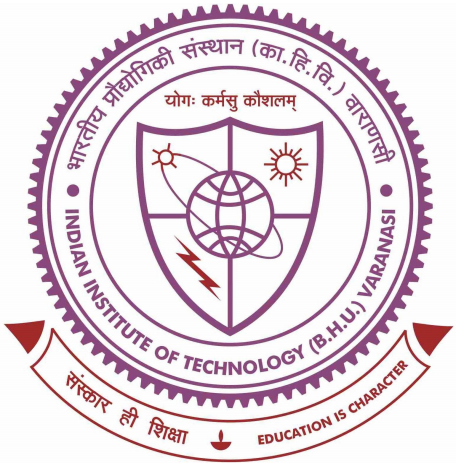
Approximation of Functions by Positive Linear Operators and their Applications in Integral Equations



Thesis submitted in partial fulfillment

for the Award of Degree

DOCTOR OF PHILOSOPHY

by

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DEPARTMENT OF MATHEMATICAL SCIENCES

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It is certified that the work contained in this thesis titled "***Approximation of Functions by Positive Linear Operators and their Applications in Integral Equations***" by ***Abhishek Senapati*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy, and SOTA for the award of a Ph.D. degree.

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I, ***Abhishek Senapati***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. Tanmoy Som*** from ***January 2020*** to ***August 2024*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports, dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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Dedicated to

My Mom Mrs. Mamata Senapati

My Father Mr. Purna Chandra Senapati

&

My Sister Maheswari Senapati

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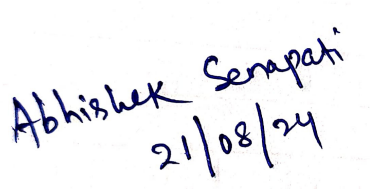

(Abhishek Senapati)

List of Figures

List of Tables

Symbols

$\mathbb{N}$	Collection of all natural numbers
$\mathbb{R}$	Collection of all real numbers
$\mathfrak{C}(X)$	Space of all real-valued continuous functions defined on X
$\mathfrak{C}_B(X)$	Space of all real-valued bounded continuous functions defined on X
$\mathfrak{C}_B^n(X)$	Space of all real valued $n-$times continuously differentiable function in $\mathfrak{C}_B(X)$ $(n \in \mathbb{N})$
$\mathfrak{U}(X)$	Space of all uniformly continuous functions in $\mathfrak{C}_B(X)$
$L^p(X)$	Space of all real-valued p-integrable functions on X, for $1 \leq p \leq \infty$
$\mathcal{AC}_l(X)$	Space of all locally absolutely continuous functions on X
$\lvert\cdot\rvert$	modulus
$\lVert\cdot\rVert$	sup norm
$\square$	End of the proof