Contents

Symbols x

List of Figures xvii

List of Tables xix

Preface xxi

1 Introduction 1
- 1.1 Fractional Calculus . . . 1
- 1.2 Preliminaries . . . 2
  - 1.2.1 Basic Definitions . . . 2
  - 1.2.2 Prabhakar Fractional Derivatives and Integrals . . . 3
- 1.3 Model Problems . . . 5
  - 1.3.1 Time Fractional Advection–Diffusion Equation . . . 5
  - 1.3.2 1D Time Fractional Reaction–Diffusion Equation . . . 6
  - 1.3.3 2D Time Fractional Reaction–Diffusion Equation . . . 7
- 1.4 Literature Review . . . 7
  - 1.4.1 Literature Review on Time Fractional Advection–Diffusion Equation . . . 10
  - 1.4.2 Literature Review on Time Fractional Reaction–Diffusion Equation . . . 11
- 1.5 Motivation and Objectives of the Thesis . . . 12
  - 1.5.1 Motivation . . . 12
  - 1.5.2 Objectives . . . 13
- 1.6 Outline of the Thesis . . . 14

2 Approximation of Caputo–Prabhakar Derivative With Application in Solving Time Fractional Advection–Diffusion Equation 17
- 2.1 Introduction . . . 18

2.2 Approximation of Caputo–Prabhakar Derivative . . . 18
2.2.1 Numerical Scheme 1 ($NS1$) . . . 19
2.2.2 Numerical Scheme 2 ($NS2$) . . . 21
2.3 Error Analysis of the Schemes . . . 25
2.3.1 Error Analysis of $NS1$ . . . 25
2.3.2 Error Analysis of $NS2$ . . . 26
2.4 Application . . . 27
2.4.1 Numerical Scheme 1 ($NS1$) . . . 29
2.4.2 Numerical Scheme 2 ($NS2$) . . . 34
2.5 Numerical Experiments . . . 39
2.6 Conclusions . . . 52

3 High-Order Approximation of Caputo–Prabhakar Derivative With Application in Solving Linear and Nonlinear Fractional Diffusion Models 55
3.1 Introduction . . . 56
3.2 Numerical Scheme . . . 57
3.2.1 Error Analysis of the Scheme . . . 64
3.2.2 Numerical Experiments . . . 68
3.3 Application-1 . . . 73
3.3.1 Stability Analysis . . . 76
3.3.2 Uniqueness and Existence . . . 78
3.3.3 Convergence Analysis . . . 79
3.3.4 Numerical Experiments . . . 81
3.4 Application-2 . . . 85
3.4.1 Stability Analysis . . . 87
3.4.2 Convergence Analysis . . . 89
3.4.3 Numerical Experiments . . . 90
3.5 Conclusions . . . 95

4 A Fourth Order Accurate Numerical Method for Nonlinear Time Fractional Reaction–Diffusion Equation on a Bounded Domain 97
4.1 Introduction . . . 98
4.2 Notations and Preliminary Lemmas . . . 99
4.3 Construction of the Scheme . . . 103
4.3.1 An Iterative Algorithm for the Nonlinear Reaction Term . . . 105
4.4 Numerical Analysis . . . 106
4.4.1 Stability Analysis . . . 107
4.4.2 Convergence Analysis . . . 109
4.5 Numerical Experiments . . . 111
4.6 Conclusions . . . 123

5 A Newton Linearized Two-Level Alternating Direction Implicit Scheme for Two-Dimensional Nonlinear Time Fractional Reaction–Diffusion Equation on a Bounded Domain **125**
5.1 Introduction . . . 125
5.2 Fully Discrete Alternating Direction Implicit Scheme . . . 127
5.3 Stability and Convergence of the Alternating Direction Implicit Scheme 132
5.3.1 Stability Analysis . . . 133
5.3.2 Error Estimate . . . 135
5.4 Numerical Experiments . . . 139
5.5 Conclusions . . . 144

6 Conclusions and Future Scopes **145**
6.1 Conclusions . . . 145
6.2 Scope for Future Work . . . 147

Bibliography **151**

Publications **161**