Chapter 2

Shock wave solution for the planar, cylindrically, and spherically symmetric flows of non-ideal relaxing gas *

2.1 Introduction

The present chapter concerns with the study of solution of the problem of propagation of shock wave for planar, cylindrically symmetric, and spherically symmetric flows in a one-dimensional non-ideal relaxing gas. Many phenomenon in the area of astrometry, aerographic, explosions, aerostatics, etc. are modelled by hyperbolic

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PDEs and they have extensive use in several real circumstances when the motion of wave is included, some of the applications are oceanology, earthward motion, planetary motion, electrodynamics, quantum-theory, geophysics, and limnology. The salient feature of the PDEs system is that their solution encounters basal waves such as shock waves. The shock wave's study was done by its applicability in numerous fields such as particle physics, quantum mechanics, nucleonics, astronomical physics, geophysics, and various branches of engineering. Shocks are helpful in generating energy and their study in non-ideal gas has obtained significance in many industrial applications such as thermonuclear reactions, chemical actions, natural processes, aeronautical engineering, and aviation technology, fusion, and fission, etc. The fact-finding of non-ideal relaxing gas phenomena with shock waves is more complicated in comparison to an ideal gas. Hence, because of the effect of non-idealness, the relaxing gas flow parameter carries out remarkable results. Schiffer et al. [70] studied one-dimensional granular crystals interaction of HNSWs with adjacent elastic solids having a spherical void. Boillat and Ruggeri [71] studied the characteristic shock structure using Born Infeld theory discussing the spin of an electron. Tomar et al. [72] calculated similarity exponent with the help of Guderley's theory and demonstrated the application of perturbation series technique.

In continuum mechanics, non-ideal compressible fluids take part in a significant way in numerous fields of applications, including astrolithology, selenology, oceanography, hypersonic aerodynamics, astrochemistry (solar-system, interstellar medium), and far more. In various theoretical and practical studies, it is observed that waves bring out distinct behavior for distinct values in non-ideal gases (such as Helium, Carbon Monoxide, Ethane). In gas dynamics, when non-idealness is considered with relaxation, the system becomes more complicated than classical gas dynamics case because of non-linear coupling between them and non-existence of the self-similar nature of the classical flow field. Sahu [73] discussed the effect of various parameters like gravitation parameter, non-idealness, Cowling Number along with radiative heat transfer parameter on self-gravitating non-ideal gas under magnetic field. G. Nath and A. Devi[74] obtained the similarity solution for shocks in non-ideal medium under the effect of various parameters. Using the Lie group symmetry method, solution is obtained for shocks with magnetic field by Gupta et al. [75]. The authors in Zhao et al.[76], Jena and Sharma [77], Sharma [8], Singh et al. [78], Wang et al.[79], Zafar and Sharma [80] discussed many interesting problems related to the influence of non-idealness on the solution of shocks using several approaches. Recently Singh et al. [81] analyzed the progressive behavior of imploding strong shocks in non-ideal medium.

At high temperature, when gas is pressurized by external push or pull, its internal energy causes initially to rise translation energy. It is followed by relaxation from where modes are transformed from translational to rotational and translational to vibrational as long as it reaches the state of equilibrium [[8],[82],[83],[84]]. An important characteristic of relaxing gas is that the behavior of small amplitude motion is governed using Burgers equation whose solution depicts the property of convective steepening which gets diffused by diffusive nature of relaxation. Also, system governing the relaxing gas motion makes it important for more exhaustive study. Kustova et al. [85] studied shock wave structure and relaxation process of carbon dioxide under weak and strong approximations. Arima et al. [86] constructed rational extended thermodynamics (RET) theory of dense polyatomic gases by taking into account the experimental evidence that the relaxation time of molecular rotation and that of molecular vibration are quite different from each other.

Different studies on interesting features of shocks in non-ideal relaxing medium are carried out by several researchers. Ockendon and Spence [87] studied the evolution of waves by considering the effect of viscosity and heat conduction. Sharma and Radha [88, 89] have discussed the impact of relaxation on the evolutionary behavior of shocks of arbitrary strength. Further, Jena and Sharma [83] studied the existence and uniqueness of reflected and transmitted shocks in relaxing gas. The study of the solution of non-linear waves in relaxing gas with non-idealness is typical problem from the mathematical point of view. Many researchers have studied the non-linear wave solution in non-ideal relaxing gas using several approaches like, Singh and Jena [90] studied the propagation of the central expansive wave in non-ideal relaxing medium using perturbation method, and Siddiqui et al. [91] used HAM (Homotopy Analysis Method) to study the effect of relaxation and non-linearity on small amplitude waves in relaxing gas. Also, the similarity solution of an imploding shock for radially symmetric flow using Lie group method was studied by Arora et al. [92]. Propagation of weak and strong shock waves in non-ideal relaxing gas using systematic approximation method are analyzed by Shah and Singh [93]. The process of growth and decay phenomenon of non-linear waves in various gas dynamics regimes have been studied by the authors Chaturvedi et al. [94, 95], Gupta et al. [96], Srivastava et al. [97].

The present chapter is related to the study of the growth and decay of shocks in a van der Waals gas. Here, we use a simple relaxing gas model which gives detailed description of all the required characteristics. The quantity $H(p, \rho, \sigma)$, which is present in the governing equations, denotes the rate of change of vibrational energy σ, is a known function. The situation $H = 0$ corresponds to a physical process involving no relaxation; indeed, it includes both the cases in which the vibrational mode is either inactive or follows the translational mode according as the flow is either frozen

($\sigma = constant$) or in equilibrium ($\sigma = \overline{\sigma}$), where $\overline{\sigma}$ is the equilibrium value of σ evaluated at local p and ρ. As the relaxation mechanism makes the interaction problem considerably cumbersome, we felt that a simple analysis applicable to planar and radially symmetric motions, where we can use special solutions, would be of interest to elucidate the effects of relaxation on the wave patterns, that finally develop. As the governing system with relaxation is still hyperbolic, in order to study the shock interaction problem, we look for a particular solution of the governing system through which the shock propagates, and investigate the effects of initial discontinuities associated with the incident wave, the geometry of the fluid flow together with the background state at the rear of shock, and the relaxation mechanism present in the flow on the evolutionary behaviour of a characteristic shock and the reflected and transmitted waves after the collision. To our knowledge, a comprehensive analytical and numerical account of the evolution of a characteristic shock has not been studied previously. This chapter brings out some interesting features of the evolutionary behaviour of waves in a relaxing gas.

This chapter is organised in the following manner: In Sec. 2.1, a small introduction and history of relaxing gas are given. The system of PDEs, which govern one-dimensional planar and non-planar flow in non-ideal relaxing gas and characteristic curve are presented in Sec. 2.2. In the Sec. 2.3, characteristic variables are introduced and the fundamental equations are written in the form of these new variables. In Sec. 2.4, we derive a differential equation and its solution, that investigates the shock formation process. Its behavior is discussed in Sec. 2.5. Also, effect of various parameter on formation of shocks and its deformation is menstioned in this section. The last Sec. contains the conclusions of this study.

2.2 Governing equations and its characteristics

We consider one-dimensional flow in non-ideal relaxing gas describing inviscid planar(m=0), cylindrically symmetric(m=1) and spherically symmetric(m=2) flows. The governing system of equations are expressed as (Pandey and Sharma [77])

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} + \frac{m\rho u}{x} = 0,\\
&\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} = 0,\\
&\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho a^2\left(\frac{\partial u}{\partial x} + \frac{mu}{x}\right) = -(\gamma-1)\rho H,\\
&\frac{\partial \sigma}{\partial t} + u\frac{\partial \sigma}{\partial x} = H,
\end{aligned} \tag{2.1}$$

where ρ, x, γ, t, p, σ, u, b, are density, spatial coordinate, ratio of specific heats, time, pressure, vibrational energy, particle velocity, van der Waals excluded volume which lies between $0.9\times10^{-3} \leq b \leq 1.1\times10^{-3}$ (Pandey and Sharma [77]) respectively. The case $b = 0$ corresponds to ideal relaxing gas (Scott and Johannesen[82]), a is the speed of sound defined as

$$a^2 = \frac{\gamma p}{(1-\overline{b})\rho}, here \quad \overline{b} = b\rho.$$

The quantity H, which depends upon p, ρ, and σ denotes the rate of change of vibrational energy and stated as

$$H = \frac{\overline{\sigma} - \sigma}{\tau},$$

where

$$\overline{\sigma} = \sigma_e + c(p\frac{(1-b\rho)}{\rho} - (1-b\rho_e)\frac{p_e}{\rho_e}).$$

Here, the suffix e denotes initial equilibrium reference state; τ is the relaxation time and c symbolizes the ratio of vibrational specific heat to the specific gas constant. We have taken the following equation of state:

$$p = \frac{\rho RT}{1 - b\rho},$$

where T and R are temperature and specific gas constant respectively.

We can write equation (2.1) as follows

$$U_t + AU_x + B = 0, \tag{2.2}$$

here U and B are 4×1 ordered column vectors and A is 4×4 ordered matrix, respectively, given below

$$U = \begin{pmatrix} \rho \\ u \\ p \\ \sigma \end{pmatrix}, \quad B = \begin{pmatrix} \frac{m\rho u}{x} \\ 0 \\ \frac{m\rho a^2 u}{x} - (\gamma - 1)\rho H \\ H \end{pmatrix}, \quad \text{and} \quad A = \begin{pmatrix} u & \rho & 0 & 0 \\ 0 & u & 1/\rho & 0 \\ 0 & \rho a^2 & u & 0 \\ 0 & 0 & 0 & u \end{pmatrix}.$$

Here function $U(x,t)$ is continuous everywhere in the characteristic plane excluding characteristic curve $C(t)$ and satisfies equation (2.2). Across $C(t)$, U remains continuous, but the derivatives U_x and U_t are discontinuous. Now, along $C(t)$, we get

$$\frac{\partial}{\partial t}[U] = [U_t] + \frac{dC(t)}{dt}[U_x], \tag{2.3}$$

where $\frac{\partial}{\partial t}$ denotes the partial derivative with respect to time t.
As U is a continuous function, so $[U] = 0$.

Taking jump in (2.2) and using (2.3) with $[U] = 0$, we get

$$\left(A - \frac{dC}{dt} I\right) [U_x] = 0, \tag{2.4}$$

where I is the identity matrix of order 4. From (2.4), it is observed that $\frac{dC}{dt}$ is an eigenvalue of the matrix A along the characteristic curve $C(t)$. The characteristic curves corresponding to (2.2), are

$$\frac{dx}{dt} = u \pm a \qquad \text{and} \qquad \frac{dx}{dt} = u\ , \tag{2.5}$$

which represent waves propagating in positive and negative directions along x axis and path of the particle respectively.

2.3 Shock waves in characteristic plane

To transform the governing equations, we introduce ξ and ϕ as the new independent variables, where ξ and ϕ are particle and wave tag respectively. ξ is constant along the particle path $\frac{dx}{dt} = u$, and ϕ is constant along the characteristic $\frac{dx}{dt} = u + a$. Therefore, if the characteristic wavefront moves through a particle at time t^*, then the particle tag ξ can be found as $\xi = t^*$, and if outgoing wave is generated by the piston at time $t^{'}$, then wave tag ϕ can be written as $\phi = t^{'}$.

Hence, for every pair (ϕ, ξ), a pair (x, t) can be obtained in such a way that $x = x(\phi, \xi)$, $t = t(\phi, \xi)$. Now, ϕ and ξ satisfy the following conditions

$$x_\phi = u t_\phi, \qquad x_\xi = (u + a) t_\xi. \tag{2.6}$$

With the help of (2.6), we can transform U_t and U_x as

$$U_t = \frac{U_\xi x_\phi - U_\phi x_\xi}{J}, \qquad U_x = \frac{U_\phi t_\xi - U_\xi t_\phi}{J}, \tag{2.7}$$

where $J = \dfrac{\partial(x,t)}{\partial(\phi,\xi)} = -at_\phi t_\xi$, is the Jacobian of transformation.
By using equation (2.7), system (2.1) can be written as

$$a\rho_\phi t_\xi - \rho\left(u_\phi t_\xi - u_\xi t_\phi - \frac{muat_\phi t_\xi}{x}\right) = 0, \tag{2.8}$$

$$a\rho u_\phi t_\xi - p_\phi t_\xi + p_\xi t_\phi = 0, \tag{2.9}$$

$$ap_\phi t_\xi - \rho a^2\left(u_\phi t_\xi - u_\xi t_\phi - \frac{muat_\phi t_\xi}{x}\right) = -(\gamma-1)\rho H a t_\phi t_\xi, \tag{2.10}$$

$$\sigma_\phi = -t_\phi H(\rho, p, \sigma). \tag{2.11}$$

Using eqs. (2.9) – (2.11) in (2.8), we obtain

$$p_\xi + \rho a u_\xi + \frac{m\rho a^2 u t_\xi}{x} = -(\gamma-1)\rho H(\rho, p, z)t_\xi. \tag{2.12}$$

The subscripts ϕ and ξ represents the partial derivative with respect to ϕ and ξ, respectively.
The boundary conditions at $\phi = 0$ are

$$[p] = 0,\ \ [\rho] = 0,\ \ [u] = 0,\ \ [\sigma] = 0,\ \ t = \xi. \tag{2.13}$$

At the front of the wave, it is given that the flow is uniform and at rest, therefore we get

$$p_\xi = 0,\ \rho_\xi = 0,\ u_\xi = 0,\ \sigma_\xi = 0 \ \text{ and } \ t_\xi = 1 \quad \text{at } \ \phi = 0. \tag{2.14}$$

On insertion of (2.13) and (2.14) in (2.11), (2.9) and (2.6), and evaluating at $\phi = 0$, yields

$$\rho_\phi = \left(\frac{\rho_0}{a_0}\right) u_\phi, \tag{2.15}$$

$$p_\phi = \rho_0 a_0 u_\phi, \tag{2.16}$$

$$x_\phi = 0, \quad x_\xi = a_0. \tag{2.17}$$

Here, the flow variables evaluated at the front of the shock are signified by the subscript '0'. Using (2.14) in (2.7), we get

$$\left[\frac{\partial u}{\partial x}\right] = Y = -\frac{u_\phi}{a_0 t_\phi}, \quad \text{at } \phi = 0, \tag{2.18}$$

where Y signifies the amplitude of the shock wave at $\phi = 0$.

2.4 Shock wave solution

In this section, we determine the relations for the dependence of u_ϕ and t_ϕ on time, and with the help of these relations, we find the solution to the problem. Differentiating equation (2.6) and (2.12) with respect to ϕ and combining with ξ derivative of (2.16), at $\phi = 0$, we get

$$\frac{t_{\phi\xi}}{t_\phi} = \frac{\gamma + 1}{2(1 - \bar{b})} Y, \tag{2.19}$$

$$\frac{u_{\phi\xi}}{t_\phi} = \left(\frac{\gamma - 1}{2a_0^2} H_0 \left(\frac{1 - \gamma - 2b\rho_0}{2(1 - \bar{b})} H_0 + (H_{\rho_0} + a_0^2 H_{p_0})\rho_0\right) + \frac{m a_0^2}{2\xi}\right) Y. \tag{2.20}$$

Differentiating equation (2.18) with respect to ξ and using equations (2.19) and (2.20), we get

$$\frac{dY}{d\xi}+\left(\frac{\gamma-1}{2(1-\bar{b})a_0^2}\left((\frac{1-\gamma-2b\rho_0}{2})H_0+(H_{\rho_0}+a_0^2H_{p_0})\rho_0(1-\bar{b})\right)+\frac{ma_0}{2\xi}\right)Y+\left(\frac{\gamma+1}{2(1-\bar{b})}\right)Y^2=0 \quad (2.21)$$

at $\quad \phi=0.$

Now, to transform the governing equations into a system of dimensionless partial differential equations, we introduce η, μ, and δ as

$$\eta=\frac{Y}{Y^*},\ \ \mu=\frac{\xi-\xi^*}{2\xi^*}\ \text{ and }\ \delta=Y^*\xi^*. \quad (2.22)$$

Here, δ, η and μ are the initial disturbance, wave amplitude and time respectively. The superscript '*' has been used to denote the parameter's value at $t=t^*$. Considering equation (2.22), equation (2.21) can be reduced to the following dimensionless form

$$\frac{d\eta}{d\mu}+\left(\frac{R_0}{(1-\bar{b})}+\frac{m}{2\mu+1}\right)\eta+\frac{\delta(\gamma+1)}{(1-\bar{b})}\eta^2=0, \quad \text{at } \phi=0, \quad (2.23)$$

where $R_0=\frac{(\gamma-1)}{a_0^2}(\frac{(1-\gamma-2b\rho_0)(H_0)}{2})+(H_{\rho_0}+a_0^2H_{p_0})(1-\bar{b}))\xi^*$, which denotes relaxation parameter. Equation (2.23) is a Bernoulli differential equation with η and μ as dependent and independent variables respectively. The solution of equation (2.23) is given as

$$\eta=\left\{(2\mu+1)^{\frac{m}{2}}e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu}\left(1+\left(\frac{(\gamma+1)\delta}{(1-\bar{b})}J(\mu)\right)\right)\right\}^{-1}, \quad (2.24)$$

where $J(\mu)=\int_0^{\mu}\frac{-e^{\left(\frac{R_0}{(1-\bar{b})}\right)s}}{(2s+1)^{\frac{m}{2}}}ds.$

From equations (2.18) and (2.24), it is clear that, for the formation of shock we must have $t_\phi=0$, i.e.

$$1 + \left(\frac{(\gamma + 1)\delta}{(1 - \bar{b})} J(\mu) \right) = 0. \tag{2.25}$$

From equation (2.25) it is clear that the compressive waves ($\delta < 0$) terminate into the shock wave.

2.5 Results and discussion

Now, we will discuss the growth and decay of shocks along with their formation. For better view, we consider three cases for three distinct values of m (i.e. $m = 0$ for planar, $m = 1$ for cylindrically symmetric, and $m = 2$ for spherically symmetric) and analyze the impact of parameters.

Case I. Planar flow ($m = 0$)**:**

By substituting $m = 0$ in (2.24), we get

$$\eta = \left\{ e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu} + \frac{\delta(\gamma + 1)}{R_0} \left(e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu} - 1 \right) \right\}^{-1}. \tag{2.26}$$

Using equation (2.22), yields

$$Y = \frac{Y^*}{e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu} + \frac{\delta(\gamma+1)}{R_0} \left(e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu} - 1 \right)}. \tag{2.27}$$

Fig.2.1 displays solution curve for $\delta < 0$ and it shows that in the presence of relaxing gas parameter, growth of the propagating waves is slowed down and an increase in its value shows delay in shock formation, where $R_0 = 0$ corresponds to the non-relaxing gas. The fig.2.2 depicts the decay of expansive wave, which advances in the presence of relaxing gas parameter.

Fig.2.3 presents the behavior of solution curve for relaxing and non-relaxing gas, in

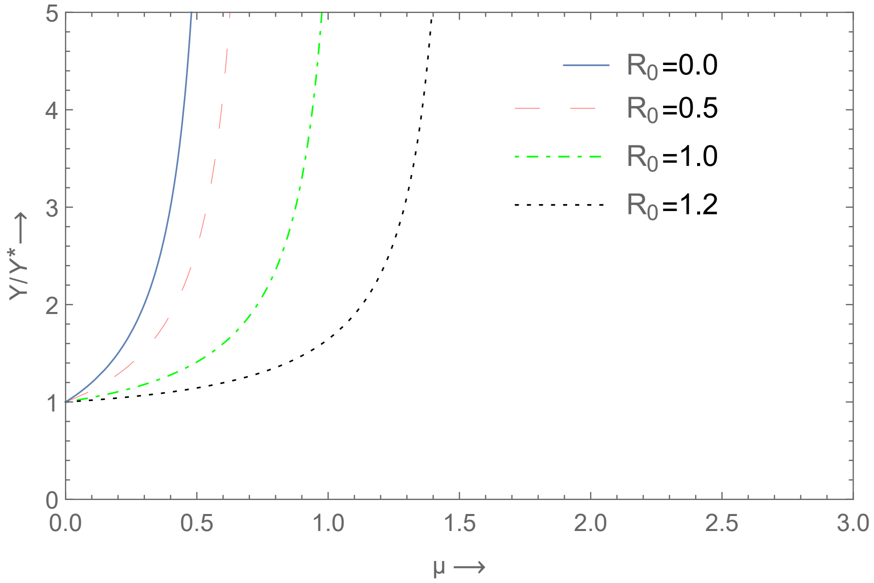

FIGURE 2.1: Effect of relaxing gas parameter R_0 for $\delta < 0$ with $m = 0, \gamma = 1.67, \delta = -0.5, \bar{b} = 0.2$.

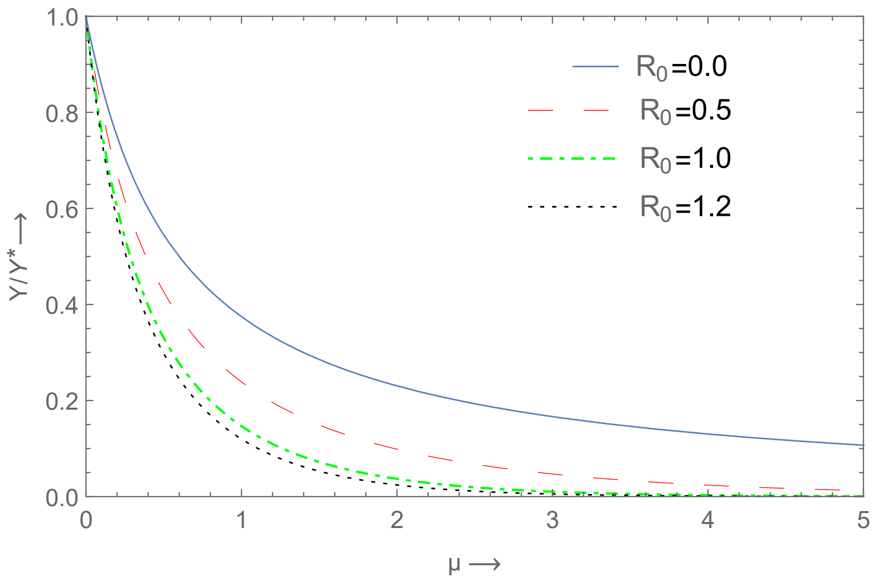

FIGURE 2.2: Effect of relaxing gas parameter R_0 for $\delta > 0$ with $m = 0, \gamma = 1.67, \delta = 0.5, \bar{b} = 0.2$.

planar case with distinct values of non-ideal parameter $\bar{b}$. With an increase in the value of $\bar{b}$, growth of propagating waves decreases. So the outgrowth of compressive wave in non-ideal relaxing gas is lower than that of ideal relaxing gas. The fig.2.4 shows that the decay rate of an expansive wave is slow in relaxing gas as compared to non-relaxing gas.

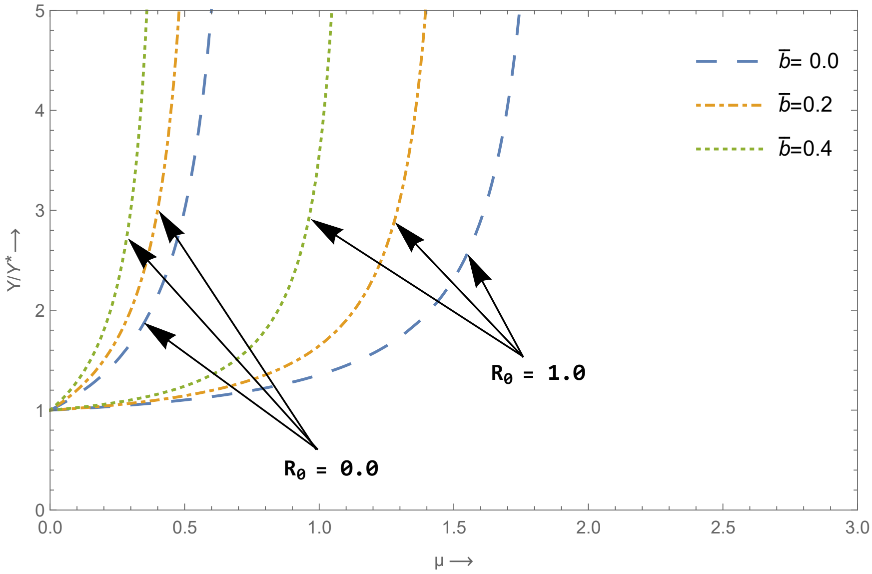

FIGURE 2.3: Impact of non-ideal parameter $\bar{b}$ for $\delta < 0$ in planar relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$.

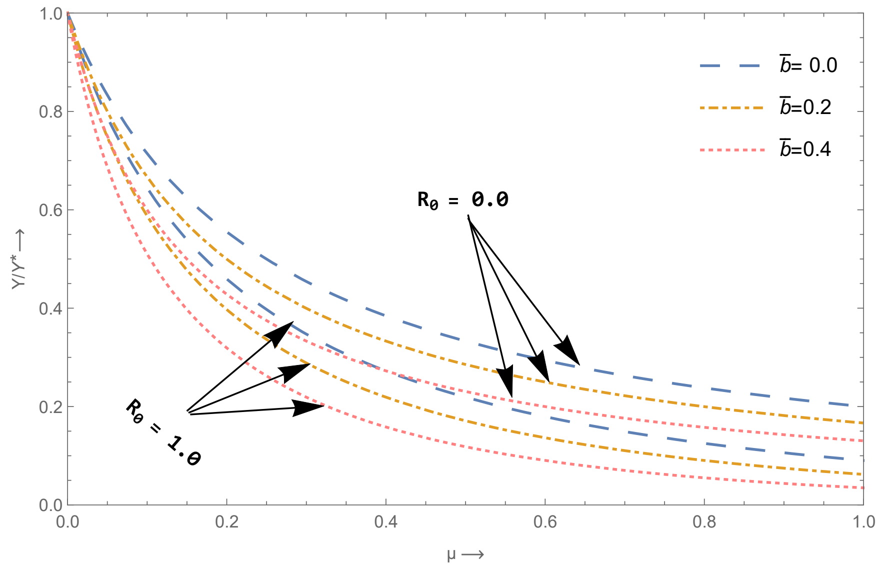

FIGURE 2.4: Impact of non-ideal parameter $\bar{b}$ for $\delta > 0$ in planar relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$.

Case II. Cylindrically symmetric flow ($m = 1$)**:**

For cylindrically symmetric flow, the solution of (2.23) is given as

$$Y = \frac{Y^*}{\sqrt{(2\mu+1)}e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu}\left(1 + \left(\frac{(\gamma+1)\delta}{(1-\bar{b})}J(\mu)\right)\right)}, \tag{2.28}$$

where $J(\mu) = \int_0^\mu \frac{-e^{\left(\frac{R_0}{(1-\bar{b})}\right)s}}{\sqrt{(2\mu+1)}}ds$.

In cylindrically symmetric flow case, a similar phenomenon is obtained for the propagating wave for $\delta < 0$ and $\delta > 0$ as in the planar case which can be seen by the solution curves depicted in Figs.2.5. The effect of the relaxing gas parameter in non-ideal and ideal relaxing gases on the compressive wave and the expansive wave is shown in Fig.2.5 and Fig.2.6, respectively. The shock formation of compressive wave is delayed and the decay rate of an expansive wave is accelerated in the presence of non-ideal parameters in relaxing gas (see Figs.2.5,2.6 respectively). We observed that the relaxing gas parameter R_0 together with the non-ideal parameter

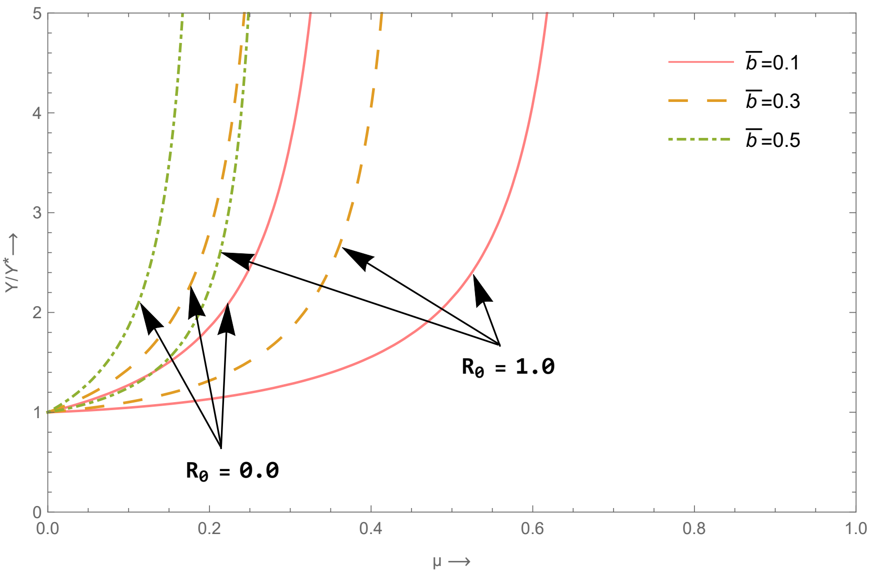

FIGURE 2.5: Impact of non-ideal parameter $\bar{b}$ for $\delta < 0$ in cylindrically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$.

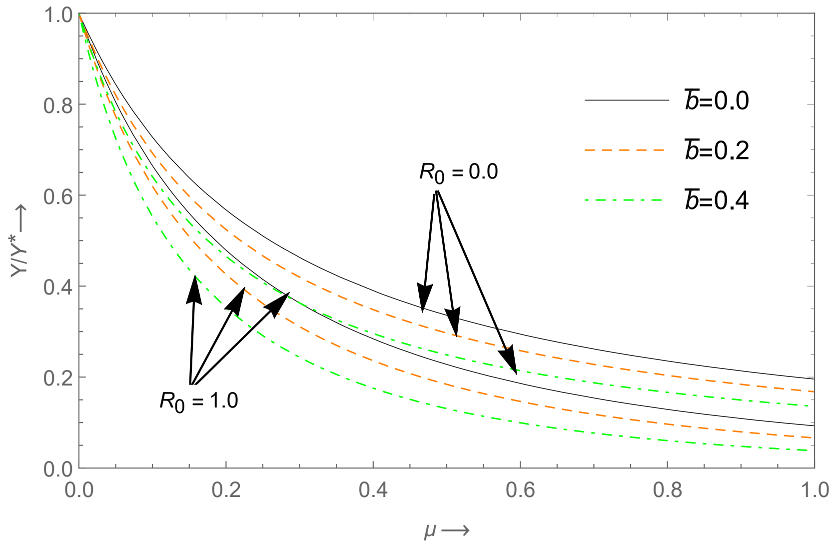

FIGURE 2.6: Impact of non-ideal parameter $\bar{b}$ for $\delta > 0$ in cylindrically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$.

$\bar{b}$ magnifies the flattening of expansive waves and shock formation time is reduced in relaxing gas flow.

Case III. Spherically symmetric flow ($m = 2$)**:**

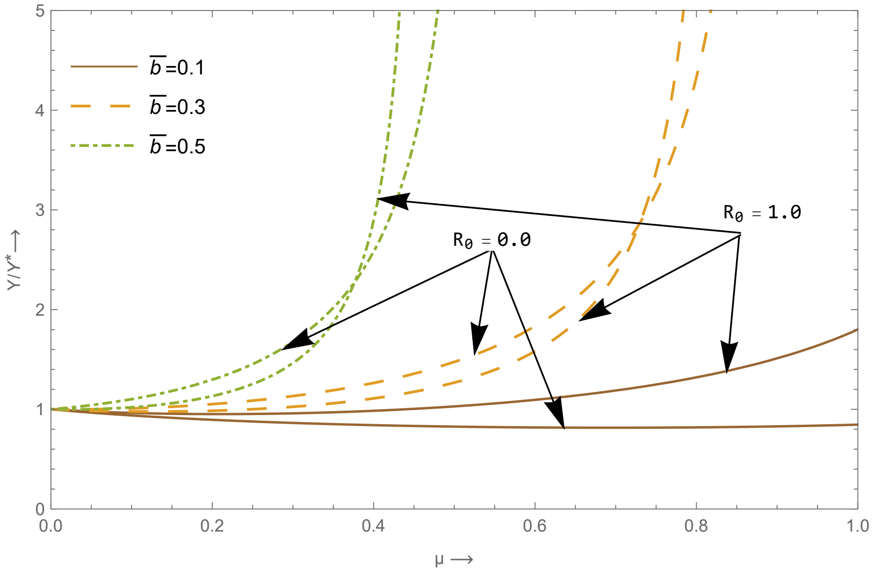

FIGURE 2.7: Impact of non-ideal parameter $\bar{b}$ for $\delta < 0$ in spherically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$.

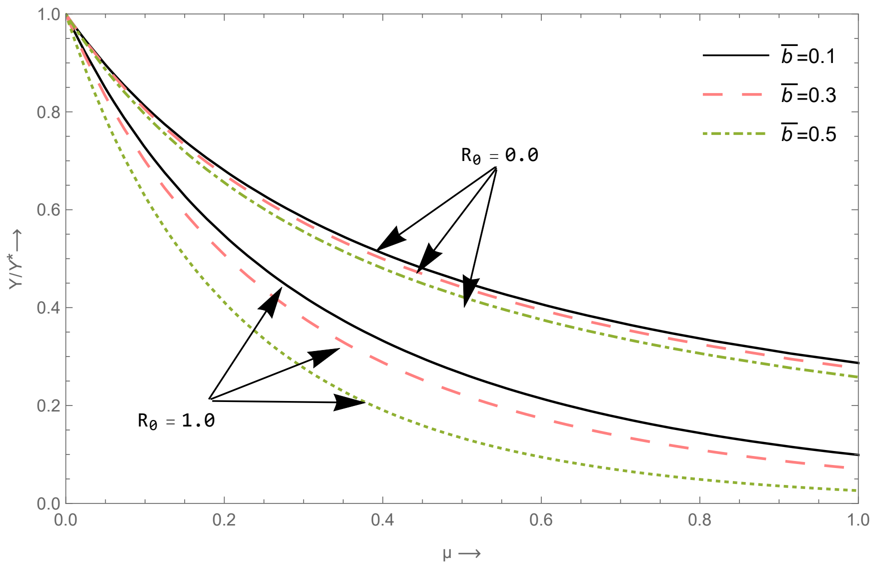

FIGURE 2.8: Impact of non-ideal parameter $\bar{b}$ for $\delta > 0$ in spherically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$..

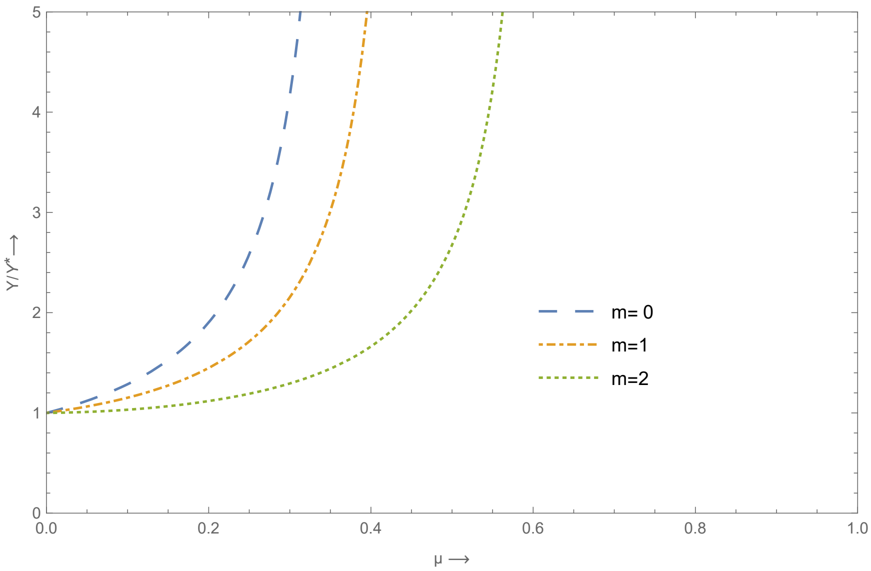

FIGURE 2.9: Comparative analysis of the geometry of non-ideal relaxing gas flow for $\gamma = 1.67, \delta = -1, \bar{b} = 0.2$.

For spherically symmetric flow, the solution of (2.23) is given as

$$Y = \frac{Y^*}{(2\mu+1)e^{\left(\frac{R_0}{(1-\bar{b})}\right)\mu}\left(1+\left(\frac{(\gamma+1)\delta}{(1-\bar{b})}J(\mu)\right)\right)}, \tag{2.29}$$

where $J(\mu) = \int_0^\mu \frac{-e^{\left(\frac{R_0}{(1-\bar{b})}\right)s}}{(2\mu+1)}ds$.

It is also obtained that the propagating wave for $\delta < 0$ and $\delta > 0$ in spherically symmetric flow case have similar phenomenon as in the planar and cylindrically symmetric flows case in relaxing gas flow with non-ideal parameter which can be seen by the solution curves presented by Figs. 2.7, 2.8.

Fig.2.9 shows the comparative analysis for all cases, which depicts that in case of spherically symmetric flow, compressive wave grows later which slows down the formation of shocks. For better view, we can say compressive wave terminates into shock wave earlier in other cases(i.e. in planar, cylindrically symmetric cases). (see Fig. 2.9)

2.6 Conclusion

In this chapter, we investigated one-dimensional flow consisting of non-ideal relaxing gas with van der Waals equation of state to study the solution of shocks for planar, cylindrically symmetric and spherically symmetric flows. Transport equations for characteristic shock are obtained explicitly. The nature of the solution obtained is varied in physical plane in contrast to characteristic plane. The impact of the relaxation parameter on the solution of the shock wave is examined and it is analyzed that the presence of relaxing gas parameter causes to accelerate the decay process of expansive wave and slow down the growth rate of compressive waves. In all cases, solution's nature completely depends upon relaxing gas parameter and non-idealness. Generally, it is observed that the shock occurs early for planar cases as compared to cylindrically and spherically symmetric cases shown in Fig 2.9.

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