Chapter 7

Conclusion and Future Scope

7.1 Conclusions

This section is the terminating part of the work accomplished in the entire thesis. The thesis is classified into five independent working chapters comprising a total of five problems related to various 2D quasilinear hyperbolic problems. In the second and third chapters, the existence of simple waves is proved in 2D compressible flow in a non-ideal MHD system. In the next chapter, exact closed-form solutions of the 1D genralised Riemann problem for a generalized Chaplygin gas with Coulomb friction are obtained using the differential constraint method.

For studies in Chapters 5 and 6, weak shock diffraction-reflection phenomena are examined, and the graphs have been plotted using the software MATHEMATICA. The diffracted shock and expansion wave profile behaviour has been studied graphically under the influence of various parameters considered. Some of the significant highlights, as well as concluding remarks, observed from the overall investigation have been encapsulated below.

- **Chapter 1** introduced the thesis with fundamental definitions. The past and recent works on the existence of simple waves, the Riemann problem, and weak shock diffraction-reflection are presented.
- **Chapter 2** deals with finding the simple waves and the existence of the characteristic decomposition of the 2D compressible flow in a non-ideal MHD system. In this study, we used characteristic decomposition to elucidate the presence of simple waves within a two-dimensional compressible flow governed by a non-ideal magnetohydrodynamics system. The existence of simple waves is shown for steady, pseudo-steady, and full magnetohydrodynamics systems.
- **Chapter 3** unfolds finding the simple waves and the existence of the characteristic decomposition of the 2D compressible flow in a non-ideal MHD system using a sufficient condition. The existence of simple waves is shown for steady, pseudo-steady, and full magnetohydrodynamic systems.
- **Chapter 4** finds the closed-form solutions for the one-dimensional genralised Riemann problem of a generalized Chaplygin gas with Coulomb-type friction using the differential constraint method. Further, the compatibility conditions and first-order differential constraints are established, yielding an exact solution that extends the classical simple wave to include non-homogeneous (source-term) effects. The genralised Riemann problem solutions are fully characterized for piecewise-linear initial data, including a closed-form rarefaction-wave solution for the nonlinear problem.
- **Chapter 5** examines the effect of extended Chaplygin gas when a weak shock hits the right-angled wedge. The reflection-diffraction phenomenon has been discussed. The nonlinear behaviour has been examined at the different boundaries of the considered region. Graphical illustrations from this study establish

that it becomes stronger in the extended Chaplygin gas than in the ideal gas, and the rarefaction wave (expansion wave) is weakened while the extended Chaplygin gas effects simultaneously strengthen the diffracted shock.

- **Chapter 6** unravels the analysis of the reflection-diffraction phenomenon when a weak shock hits a rigid wedge at an arbitrary angle. The linear and nonlinear behaviour has been examined in the various regions and on the boundaries of the flow region. It is quite evident from the graphs that the extended Chaplygin gas weakens the rarefaction wave while strengthening the diffracted shock compared to the ideal gas.

7.2 Future Scope

There are some interesting and challenging research problems that can be studied in the future.

- In Chapters 2 and 3, we established the existence of simple waves and characteristic decompositions for two-dimensional, non-ideal MHD flows under steady, pseudo-steady, and fully unsteady assumptions. In future work, we may generalize these results to three-dimensional non-ideal MHD systems, which would capture more realistic astrophysical or engineering configurations. We may incorporate additional physical effects, such as viscosity, heat conduction, or variable electrical resistivity, to investigate how the existence and structure of simple waves change when the flow departs further from ideal MHD. Study of multiphase or multi-component MHD flows (e.g., dusty-plasma or plasma neutral mixtures) may be focused to see whether characteristic decomposition theory can be extended to systems with interphase coupling.

- Chapter 4 derived closed-form solutions for the one-dimensional generalized Riemann problem (GRP) of a generalized Chaplygin gas with a Coulomb-type friction term using the differential constraint method (DCM). Possible future investigations may consider replacing the Chaplygin-type equation of state by more general or multi-parameter EOS (e.g., van-der Waals Chaplygin hybrid models or equation of state with multiple inflexion points) to see how friction affects solution structure and whether exact-type solutions remain obtainable via DCM. We may extend this analysis to multi-dimensional Riemann problems (e.g., two or three-dimensional discontinuities) under generalized Chaplygin or van der Waals–type gas with source terms, determining whether DCM can still yield closed-form rarefaction and shock interactions. Incorporation of additional source terms, such as radiation, magnetic fields, or chemical reactions, may be considered to evaluate how these non-homogeneous effects modify compatibility conditions and whether new families of exact or approximate solutions arise.

- In Chapters 5 and 6, we examined shock reflection–diffraction over right-angled and arbitrary-angle wedges with a more realistic extended Chaplygin gas. We constructed local self-similar solutions in the hyperbolic region and demonstrated how extended Chaplygin effects strengthen the diffracted shock and weaken the rarefaction wave. Possible future directions include constructing a global smooth supersonic solution of the modified problem for relativistic Euler–Chaplygin flows up to a specified positive characteristic curve, by posing and solving a free boundary problem that influences airfoil symmetry and modern elliptic–hyperbolic coupling techniques.

Although the presented works in this thesis produced closed-form, self-similar solutions and local expansions, future research could involve implementing numerical schemes (e.g., ENO/WENO or adaptive finite-volume methods) that incorporate extended Chaplygin or van der Waals equations of state with friction, to compare numerical results with the analytic predictions for reflection–diffraction angles, wave strengths, and wavefront curvature.