CHAPTER 7

SUMMARY OF THE THESIS AND FUTURE SCOPE

7.1 Summary

In the field of materials science and engineering and in geophysics, there are many situations where accurate predictions of the thermal and mechanical responses of materials and structures are essential. The subject "coupled thermoelasticity" provides a framework for understanding such situations as it explores the mutual interactions between the thermal and mechanical fields in elastic materials subjeted to thermomehanical loading effects. Thermoelasticity has therefore been the centre of active research during last several years. After the developement of the first comprehensive model of thermoelasticity given by Biot (1956), several models of coupled thermoelasticity have come into existence during last few decades to tackle the coupling effects of thermal and mechanical fields in elastic material in various alternative way. The objective of the present thesis is to examine some aspects related to the thermomechanical responses due to some impulsive forces and heat sources present in elastic medium. Focus has been made to understand the effects in the frame of two newly introduced thermoelasticity theories, namely modified Green-Lindsay (MGL) theory and Quintanilla-Moore-Gibson-Thompson (QMGT) thermoelasticity theory. The primary contribution of this thesis is categorised into two distinct parts:

I: Study on Modified Green-Lindsay (MGL) thermoelasticity and its applications

The primary portion of this study consists of **Chapters 2-5**, which concentrate on study of the recently formulated generalized thermoelasticity theory (MGL) proposed by Yu et al. (2018). The constitutive equations incorporate the terms that provide a connection between the strain rate, rate of temperature change, and stress. From a practical point of view, these equations take into account the fact that the material response depends not only on temperature changes but also on how quickly the deformation is applied. This theory is also referred to as the strain and temperature rate-dependent thermoelasticity theory. **Chapter 2** focuses on conducting an investigation of thermal and mechanical response shown by a linear, homogeneous and isotropic unbounded elastic medium containing a cylindrical cavity when the boundary is subjected to thermal shock. In order to investigate the influence of strain rate terms, we merge the governing equations of MGL thermoelasticity theory with two other generalized thermoelasticity theories (specifically GL and LS theories). The analytical derivation for solution of various field variables are obtained by the execution of Laplace transform technique followed by short-time approximation method. In this study, we locate and evaluate the points of discontinuity and examine the solutions of the field variables individually within the framework of three distinct models. The MGL model appears to effectively resolve the problem of unrealistic prediction of discontinuity in the displacement field, a limitation that is observed in case of GL model. However, it is crucial to acknowledge that each field variable in the context of MGL model is classified as a combination of two components: a diffusive component and a wave component. Moreover, the solution suggests that field variables exhibit an unbounded speed, analogous to the behaviour observed in the visco-thermoelastic framework. This implies that the influence of disturbances are immediately propagated across the entire material. This observed behaviour of the solution in context of MGL model appears to be impractical, in contrast to the LS and

GL models which predict finite speed for both elastic and thermal waves, resulting in limited areas of influence.

The next part of the work is discussed in **Chapter 3**, which comprises two subchapters. We go deeper into the analysis of MGL theory by addressing two different problems related to thermoelastic interactions caused by various heat sources and discuss the analytical as well as numerical results of the problems. A detailed analysis of the results predicted by this model is reported**.** The first problem in this chapter as presented in **Subchapter 3.1** deals with the impact of line heat source which is located at the center (origin) of a linear, homogeneous and isotropic unbounded elastic medium. The short-time approximated solution for MGL model reveals that each field variable consists of combination of wave term and diffusive term and it decays exponentially. The presence of the diffusive term, causes the disturbance to propagate with infinite speed. We observe the presence of a single wavefront in this model where temperature, displacement and radial stress components exhibit continuous behaviour. However, the circumferential stress component undergoes an infinite jump discontinuity at this wavefront. We compare our findings with the corresponding outcomes obtained from other established thermoelastic models like LS and GL models and highlight that a significant disagreement is observed in the prediction by the present MGL thermoelastic theory as compared to the other existing theories (LS and GL). **Subchapter 3.2** further attempts to investigate the MGL theory and solve a thermoelastic problem where an unbounded medium with a cylindrical cavity is exposed to an external moving heat source along the radial direction in absence of any body force. The inner boundary of cavity is maintained at constant temperature in the absence of traction force along the radial direction on inner surface. This problem is solved using Laplace transformation and the final results for field variables are obtained numerically using Stehfest method (1970) for inversion of Laplace transforms. The predictions are analyzed via graphical results to mark the effects of the relaxation time parameters, which is characteristic of the present generalized thermoelasticity theory. A prominent impact of these parameters is observed in all the physical fields, i.e., displacement, temperature and stress fields. The observations for MGL model are compared graphically with the outcomes of LS and GL models to notice the differences in results. Further, the impact of velocity of heat source is examined.

The biomedical field relies on accurate models as they enable a comprehensive understanding of responses of biological tissues. These models are indispensable for various applications such as medical diagnostics, therapies, and the design of medical devices. The **Chapter 4** discusses the implementation of MGL thermoelasticity theory to enhance our understanding of the coupled thermal and mechanical phenomena in biomedical cases. The objective of this study is to establish a theoretical framework using the MGL thermoelastic model to analyse the stresses and temperature generated in each layer of the triple-layered skin tissue when subjected to thermal loading. The present problem is formulated in a unified way to derive the governing equations and constitutive relations under the MGL model and DPL thermoelastic model that incorporate two phase-lag times. We employ Laplace transform technique and subsequently implement Stehfest algorithm (1970) to compute the temperature, displacement and thermal stress distributions in the skin tissue. The MGL model reveals distinct characterstic for temperature and thermal stress distribution in tissues when compared to DPL heat conduction model. The MGL model precisely indicates temperature oscillation during initial time stages, as observed in the experiments conducted by Mitra et al. (1995) and Liu et al. (1996; 1997). This suggests that MGL model may be more effective than DPL model in studying the thermoelastic behaviour of skin tissue. This work aims to provide valuable insights for experts in the bio-medical field. It focuses on the selection of an appropriate and effective generalized thermoelastic model for treating skin tissue diseases caused by thermal loading.

Chapter 5 derives the formulation of boundary integral equations for a homogenous isotropic thermoelastic medium with mixed thermal and mechanical boundary conditions. This formulation is presented within the framework of MGL theory. The boundary integral formulation (BIE formulation) is recognised as an alternative and efficient approach for the modeling and analysis of thermoelastic phenomena. The implementation of this methodology avoids discretizing the complete domain, leading to simplified numerical calculations, enhanced efficacy, and increased precision, especially in scenarios encompassing intricate geometries. Therefore, the BIE formulation have significant importance in the numerical treatment of coupled thermoelastic problems, particularly in cases when analytical solution for the problem is challenging. In this chapter, the BIE formulation is derived by establishing significant findings. The fundamental solutions in the Laplace transform domain are derived when the body is exposed to two distinct scenarios: one involving a concentrated heat source and the other involving a body force acting in a specific direction. Thus, a reciprocal relationship is derived between the field variables for these two systems of causes. Moreover, the field variables are expressed in integral form, taking into account the boundary conditions, using the reciprocity relation. Finally, we demonstrate the application of our BIE fomuation and discuss the aspects of numerical execution using the boundary element approach.

II: Study on Quintanilla-Moore–Gibson–Thompson (QMGT) Thermoelasticity

The second part of the thesis includes **Chapter 6**, which highlights the characteristics of recently developed generalized thermoelasticity theory proposed by Quintanilla (2019). This theory is the combination of the Lord-Shulman and Green-Nagdi thermoelasticity theory of type III. The objective of this chapter has been the analysis of the impact of a continuous line heat source on a linear, isotropic, elastic unbounded medium within the framework of the QMGT model and compare it with other generalized thermoelasticity theories. The displacement field shows continuous nature. On the other hand, it is important to note that the temperature and stress components exhibit an infinite singularity at the location of the source, as well as a discontinuity at both wavefronts. The analytical solutions derived for the (QMGT) model indicate that the solution for each field is composed of a superposition of two waves. These waves propagate with a finite velocity and exhibit exponential decay over a short period of time. The LS, GL, and QMGT models claim that the impact of a continuous line heat source is limited to a time-varying bounded area in the vicinity of the heat source. However, this behaviour differs from that observed in the MGL model.

7.2 Future Scope

generalized thermoelasticity is an advanced version of classical thermoelasticity that incorporates additional factors to improve the precision of material behaviour response, especially in scenarios where classical models may be unsatisfactory. The theory includes non-local effects and relaxation times into the constitutive equations, providing a more accurate structure for explaining the thermal and mechanical behaviour of materials. In recent years, there has been a significant focus on describing the behavior of materials at small scales, including micro and nanostructures. This is because classical models may not accurately describe these materials due to the prevalence of non-local effects. This research field is necessary for advancing industries and equipment in modern microfabrication technology. The investigation of fractures and failures in solids holds considerable significance in various industries, such as the manufacturing of electronic components, geophysics, and earthquake engineering. The application of generalized thermoelasticity has been implemented to investigate the thermal and mechanical characteristics of biological tissues, such as hyperthermia treatments and medical imaging etc., where accurate modeling of heat transmission and stress distribution is crucial. Hence, it is considered worthwhile to continue further investigation in this particular area of research. Various generalized thermoelastic theories have been formulated, including the Modified Green-Lindsay model, the Quintanilla-Moore-GibsonThompson model, the Green-Naghdi theory, the dual phase-lag theory, the Lord Shulman model, the Green-Lindsay theory and many other thermoelasticity theories. The investigation of several models has led academics to focus on thermoelasticity theories, which have garnered considerable interest due to their ability to highlight unique properties. The present study also includes an in-depth review of some of these models. Some discrepencies are further observed by these models. It is therefore concluded that further study in both theoretical and experimental directions is necessary to identify the most appropriate model for accounting for the coupling effects of thermal and mechanical fields for different practical scenarios. While there have been a limited number of studies conducted in this area, they are insufficient to adequately address the correlated system of equations. When analysing thermal stresses in elements, it is essential to consider certain properties of the material. These properties include (i) the temperature-dependent nature of material properties, (ii) the time-dependent strain and stress response of materials, and (iii) the decrease in yield stress of the material as temperature increases. Despite extensive research on heat transfer in living tissues, the complexity of biological systems prevents researchers from reaching a definitive conclusion on the most accurate model for describing heat conduction in biological tissue. Further research is required to explore different theories of thermoelasticity, while considering the mentioned features. The investigation of heat conduction issues is approached from several perspectives, including general, analytic, and numerical viewpoints. This approach allows researchers to explore different aspects of the problem and obtain valuable insights. General and analytical solutions serve as crucial tools for doing numerical analysis in this field. The idea of thermoelasticity has applications in several engineering disciplines and technological advancements, including areas such as acoustic, chemical, mining, and nuclear engineering.

Further thermoelasticity is a fundamental concept that underlies several scientific disciplines, such as visco-thermoelasticity, aero-thermoelasticity, magneto-thermoelasticity, poro-thermoelasticity, thermo-piezoelectric theory, and bio-thermoelasticity. This concept is often implemented in real applications. Recently, various models of thermoelasticity as discussed in the present thesis have been extended by employing the concept of fractional order derivatives and memory dependent derivatives. Detailed investigation of such models in order to understand their applications in real world problems is yet to be carried out. The completion of the thesis is accompanied by strong anticipation that the points mentioned here will be duly acknowledged and considered in subsequent endeavors.