below:

$$C = \eta(h_s) \quad \text{Eq.5. 33}$$

$$R = \frac{2\sqrt{h_s}}{\beta} \quad \text{Eq.5. 34}$$

5.4 Process Identification

The experimental setup is operated in two modes:

- (a) Open loop/Manual Mode: In this mode, the experimental setup is operated to study the open loop response of liquid level in the annular flow conical tank subject to step changes in inlet volumetric flowrate (forcing function). This is effected by providing a step change in the Controller Output (OP %) which in turn provides the step change in the inlet volumetric flowrate through the final control element (pneumatic control valve).
- (b) Closed loop/Auto Mode: In this mode, the experimental setup is operated to study the closed loop response of liquid level in the annular flow conical tank subject to step changes in set point, using a standard PID Controller, and

5.4.1 Identification from Open loop Steady State Experimental Data

Various parameters of the annular flow conical tank liquid level setup are identified from the experimental results of open loop responses, as shown in the following sections.

5.4.1.1 Experimental Procedure

To begin with, the experimental setup was operated in the manual mode. Since the controller output signal (measured as the percentage of the maximum value) is the input signal to the final control element (pneumatic valve), the linear range of operation of the control valve was obtained in the controller output (OP%) range of 42% to 80%. The relationship between controller output and liquid inlet volumetric flowrate is obtained from the experimental data, as shown in equation 5.35 and Figure 5.4.

$$F_i = 153.97 - 1.40(OP\%) \quad \text{Eq.5. 35}$$

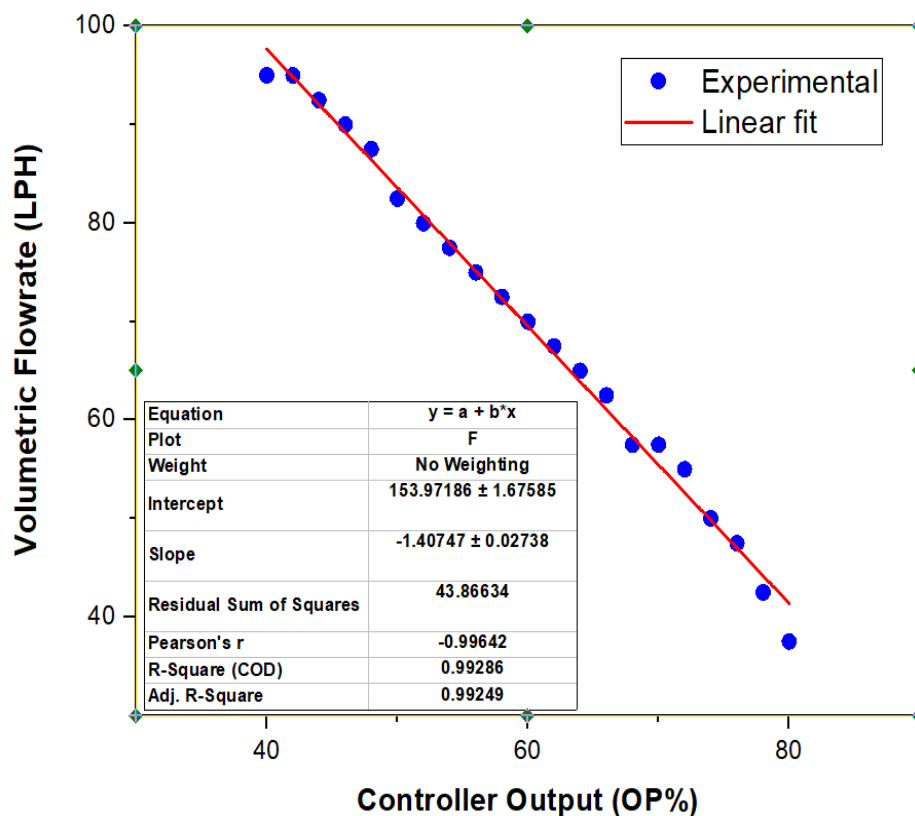


Figure 5. 4 Relationship between Volumetric Flowrate and Controller Output

An analogous relationship between the inlet volumetric flowrate and the pneumatic valve stem pressure is obtained as shown in equation 5.36 and Figure 5.5.

$$F_i = 198.73 - 11.93P \quad \text{Eq.5. 36}$$

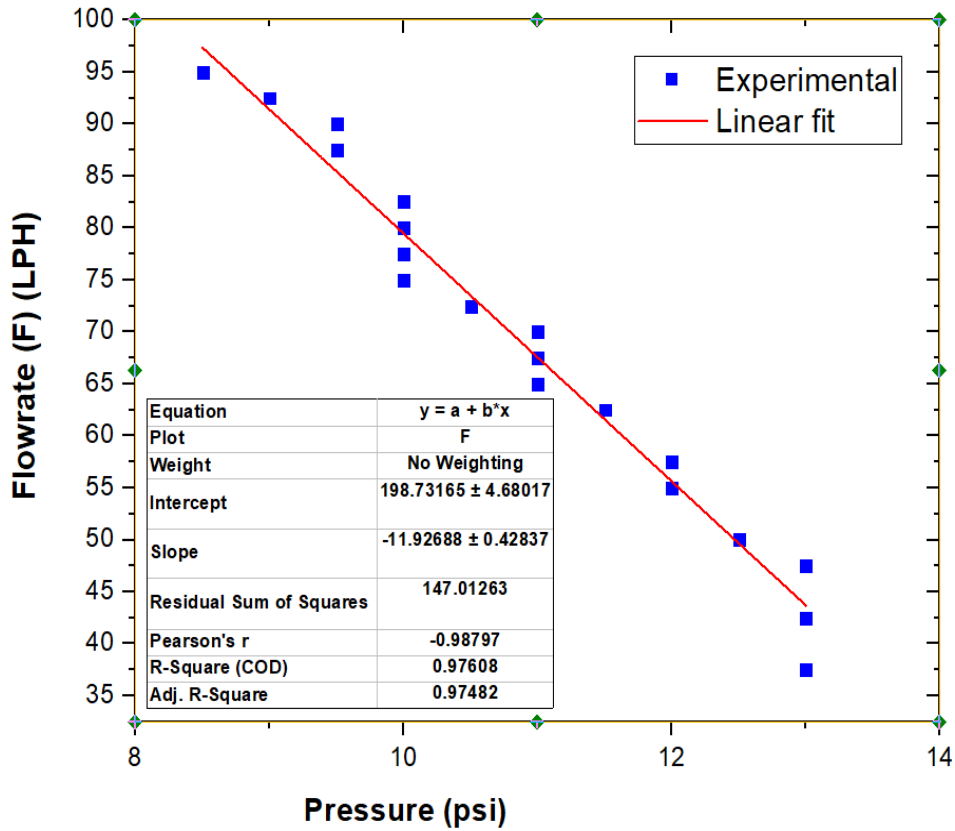


Figure 5. 5 Relationship between Volumetric Flowrate And Pneumatic Valve Stem Pressure

Similarly, a relationship between the pneumatic valve stem pressure and the controller output is obtained as shown in equation 5.37 and Figure 5.6.

$$P = 3.88 - 0.12(OP\%)$$

Eq.5. 37

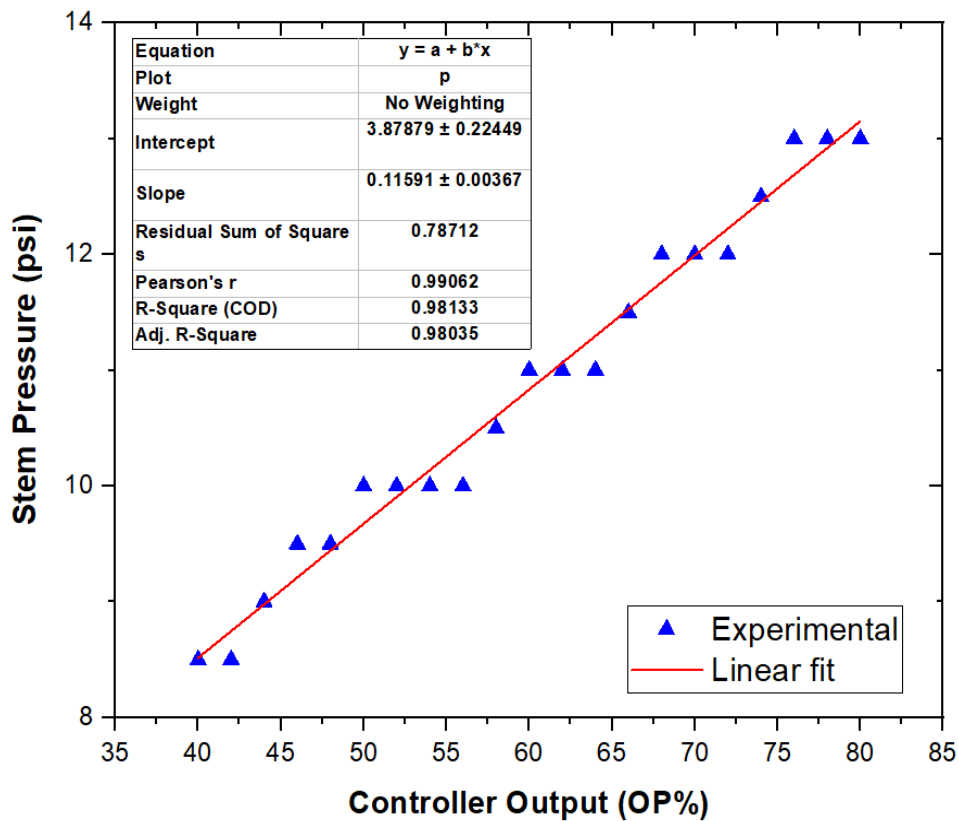


Figure 5. 6 Relationship between Pneumatic Valve Stem Pressure and Controller Output

5.4.1.2 Estimation of Nonlinear Flow Resistance

The physical setup was operated at various steady state operating points by changing the controller output in the range of 42% to 80%. The corresponding steady state inlet volumetric flowrates were calculated and the steady state heights were measured. Based on the steady state experimental data, the steady-state input-output curve is plotted, as shown in Figure 5.7. The (outlet) nonlinear flow resistance, β (defined in equation 5.15) is estimated from the slope of linear fit of the steady state input-output curve. The estimated value of beta is $21.17 \text{ LPH} / \sqrt{\text{cm}}$.

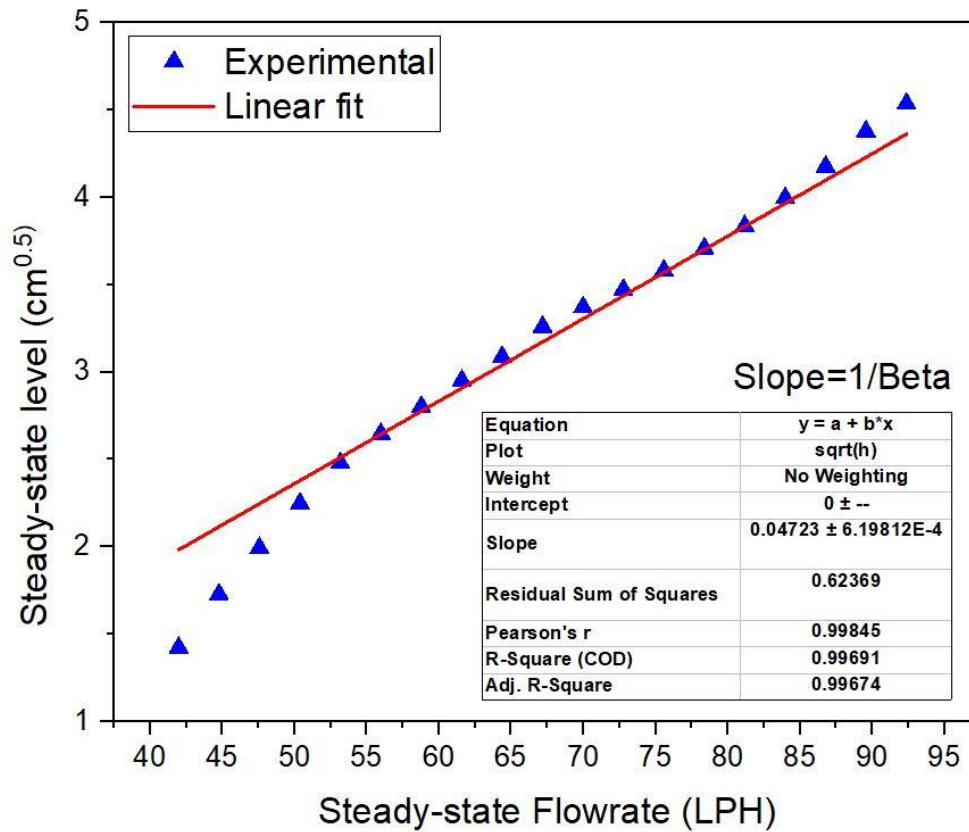


Figure 5. 7 Estimation of Nonlinear Flow Resistance

5.4.1.3 Estimation of Steady State Gain and Time Constant

The other process parameters like the capacitance, resistance (steady state gain) and time constant of the annular flow conical tank liquid level nonlinear process are calculated based on the experimental steady state data and the estimated value of beta, as shown in Table 5.2. The variation of time constant and steady state gain of the process, with respect to the height of liquid level is demonstrated in Figure 5.8 and 5.9 respectively.

Table 5. 2 Estimation of Process Parameters from Experimental Steady State Data.

Controller Output OP%	Steady State Inlet Volumetric Flowrate F_{is} (LPH)	Steady State Height h_s (cm)	Capacitance (Cross sectional area of annulus) $\eta(h)$ (cm²) (Equation 5.22)	Resistance (Steady state gain) $K_p = \frac{2\sqrt{h_s}}{\beta}$ (Equation 5.34)	Time constant $\tau_p = \eta(h_s) \frac{2\sqrt{h_s}}{\beta}$ (sec) (Equation 5.32)
44	92.37	20.6	56.83	0.43	87.68
46	89.57	19.2	54.71	0.41	81.46
48	86.77	17.4	51.87	0.39	73.68
50	83.97	16.0	49.17	0.38	66.83
52	81.17	14.7	46.71	0.36	60.94
54	78.37	13.7	44.71	0.35	56.38
56	75.57	12.8	42.71	0.34	52.02
58	72.77	12.1	40.96	0.33	48.36
60	69.97	11.4	39.35	0.32	45.11
62	67.17	10.6	37.55	0.31	41.62
64	64.37	9.5	34.82	0.29	36.56
66	61.57	8.7	32.68	0.28	32.80
68	58.77	7.9	30.36	0.26	28.93
70	55.97	7.0	28.02	0.25	25.22
72	53.17	6.1	25.54	0.23	21.53
74	50.37	5.1	22.30	0.21	17.05
76	47.57	4.0	18.96	0.19	12.85
78	44.77	3.0	15.80	0.16	9.28
80	41.97	2.0	12.63	0.13	6.11

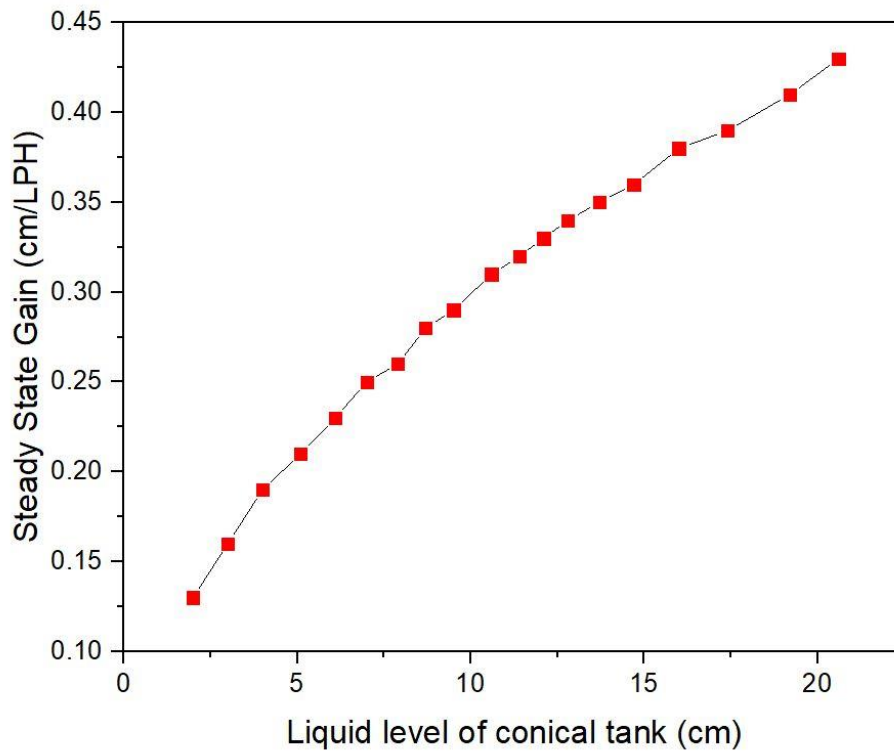


Figure 5. 8 Variation of Steady State Gain with respect to Height

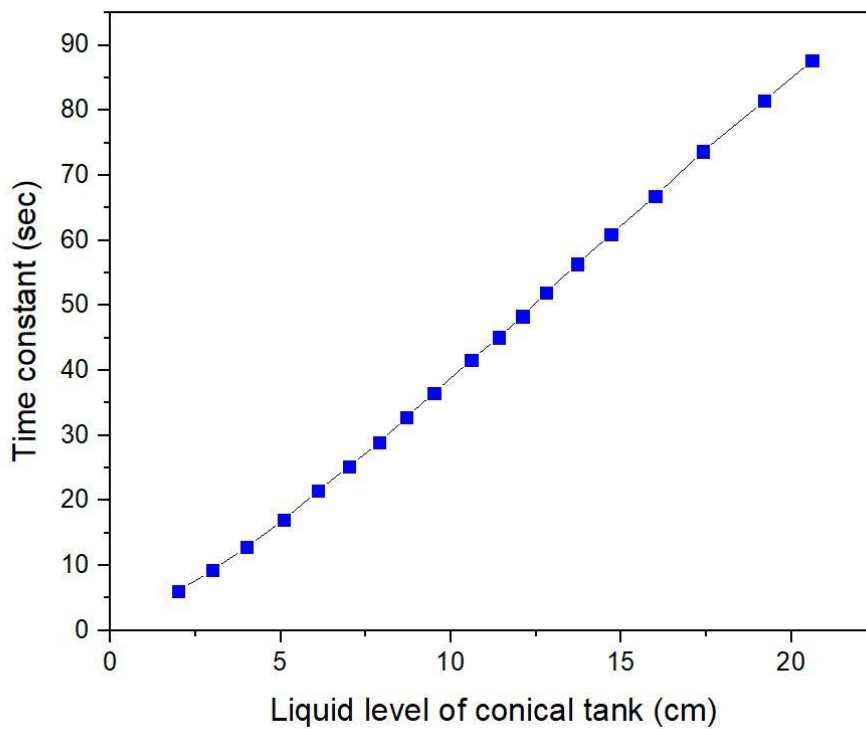


Figure 5. 9 Variation of Time Constant with respect to Height

5.4.2 Identification from Open loop Step response Experimental Data

In order to obtain the open loop step responses, the process was subjected to a sequence of small step changes in the controller output (OP%) and the variation of liquid level (height) with respect to time was recorded.

The experimental data is used for the identification of process parameters. Even though the process is inherently first order, but experimental data revealed that there is a delay of approximately 5 seconds in the process output. This is attributed to the dynamics associated with the other components in the control loop, like the final control element, measuring element etc. The experimental data was therefore used to identify a First Order plus Dead Time (FOPDT) model, based on the methodology discussed in section 3.4.1. Comparison of the identified First Order plus Dead Time (FOPDT) model with the experimental data is shown in Figure 5.10

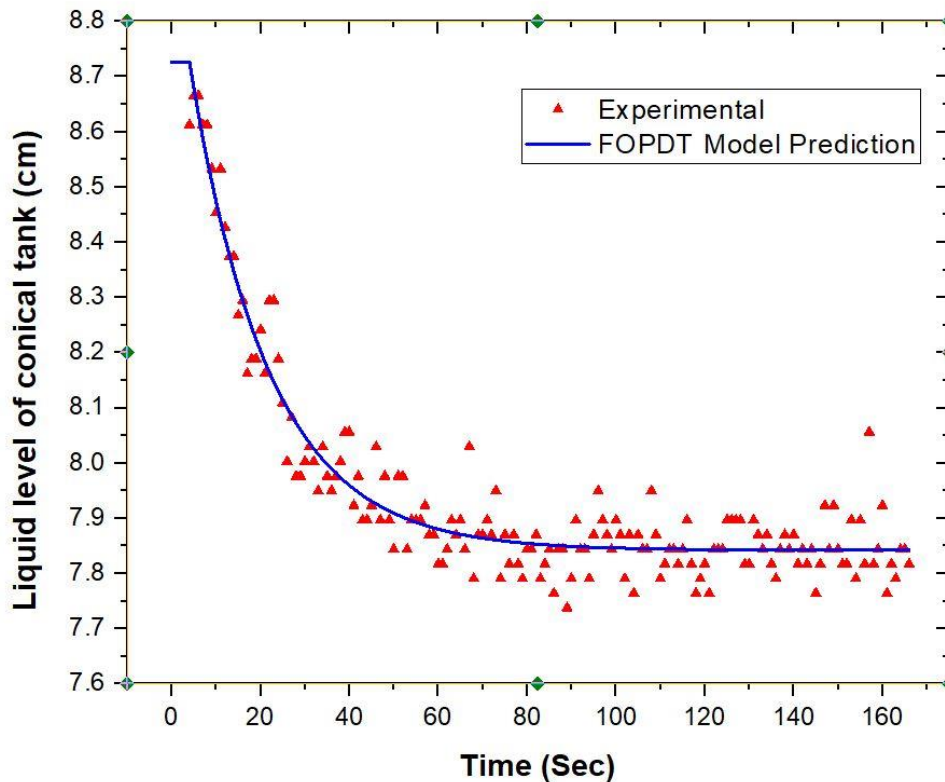


Figure 5. 10 Open Loop Step Response of Liquid Level in Conical Tank Process

5.5 Controller Design

5.5.1 Controller Design based on Cohen and Coon Method

The open loop step response experimental data (process reaction curve) is used to design a conventional PID controller based on Cohen and Coon settings (Table 3.4). The process transfer function parameters are taken from the FOPDT model identified earlier. In order to study the servo problem, the setup was operated in auto mode.

5.5.2 IMC based PID Controller Design

An Internal Model Control (IMC) based PID controller is designed to overcome the limitations of conventional PID controller. Based on the equations 3.48-3.50, the parameters of Internal Model Control (IMC) based PID controller, corresponding to four different steady state operating points are shown in Table 5.3 The tuning parameter λ is set equal to the time constant of the process.

Table 5. 3 IMC based PID Controller Parameters Corresponding to Different Steady States

Steady State Height of Liquid Level h_s (cm)	Process Transfer Function	IMC based PID controller parameters			
		λ	K_c	τ_I	τ_D
6.2	$\frac{0.23}{21.53s + 1} e^{-5s}$	21.53	4.35	24.03	2.24
10.6	$\frac{0.31}{41.62s + 1} e^{-5s}$	41.62	3.23	44.12	2.36
14.7	$\frac{0.36}{60.94s + 1} e^{-5s}$	60.94	2.78	63.44	2.40
20.6	$\frac{0.43}{87.68s + 1} e^{-5s}$	87.68	2.33	90.18	2.43

5.6 Closed Loop Response

The closed loop block diagram of Internal Model Control (IMC) based PID control system is shown in Figure 5.11.

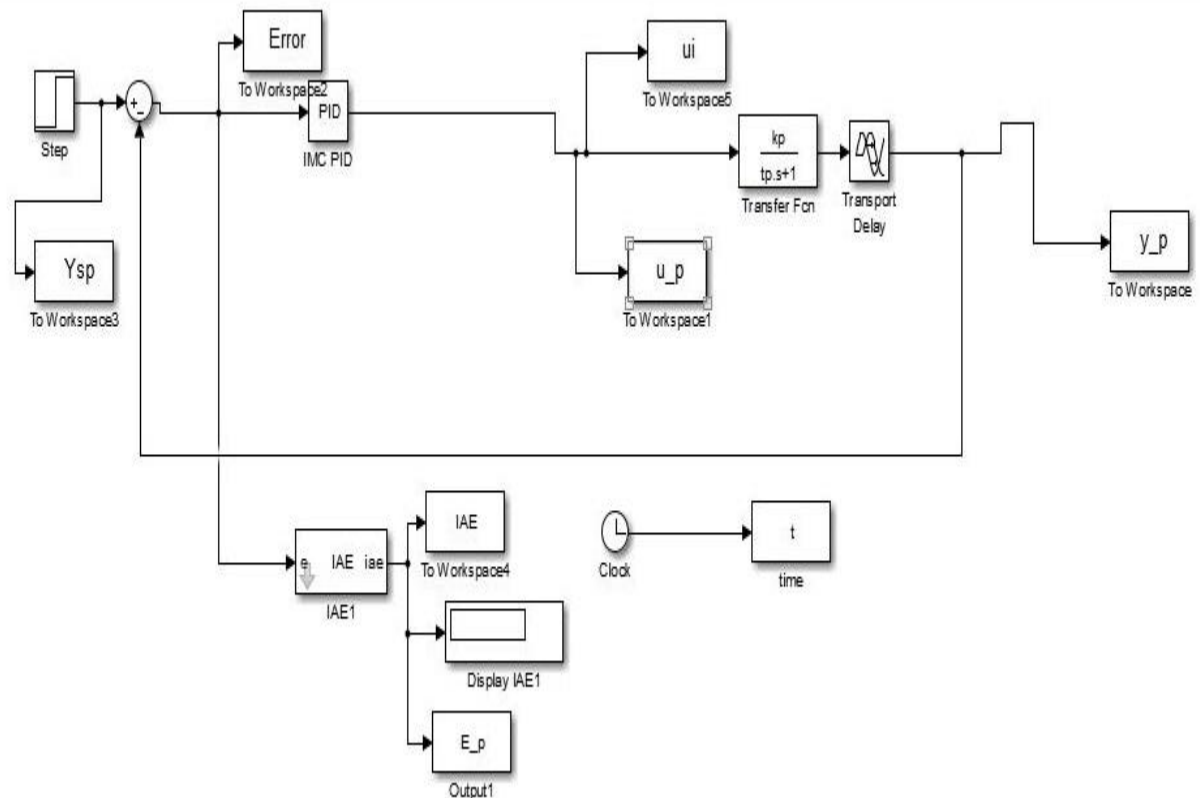


Figure 5. 11 Closed loop block diagram of IMC based PID controller

In order to compare the performance of Internal Model Control (IMC) based PID controller with the conventional PID controller (based on Cohen and Coon method), the experimental setup was operated in auto mode. Figure 5.12 shows that the Internal Model Control (IMC) based PID controller provides more stabilized and superior performance.

Table 5. 4 Quantitative Performance Indices for IMC based PID Controller.

IMC based PID Controller		ISE	IAE	ITAE	Overshoot %	Rise Time (Sec)	Settling Time (Sec)
Designed at 6.1cm	Tested at 6.1 cm	169.21	473.09	3802.38	81.56	9	62
	Tested at 10.6 cm	550.57	734.53	6499.44	80.48	9	55
	Tested at 14.7 cm	613.01	796.31	7594.3	93.98	9	45
	Tested at 20.6 cm	654.45	887.93	8348.50	80.89	9	35
Designed at 10.6 cm	Tested at 10.6 cm	169.21	473.09	3802.38	32.82	9	34
	Tested at 6.1 cm	361.60	461.73	3600.37	81.86	9	40
	Tested at 14.7 cm	345.46	456.92	3540.467	60.30	10	50
	Tested at 20.6 cm	361.60	461.73	3600.37	81.86	9	40
Designed at 14.7 cm	Tested at 14.7 cm	169.21	473.09	3802.38	78	9	61
	Tested at 6.1 cm	638.12	873.14	8125.71	77.59	9	66
	Tested at 10.6 cm	732.64	1005.73	9397.98	77.05	9	71
	Tested at 20.6 cm	593.74	776.22	7211.60	90.35	9	37
Designed at 20.6 cm	Tested at 20.6 cm	182.6327	631.46	5570.29	94.51	8.5	45
	Tested at 6.1 cm	1597.99	2499.50	46431.98	96.77	8	114
	Tested at 10.6 cm	1597.99	2499.59	46430.58	96.77	8	102
	Tested at 14.7 cm	1857.39	2818.19	57305.84	109.8	10	109

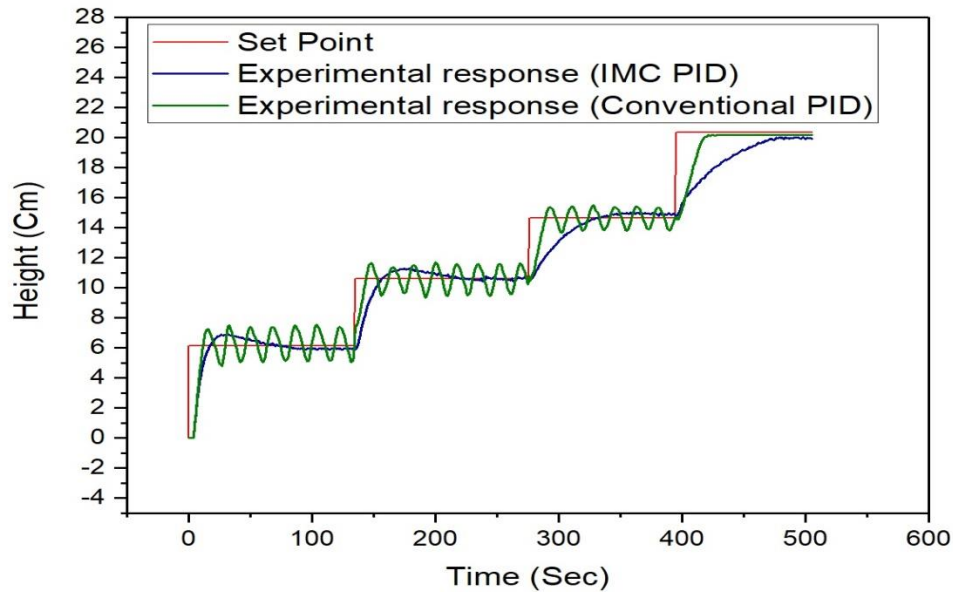


Figure 5.12 Comparison of Experimental Closed Loop Responses

The closed loop performance of IMC based PID controller to servo problem is studied at four different steady state operating points, in terms of various quantitative performance indices such as ISE, IAE, ITAE, rise time, settling time and percentage overshoot as shown in Table 5.4. Since the annular conical tank liquid level process is nonlinear and has variable gain and time constant corresponding to different liquid levels, there is variation in the performance of the IMC based PID controller when tested for set point changes at different steady states.

5.7 Conclusion

In the present work, the process dynamics and control of a variable gain variable time constant conical tank liquid level experimental process is studied. Mathematical model of the process is developed and the process parameters are identified using the experimental data of open loop step responses. An IMC based PID controller is designed and implemented on the experimental setup to study the closed loop performance.