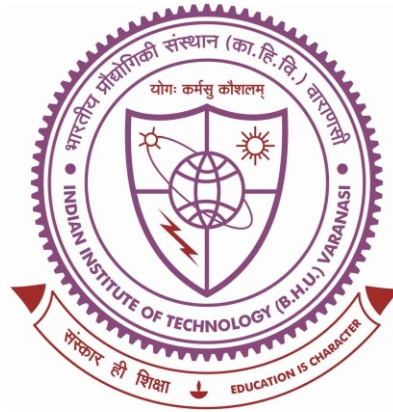


Flow Dynamics of Oesophageal Swallowing in Normal and Herniated States



Thesis submitted in partial fulfillment for the
Award of Degree

Doctor of Philosophy

By

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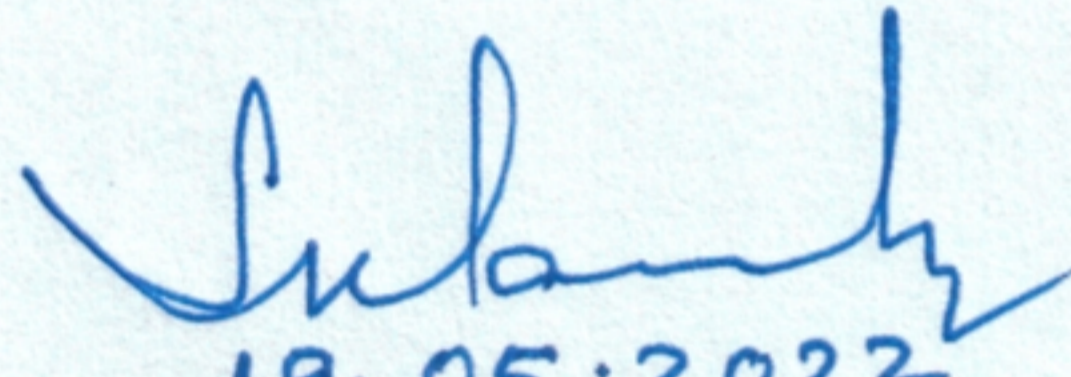
This thesis is dedicated to my parents.

For their endless love, support and encouragement

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Contents

List of Figures	xi
Preface	xvii
Abstract	xix
1 Introduction	1
1.1 Physiological systems associated with peristalsis	2
1.1.1 Oesophagus	2
1.1.2 Intestine	4
1.1.3 Ureter	6
1.1.4 Vas deferens	7
1.2 Peristaltic pump	7
1.3 Oesophageal diseases	8
1.3.1 Hiatus hernia	8
1.3.1.1 Sliding hiatus hernia	8
1.3.1.2 Para-oesophageal hiatus hernia	9
1.3.1.3 Symptoms of hiatus hernia	9
1.3.1.4 Cause of hiatus hernia	10
1.3.2 Gastro-oesophageal reflux disease	10
1.3.3 Barrett's oesophagus	11
1.3.4 Achalasia	11
1.3.5 Nutcracker oesophagus	12
1.3.6 Oesophageal cancer	12
1.3.6.1 Signs and symptoms of oesophageal cancer	13
1.3.6.2 Treatment options of oesophageal cancer	13
1.4 Characteristic of physiological fluids	13
1.5 Hypothesis of the research	14

2	Review of the literature	17
2.1	Review of the literature	17
2.1.1	Short study for Newtonian fluids	18
2.1.2	Non-Newtonian flows	19
2.1.3	Flows with particulate suspensions: application to diseased ureter	21
2.1.4	Multi-layered flows: application to intestinal flows	23
2.2	Flows in vessels of finite length: application to swallowing in oesophagus	23
2.2.1	Flows with dilating wave amplitude: application to sliding hiatus hernia	24
2.2.2	Heat transfer in peristaltic transport: application to cryosurgery	26
2.2.3	Flows in asymmetric channels	28
2.2.4	Flows in non-uniform vessels: application to vas deferens and uterine cavity	29
2.2.5	Flows of nano-fluids	30
2.2.6	Flows in elastic tubes	32
2.2.7	Flows in porous media	33
2.2.8	Electro-osmosis/electric double layer/electro-kinetic flows	35
3	Transportation of micro-polar fluid by dilating peristaltic waves: Application to flows in normal oesophagus	37
3.1	Introduction	37
3.2	Formulation of the problem	40
3.3	Solution of the problem	44
3.4	Reflux limit	52
3.5	Results and Discussion	56
3.5.1	Effect of dilating wave amplitude on pressure	56
3.5.2	Effect of Coupling Number on pressure	57
3.5.3	Effect of micro-polar parameter on pressure	59
3.5.4	Effect of dilating wave amplitude on wall shear stress	62
3.6	Effect of dilating wave amplitude on reflux	62
3.7	Conclusion and Physical Interpretation	64
4	Transportation of particulate suspension in a Newtonian fluid by dilating peristaltic waves in a tube of uniform cross-section: Application to flows in normal oesophagus	67
4.1	Introduction	67
4.2	Mathematical Formulation	70
4.3	Perturbation Solution	74
4.3.1	Solution for the zeroth order system	79

4.3.2	Solution of the first-order system	81
4.3.3	Solution for the second-order system	83
4.3.4	Stream function	90
4.4	Discussions and Results	91
4.5	Conclusions	100
5	Transportation of micro-polar fluids by means of dilating peristaltic waves in a tube of non-uniform cross-sectional area: Application to sliding hiatus hernia	103
5.1	Introduction	104
5.2	Mathematical Formulation	108
5.3	Solution of the Problem	112
5.4	Result and discussions	118
5.4.1	Effect of dilating wave amplitude on pressure	119
5.4.2	Effect of the gradient parameter on pressure	120
5.4.3	Effect of coupling number on pressure	124
5.4.4	Effect of the micropolar parameter on pressure	125
5.5	Conclusion and physical interpretations	126
6	Transportation of micro-polar fluids by means of peristaltic waves of dilating amplitude in a tube of exponentially changing cross-sectional area: Application to sliding hiatus hernia	131
6.1	Introduction	132
6.2	Problem Formulation	135
6.3	Mathematical Analysis	140
6.3.1	Time averaged volume flow rate	146
6.3.2	Reflux limit	149
6.4	Numerical results and discussion	153
6.4.1	Divergence along the entire length of the tube	154
6.4.1.1	Effect of wave amplitude dilation	154
6.4.1.2	Effect of Coupling Number	154
6.4.1.3	Effect of micropolar parameter	157
6.4.1.4	Effect of gradient parameter	157
6.4.1.5	Effect on wall shear stress	160
6.4.2	Divergence limited to a small length near the other end	163
6.4.3	Effect of wave amplitude dilation on reflux	166
6.5	Conclusion and physical interpretations	169
7	Analysis of unsteady two-phase peristaltic flow in a tube of exponentially changing cross sectional area: Application to sliding hiatus hernia	177
7.1	Introduction	177

7.2	Mathematical Formulation	180
7.3	Method of Solution	185
7.3.1	Solution for the zeroth order system	190
7.3.2	Solution of the first-order system	192
7.3.3	Solution for the second-order system	194
7.4	Results and discussion	201
7.4.1	Analysis of flow in the relaxed and contracted regions	202
7.4.2	Impact of the gradient parameter on flow	202
7.4.3	Effect of the wave-amplitude dilation on flow	204
7.4.4	Effects of the gradient parameter on the radial profile of the axial velocity	216
7.4.5	Effect of the dilation parameter on the radial profile of the axial velocity	218
7.5	Conclusions	218
8	Conclusions and further scope of the work	221
8.1	Further scope of the study	225
	References	227
	List of Publications	257

List of Figures

1.1	Swallowing and regurgitation of cow (Courtesy https://www.futurelearn.com/info/courses/climate-smart-agriculture/0/steps/26577)	2
1.2	The peristaltic transport through oesophagus (Courtesy http://kypho.com/deglutition.html)	5
1.3	The digestive system (Courtesy https://www.pinterest.com/pin/361765782566604002/)	6
1.4	Peristaltic pump (Courtesy https://2018.igem.org/Team:Rotterdam_HR/Hardware_Peristaltic_Pump)	7
1.5	Normal oesophagus and hiatus hernia (Courtesy: https://www.msmanuals.com/en-au/professional/gastrointestinal-disorders/esophageal-and-swallowing-disorders/h hiatus-hernia)	9
3.1	Schematic diagram of oesophagus, (based on equation (3.16)), represents the propagation of a progressive wave, where $\delta = \frac{a}{\lambda}$, Re , Q denote respectively the wave number, the Reynolds number and the volume flow rate.	42
3.2	Pressure distribution along axial distance at different time instants showing the effect of dilation parameter ω . Other parameters are taken as $l = 3$, $\phi = 0.7$, $N = 0.10$, $m = 1.0$.	58
3.3	Pressure distribution along axial distance at different time instants showing the effect of coupling number N . Other parameters are set as $l = 3$, $\phi = 0.7$, $\omega = 0.01$, $m = 1.0$.	60
3.4	Pressure distribution along axial distance at different time instants showing the effect of micro polar parameter m . Other parameters are taken as $l = 3$, $\phi = 0.7$, $\omega = 0.01$, $N = 0.50$.	61
3.5	Wall shear stress τ_w distribution along axial distance at different time instants showing the effect of dilation parameter ω . Other parameters are taken as $l = 3$, $\phi = 0.7$, $N = 0.10$, $m = 1.0$.	63
3.6	The diagram exhibits the relation between time averaged flow rate \tilde{Q} and wave amplitude ϕ showing the effect of dilation parameter ω . Other parameters are taken as $N = 0.10$, $m = 1.0$.	64

4.1	The schematic diagram of wall positions of oesophagus when a peristaltic wave of slightly dilating amplitude propagates along it with velocity c	70
4.2	((4.2a), (4.2b)) - ((4.2c), (4.2d)) Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.3$ and ((4.2a), (4.2b)) $t = 0.0$ ((4.2c), (4.2d)) $t = 0.4$. Other parameters are taken as $\delta = 0.08$, $\omega = 0.02$, $C = 0.12$, $\phi = 0.7$, $Re_0 = 5$, $\bar{Q} = 1.0$, $\bar{Q}^{(1)} = 15$, $\bar{Q}^{(2)} = 15$, $M = 1139$. ((4.2a), (4.2c)) correspond to the present analysis while ((4.2b), (4.2d)) correspond to the Pandey and Singh, 2018).	93
4.3	The radial profiles of the axial velocity respectively of the fluid and solid particles versus the tube radius at the fixed axial position $x = 0.6$ and $t = 0.4$. Other parameters are taken as $\delta = 0.08$, $\omega = 0.02$, $C = 0.12$, $\phi = 0.7$, $Re_0 = 5$, $\bar{Q} = 1.5$, $\bar{Q}^{(1)} = 20$, $\bar{Q}^{(2)} = 20$, $M = 1139$	94
4.4	Radial profiles the axial velocity respectively of (4.4a)-(4.4b) the fluid and (4.4c)-(4.4d) solid particles versus the radial distance at the fixed axial position $x = 0.3$ and time $t = 0.9$ showing the impact of amplitude dilation parameter ω . Other parameters are taken as $\delta = 0.08$, $C = 0.12$, $\phi = 0.7$, Re_0 , $\bar{Q} = 1.5$, $\bar{Q}^{(1)} = 20$, $\bar{Q}^{(2)} = 20$, $M = 1139$. Solid, dashed and dashed dotted line correspond respectively to $\omega = 0.0$, $\omega = 0.05$ and $\omega = 0.10$. Figures (4.4b)-(4.4d) correspond to Pandey and Singh (2018).	96
4.5	The impact of particulate matter concentration (i.e. the volume fraction C) on the axial velocity of the fluid plotted versus radial distance at the fixed axial position $x = 0.3$ for $\omega = 0.02$, $\phi = 0.8$, $Re_0 = 5$, $\bar{Q} = 1.5$, $\bar{Q}^{(1)} = 20$, $\bar{Q}^{(2)} = 20$, $t = 0.9$, $M = 1139$, (4.5a) $\delta = 0.08$, (4.5b) $\delta = 0.08$	97
4.6	Radial velocity profile of the fluid along the radial distance for different volume fraction of particles at $x = 0.2$ for $t = 0.3$, $\omega = 0.02$, $\phi = 0.6$, $Re_0 = 5$, $\delta = 0.08$, $\bar{Q} = 1.5$, $\bar{Q}^{(1)} = 20$, $\bar{Q}^{(2)} = 20$, $M = 1139$. Solid line, dashed line and dashed dotted line correspond respectively to $C = 0.0$, $C = 0.16$ and $C = 0.32$	97
4.7	The effect of δ on the relation between the pressure gradient and the time-averaged volume flow rate for different volume fraction of particles with $\omega = 0.02$, $\phi = 0.6$, $Re_0 = 5$, $\bar{Q}^{(1)} = 15$, $\bar{Q}^{(2)} = 15$, $M = 1139$, $x = 0.8$, $t = 0.4$, (4.7a) $\delta = 0.08$, (4.7b) $\delta = 0.08$	98
4.8	Streamlines in the fixed frame with $\omega = 0.02$, $C = 0.12$, $\phi = 0.6$, $Re_0 = 5$, $\delta = 0.08$, $\bar{Q}^{(1)} = 20$, $\bar{Q}^{(2)} = 20$, $M = 1139$, $t = 1.0$ at different time-averaged volume flow rates (4.8a) $\bar{Q} = 1.2$, (4.8b) $\bar{Q} = 3.2$, (4.8c) $\bar{Q} = 5.0$ and (4.8d) $\bar{Q} = 5.1$	99
5.1	Diagrams for (a) Normal oesophagus and (b) Oesophagus suffering from sliding hiatus hernia. (Source of figure 5.1 https://www.melbournegastroscopy.com.au/surgery-for-gastric-reflux-hiatus-hernia/)105	

5.2	The diagram, based on equation (5.12), represents the propagation of a progressive wave along the walls of the tube containing fluid, which undergoes contraction and relaxation but no expansion beyond the boundary.	110
5.3	((a)-(i)) Pressure distribution along axial distance at different time instant showing the effect of dilation parameter ω . Other parameters are taken as $l = 3, \phi = 0.7, N = 0.15, m = 1.0, b = 0.03$	121
5.4	((a)-(i)) Pressure distribution along axial distance at different time instant showing the effect of the gradient parameter b . Other parameters are taken as $l = 3, \phi = 0.7, \omega = 0.001, N = 0.15, m = 1.0$	124
5.5	((a)-(i)) Pressure distribution along axial distance at different time instant showing the effect of coupling number N . Other parameters are taken as $l = 3, \phi = 0.7, \omega = 0.001, m = 1.0, b = 0.03$	127
5.6	((a)-(i)) Pressure distribution along axial distance at different time instant showing the effect of Micropolar parameter m . Other parameters are taken as $l = 3, \phi = 0.7, \omega = 0.001, N = 0.50, b = 0.03$	130
6.1	Diagrams for (a) Normal oesophagus and (b) Oesophagus suffering from sliding hiatus hernia (Figure 6.1 is taken from Pandey and Singh (2019)).	133
6.2	Schematic diagram (based on equation 6.16) of the flow under peristaltic waves of progressively dilating amplitude in an exponentially diverging tube.	138
6.3	Pressure distribution along axial distance at different time instant showing the effect of dilation parameter ω . Other parameters are taken as $l = 3, \phi = 0.7, N = 0.20, b = 0.04, m = 1.00$	157
6.4	Pressure distribution along axial distance at different time instant showing the effect of coupling number N . Other parameters are taken as $l = 3, \phi = 0.7, \omega = 0.001, b = 0.04, m = 1.00$	160
6.5	Pressure distribution along axial distance at different time instant showing the effect of micro polar parameter m . Other parameters are taken as $l = 3, \phi = 0.7, \omega = 0.001, b = 0.04, N = 0.20$	163
6.6	Pressure distribution along axial distance at different time instant showing the effect of gradient parameter b . Other parameters are taken as $l = 3, \phi = 0.7, \omega = 0.001, N = 0.20, m = 1.00$	166
6.7	Wall shear stress τ_w distribution along axial distance at different time instant showing the effect of dilation parameter ω . Other parameters are taken as $l = 3, \phi = 0.7, N = 0.20, b = 0.04, m = 1.0$	169
6.8	Wall shear stress τ_w distribution along axial distance at different time instant showing the effect of gradient parameter b . Other parameters are taken as $l = 3, \phi = 0.7, N = 0.20, m = 1.0$	172

6.9	Pressure variation along the length of oesophagus at different time instants showing the effect of gradient parameter b when tube diverges at distal end. Other parameters are taken as $l = 3$, $\phi = 0.7$, $N = 0.20$, $m = 1.0$	175
6.10	The diagram exhibits the relation between time averaged flow rate \tilde{Q} and wave amplitude ϕ showing the effect of dilation parameter ω . Other parameters are taken as $N = 0.20$, $m = 1.0$, $b = 0.002$. . .	175
7.1	Schematic diagram of wall positions of oesophagus, based on the equation (7.9), when a peristaltic wave of slightly dilating amplitude propagates along it with velocity c	181
7.2	Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.2a) $t = 0.00$ (7.2b) $t = 0.40$ (7.2c) $t = 0.80$. Other parameters are taken as $b = 0.00$, $\omega = 0.00$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$	203
7.3	Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.3a) $t = 0.00$ (7.3b) $t = 0.40$ (7.3c) $t = 0.80$. Other parameters are taken as $b = 0.00$, $\omega = 0.025$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$	205
7.4	Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.4a) $t = 0.00$ (7.4b) $t = 0.40$ (7.4c) $t = 0.80$. Other parameters are taken as $b = 0.04$, $\omega = 0.025$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8.00$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$	206
7.5	Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.5a) $t = 0.00$ (7.5b) $t = 0.40$ (7.5c) $t = 0.80$. Other parameters are taken as $b = 0.08$, $\omega = 0.025$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$	207
7.6	Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.6a) $t = 0.00$ (7.6b) $t = 0.40$ (7.6c) $t = 0.80$. Other parameters are taken as $b = 0.08$, $\omega = 0.025$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$	209
7.7	Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.7a) $t = 0.00$ (7.7b) $t = 0.40$ (7.7c) $t = 0.80$. Other parameters are taken as $b = 0.02$, $\omega = 0.003$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$	210

- 7.8 Axial velocity profile of the fluid and solid particles versus the tube length at the fixed radial distance $r = 0.30$ and (7.8a) $t = 0.00$ (7.8b) $t = 0.40$ (7.8c) $t = 0.80$. Other parameters are taken as $b = 0.02$, $\omega = 0.04$, $\phi = 0.76$, $\delta = 0.05$, $C^{(1)} = 8.00$, $Re_0 = 8$, $\tilde{Q} = 1.60$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 2500$ 211
- 7.9 Radial velocity profile of the axial velocity of the fluid and solid particles versus the tube length at the at the fixed axial position $x = 0.651$ and (7.9a) $t = 0.00$ (7.9b) $t = 0.40$, (7.9c) $t = 0.80$. Other parameters are taken as $b = 0.004$, $\omega = 0.002$, $\phi = 0.60$, $\delta = 0.05$, $C^{(1)} = 4$, $Re_0 = 8$, $\tilde{Q} = 1.40$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 1200$ 213
- 7.10 Radial profile of the axial velocity profile of the fluid versus the radial distance at the fixed axial position $x = 0.30$ and time (7.10a) $t = 0.00$ (7.10b) $t = 0.40$, (7.10c) $t = 0.80$. Other parameters are taken as $b = 0.002$, $\delta = 0.055$, $C^{(1)} = 8$, $\phi = 0.72$, $Re_0 = 8$, $\tilde{Q} = 1.51$, $\tilde{Q}^{(1)} = 20$, $\tilde{Q}^{(2)} = 20$, $M = 1200$. Solid line, dashed line and the solid line with marker correspond respectively to $\omega = 0.00$, $\omega = 0.08$ and $\omega = 0.16$ 214
- 7.11 Radial profile of the axial velocity profile of the particles versus the radial distance at the fixed axial position $x = 0.30$ and time (7.11a) $t = 0.00$ (7.11b) $t = 0.40$, (7.11c) $t = 0.80$. Other parameters are taken as $b = 0.002$, $\delta = 0.055$, $C^{(1)} = 8$, $\phi = 0.72$, $Re_0 = 8$, $\tilde{Q} = 1.51$, $\tilde{Q}^{(1)} = 20$, $\tilde{Q}^{(2)} = 20$, $M = 1200$. Solid line, dashed line and the solid line with marker correspond respectively to $\omega = 0.00$, $\omega = 0.08$ and $\omega = 0.16$ 215
- 7.12 The impact of δ on the relation between the axial velocity of the fluid and the radial distance for different volume fractions at the fixed axial position $x = 0.30$ for (7.12a) $\delta = 0$ (7.12b) $\delta = 0.07$. Other parameters are taken as $b = 0.04$, $\omega = 0.002$, $\phi = 0.72$, $C^{(1)} = 4$, $Re_0 = 8$, $\tilde{Q} = 1.51$, $\tilde{Q}^{(1)} = 20$, $\tilde{Q}^{(2)} = 20$, $t = 0.00$, $M = 1200$. Solid line, dashed line and the solid line with marker respectively correspond to $C = 0.0$, $C = 0.32$ and $C = 0.64$ 216
- 7.13 Radial velocity profile of the fluid along the radial distance for different volume fraction of particles at $x = 0.20$ for $t = 0.30$, $b = 0.002$, $\omega = 0.001$, $\phi = 0.70$, $Re_0 = 4$, $\delta = 0.061$, $\tilde{Q} = 1.52$, $\tilde{Q}^{(1)} = 20$, $\tilde{Q}^{(2)} = 20$, $M = 30$. Solid line, dashed line and the solid line with marker correspond respectively to $C = 0.00$, $C = 0.183$ and $C = 0.366$. 217
- 7.14 The impact of gradient parameter b on radial profile of the axial velocity profile of the fluid versus the radial distance at the fixed axial position $x = 0.651$ and gradient parameter (7.14a) $b = 0.33$ (7.14b) $b = 0.66$. Other parameters are taken as $\omega = 0.002$, $\delta = 0.06$, $C^{(1)} = 4$, $\phi = 0.60$, $Re_0 = 8$, $\tilde{Q} = 2.4$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 1200$, $t = 0.55$ 217

- 7.15 The impact of dilation parameter ω on radial profile of the axial velocity of the particles versus radial distance at the fixed axial position $x = 0.651$ and dilation parameter (7.15a) $\omega = 0.00$ (7.15b) $\omega = 0.40$ (7.15c) $\omega = 0.80$. Other parameters are taken as $b = 0.001$, $\delta = 0.05$, $C^{(1)} = 4$, $\phi = 0.60$, $Re_0 = 8$, $\tilde{Q} = 2.4$, $\tilde{Q}^{(1)} = 18$, $\tilde{Q}^{(2)} = 18$, $M = 1200$, $t = 0.20$ 219

PREFACE

The present thesis deals with the flow dynamics of oesophageal swallowing in normal and pathological states. Kahrilas *et al.* (1995) published a report based on their experiments on healthy and non-healthy persons, which had discovered higher pressure in the distal part of the oesophagus than that in the pharyngeal part. Meanwhile Xia *et al.* (2009) submitted an anatomical report on oesophagus during swallowing. Pandey *et al.* (2017) analysed the two reports, concluded that the wavelength of the peristaltic waves increase progressively during swallowing to drag food boluses that gradually get globular. They also presented a mathematical model for swallowing of Newtonian fluids. This brought forward the short coming of numerous models which considered oesophageal swallowing by peristaltic waves of uniform amplitude. This discovery also leads to bring about changes in the fabricated peristaltic pumps designed for engineering as well as medical purposes.

Since swallowing in oesophagus is managed by progressive peristaltic waves which not only drag the fluid within the oesophagus but also create more pressure in the distal part, therefore an appropriate wall equation, which was developed by Pandey *et al.* (2017), has been considered for analyses. Such peristaltic waves have amplitudes that increase as the waves progress and finally create higher pressure in the distal part.

The food we consume is masticated in the mouth and mixed with saliva. Thus it transforms itself into a semi-fluid which is put inside the oesophagus by means of the tongue. Therefore, the fluid has been considered of non-Newtonian nature or with particulate suspension. There are models for swallowing in the

normal oesophagus under the influence of dilating peristaltic waves by considering the fluid as Newtonian [Pandey *et al.*, 2017] and non-Newtonian [Pandey and Tiwari, 2017; Pandey and Singh, 2018]. However, the non-Newtonian nature of micro-polar type was left untouched. Therefore, fluids with non-Newtonian micro-polar properties have been considered for investigating the flow dynamics of swallowing in oesophagus. An equivalent analysis for particulate suspension in a Newtonian fluid has also been attempted. These analyses are useful when the oesophagus is normal.

Oesophagus is, however, not always normal. Sometimes it suffers from some dysfunctions. Hiatus hernia is such a dysfunction which gives adverse impacts on swallowing. Circumstances are thus changed. The oesophagus does not remain uniform. For this reason, we analyse swallowing in circular cylindrical tube which diverges or converges, not necessarily from the beginning because herniation of oesophagus affects it near the lower end. The geometry of the oesophagus becomes uneven. The non-uniformity may be linear or non-linear. The detailed discussions are given in the introduction of the chapters while the aforementioned investigations are categorized chapter-wise. The conclusions drawn are found to be quite useful and may help clinicians diagnose the dysfunction and prescribe remedial measures.

ABSTRACT

The dynamics of swallowing within oesophagus has been investigated in the thesis. Chapter 1 discusses the role mathematical modelling plays in the field of applied mathematics, particular how it helps draw useful inferences after formulating a real word phenomenon mathematically. Then it describes the anatomy and physiology of oesophagus. The various types of peristalsis observed in the nature has also been described. Then is briefly described how a man-made peristaltic pump works.

A review of the literature that revolves round the topic of the thesis has been very carefully presented in Chapter 2, which contains theoretical and experimental investigations both in the chronological order. With its growth, research in peristalsis produced several off-shoots and branches.

In Chapter 3, we analytically investigate axi-symmetric flow of a micro-polar fluid induced by peristaltic waves with progressively dilating amplitude. By means of mathematical formulation we examine its impact on swallowing of single food bolus through oesophagus. The study suggests that achalasia patients should avoid the consumption of micro-polar fluids. It is also concluded that reflux action weakens with dilation of wave amplitude for micro-polar flows.

The model in Chapter 4 characterises the flow behaviour of suspended particles swallowing through oesophagus. Transport of particle-fluid mixture due to dilating peristaltic waves on a circular cylindrical tube has been investigated. It is observed that the fluid axial velocity exceeds the solid particle velocity almost everywhere. However, at the wall the axial velocity of the fluid is zero due to the no-slip condition imposed on it; but the suspended particulate material has non-zero positive axial velocity. It is further observed that the axial velocity is negative in the regions near maximum occlusions paving way to instantaneous backward flow.

In Chapter 5, we present a mathematical formulation that deals with the flow of micro-polar fluid in a circular cylindrical tube of non-uniform cross-sectional area induced by peristaltic waves of increasing amplitude. This is an intended act to model swallowing of various types of foods in a herniated oesophagus. Due to sliding hiatus hernia, the cross section of the lower oesophagus does not remain uniform. The impact of bulging, which is formed by various combinations of divergence and convergence, has been examined.

Chapter 6 models a peristaltically driven flow when the cross section of the tube changes exponentially. This may resemble the shape of the oesophagus when it suffers from sliding hiatus hernia, that is, when the abdomen protrudes through the hiatus. The fluid transported is considered micro-polar while the peristaltic waves propagating along wall too dilate exponentially. It is observed whether the tube diverges entirely or partly, pressure required for carrying the fluid is less, hence facilitating easier swallowing, which causes sliding hiatus hernia to go unnoticed.

Chapter 7 presents the analysis of the transport of a Newtonian fluid with suspended solid particles in a tube which diverges exponentially under the

influence of peristaltic waves whose amplitude too increases progressively. Besides engineering applications, this models the state of hiatus hernia of an oesophagus swallowing a mixture of fluid and solid particles.

The thesis concludes with overall conclusions and further scope of study.

