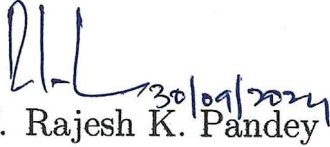


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It is certified that the work contained in this thesis titled "*Numerical Approximations of Some Fractional Sturm-Liouville Problems*" by *Eti Goel* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.



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I, *Eti Goel*, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of *Prof. Rajesh K. Pandey* from *July, 2019* to *September, 2024* in the *Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi*. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.


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