Chapter 6

Conclusion

This chapter concludes the thesis, encapsulating the key discoveries and final thoughts from the comprehensive study. Problems investigated in Chapter 2, Chapter 3 and Chapter 4 are motivated from Zagier's conjecture, speculating connection between non-trivial zeros of Riemann zeta function and Lambert series associated with a special cusp form which later proved by Hafner and Stopple. Similar result has been obtained by Chakraborty, Kanemitsu and Maji for any cusp form.

In **Chapter 1**, we have presented the essential preliminaries and relevant literature associated with the research conducted. This includes definitions of various mathematical concepts such as arithmetical function, Lambert series, Gamma function, Riemann zeta function, Meijer G-function, Whittaker function, hypergeometric function, modular forms, Jacobi forms, and Siegel modular forms. Further, we have discussed L-function associated to Fourier coefficients of these modular forms and their analytic properties.

In **Chapter 2**, we have studied the asymptotic behaviour of a Lambert series associated to Fourier Jacobi coefficients of Siegel cusp forms of weight k and degree 2; utilising Cauchy's residue theorem, analytic properties of associated Dirichlet series and bound of the defined Dirichlet series, bound for Riemann zeta function and Stirling's bound for Gamma function. Further, we have established the connection of defined Lambert series with non-trivial zeros of Riemann zeta function. To conclude the main result we have used particular value of Meijer G-function and asymptotic result of Meijer G-function in terms of Whittaker function.

In **Chapter 3**, we have generalised the result obtained in Chapter 2 by studying the asymptotic behaviour of similar Lambert series for Siegel cusp forms of different weights k_1, k_2 and degree $n > 1$, using the similar technique which we have used to prove the result in Chapter 2. Also, we have studied twist of this Lambert series by a Dirichlet character and established relation between the Fourier Jacobi coefficients and non-trivial zeros of Dirichlet L-function, using the analytic properties of Dirichlet series studied by Kohnen, Krieg and Sengupta, under the assumption of bracketing condition. Moreover, the result obtained involves Bernoulli numbers and Whittaker function.

In **Chapter 4**, we have studied the asymptotic behaviour of the Lambert series which is the constant term of the Fourier series expansion of two normalised Hecke eigenforms of weight k_1 and K_2 under the assumption of simplicity hypothesis for non-trivial zeros of Riemann zeta function. From the main result, Chakraborty, Kanemitsu and Maji's result has been obtained along with Zagier's prediction. Further, the result obtained can be viewed as a special case of the result obtained in Chapter 2, as the Fourier-Jacobi coefficients of Siegel cusp forms coincides with Fourier coefficients of normalised Hecke eigenforms for $n = 1$.

In **Chapter 5**, we have observed the oscillatory behaviour of product of L-function through the asymptotic behaviour of higher moments of generalised divisor function associated to Fourier coefficients of Rankin-Selberg L-functions associated to normalised Hecke eigenforms. We have extended the result of Zhai. He has studied summatory function over the sequence supported at sum of two squares while we have examined the summatory function over a sparse sequence obtained from binary quadrtaic form of a fixed negative discriminant utilising character sum, Deligne's bound, Weil's bound. We have obtained a better estimate which improves the result of Hua.

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