

CHAPTER 7

Effect of span length on the synergistically optimized pre-stressed concrete beams

7.1 Introduction

Pre-stressed beams are a common type of construction material used in bridges, buildings, and other structures. They are designed to carry loads while maintaining a high level of structural integrity and stability. One important factor that affects the performance of pre-stressed beams is the span length, which refers to the distance between the supports of the beam. The span length of a pre-stressed beam is an important factor to consider when designing a structure. Greater span lengths are achievable with pre-stressed beams that are not possible with reinforced cement concrete beams. As the span length increases, the beam must span a greater distance between supports, which increases the bending and deflection of the beam under load (Heyman 1998). This can lead to a decrease in the strength and stability of the beam. To counteract this, the size and number of tendons in the beam must be increased as the

span length increases thus adding to the cost of the material. Few researchers (Hawkins 1961, Aravinthan et al. 2005, Huber et al. 2020) have researched the change in strength and behaviour due to the number of spans over continuous pre-stressed concrete beams. However, there is by far no study on the behavioural changes of optimized pre-stressed beams because of span length. Hence, to prevent any failure in real life, the optimal state of pre-stressed beams ought to be examined for the deflection and stress parameters under various span lengths.

The current work tries to fill up this gap by comparing the impact of the number of spans in pre-stressed beams on the overall optimized cable layout and concrete shape as obtained using the software SSCO. The span length of a single pre-stressed beam is altered by adding supports along its length, and the effects on the optimal cable layout, concrete shape and the percentage of material saved under various loading circumstances are investigated. The obtained optimized shape is also checked on the criteria for deflection and stress. The study also works on finding out the optimum number of spans which will strike a perfect balance between the amount of material saved due to the optimized shape and the number of spans needed to be given.

7.2 Criteria for convergence

In the present study, all the models have been modelled and optimized in SSCO (Simultaneous Shape and Cable Optimization). The three criteria selected in the current study for achieving convergence are:

- i. Removal of tensile stress i.e. $\sigma_{\text{bottom/top}} \leq 0.5 \text{ N/mm}^2$
- ii. A concrete cover of a minimum of 40 mm should be maintained between the cable and concrete boundary.

- iii. The maximum deflection (δ_{\max}) shouldn't be greater than $\delta_{\text{limit}} = \text{Span}/250$ or 20 mm whichever is less.

The criteria of $W_{\text{concrete final}} < W_{\text{concrete initial}}$ has been ignored for the present study however the objective weight minimization is still considered. This is done to ensure that the execution of software doesn't stop if the objective of weight minimization is violated at any stage.

For the current work, the beam is segmented into five design elements, and the key nodes present at the bottom of the design elements are declared as the design nodes. The cable is initially laid straight, passing through the neutral axis of the rectangular beam. It is then made to follow a 3rd-order B-spline curve, which moves up and down to counter tensile stress.

7.3 Numerical Illustrations

A simply supported pre-stressed beam is taken having length = 36000 mm, depth = 1300 mm, and width = 800 mm. The beam is made up of concrete having Young's modulus (E) = 3.5×10^4 N/mm², Poisson's ratio (ν) = 0.15 and density (ρ) = 2.5×10^{-5} N/mm³. The embedded pre-stressing cable have Young's modulus (E) = 2.1×10^5 N/mm², Poisson's ratio (ν) = 0.3 and, density (ρ) = 7.85×10^{-5} N/mm³. The beam is loaded under three different loading viz. (i) three-point load (3PL), (ii) uniformly distributed load (UDL), and (iii) uniformly varying load (UVL) with changing span length using intermediate supports by dividing the 36000 mm length of the beam in (i) Single span (B^1) having one full span of 36000 mm length, (ii) Two-span (B^2) having two spans of 18000 mm length each, (iii) Three-span (B^3) having three spans of 12000 mm length each, (iv) Four span (B^4) having four spans of 9000 mm length each, and (v) Five span (B^5) having five spans of 7200 mm length each. The initial cross-section in

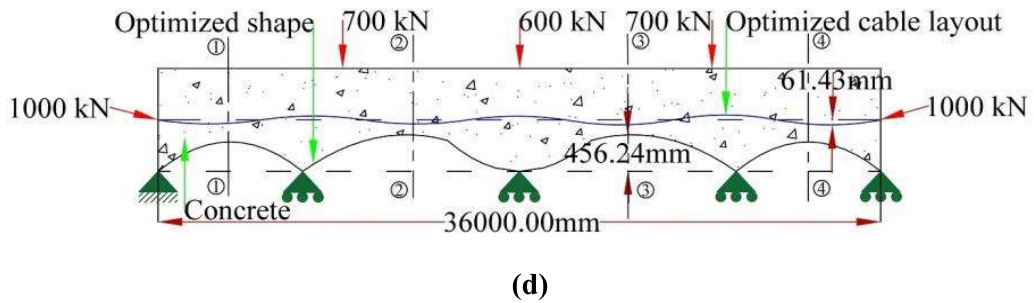
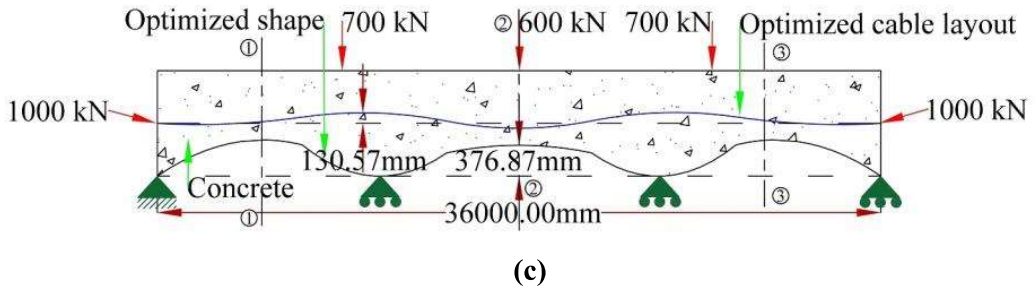
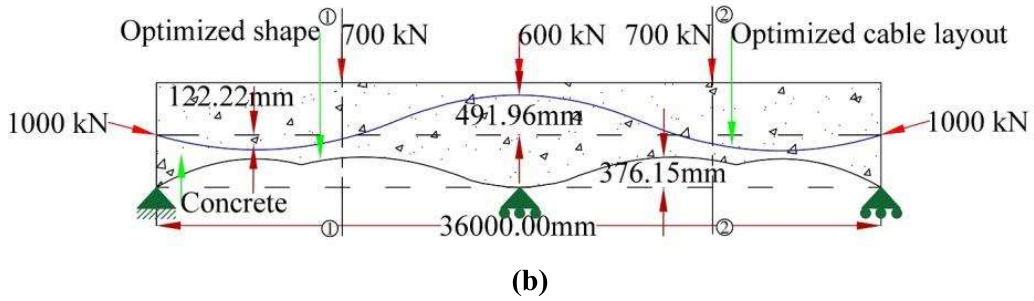
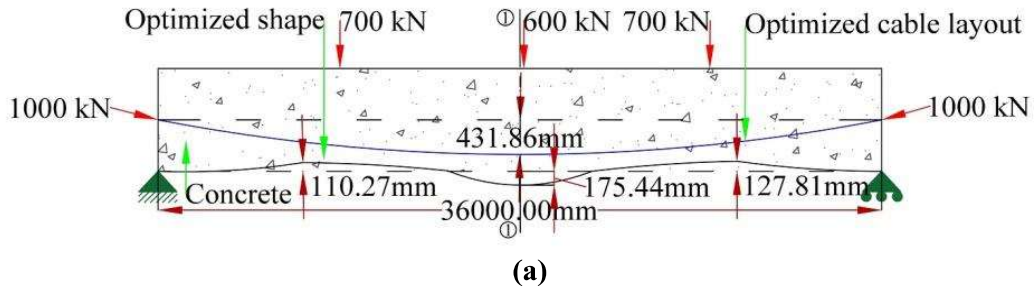
each case is kept constant, with the cable passing straight through the neutral axis. All the pre-stressed beams from B¹⁻⁵ are then optimized for cable layout and concrete shape using SSCO for the three different load cases (3PL, UDL, and UVL) individually. The optimized state is then compared for the amount of material saved, change in deflection and bending stress under each case from the initial un-optimized state. The results obtained under the 3PL, UDL, and UVL load case is compiled in Table 1, Table 2, and Table 3, respectively.

7.3.1 Load case 1: Three-point load (3PL)

In load case 1, the pre-stressed concrete beam is imposed with three-point loads (i) 700 kN at 9000 mm from the beam's left side, (ii) 600 kN at 18000 mm from the beam's left side, and (iii) 700 kN at 27000 mm from the beam's left side acting in downward ('-ve Y) direction. At the jacking end, a 1000 kN pre-stressing force is applied while taking friction loss into account. The pre-stressed beam is then optimized using SSCO, and the results are compared with the beam's original un-optimized condition over the grounds of (i) Bending stress (σ_{bottom} and σ_{top}), (ii) deflection (δ), and (iii) percentage material saved (M_{saved}) and are compiled in Table 7.1. The optimized layout of cable and concrete shape obtained for all the pre-stressed beams B¹⁻⁵ under the 3PL loading is illustrated in Figure 7.1.

Table 7.1: Load Case1: Three-point load (3PL)

Pre-stressed Beam	State	Section	Bending stress (N/mm ²)		δ (mm)	M_{saved} (%)
			σ_{bottom}	σ_{top}		
B ¹	Initial	1-1	48.72	-51.22	-285.7	01.61
	Optimized		-02.54	-40.68	-067.9	
B ²	Initial	1-1	06.91	-08.74	-07.51	19.29
	Optimized		-03.64	-11.75	-01.02	
	Initial	2-2	06.94	-08.72	-07.52	
	Optimized		-09.95	-03.22	-01.92	
B ³	Initial	1-1	01.23	-03.13	-01.52	19.57
	Optimized		-04.79	-05.49	-00.87	
	Initial	2-2	01.83	-04.30	-04.59	
	Optimized		-03.02	-06.82	00.67	
	Initial	3-3	01.28	-03.08	-01.52	
	Optimized		-04.24	-05.36	-00.99	
B ⁴	Initial	1-1	-01.42	-00.46	00.15	20.26
	Optimized		-05.08	-00.23	00.85	
	Initial	2-2	-00.83	-00.98	-00.13	
	Optimized		-04.17	-01.29	00.22	
	Initial	3-3	-00.83	-00.90	-00.11	
	Optimized		-04.16	-01.03	00.28	
	Initial	4-4	-01.31	-00.33	00.16	
	Optimized		-04.63	-00.06	00.80	
B ⁵	Initial	1-1	-01.60	-00.29	00.17	20.33
	Optimized		-05.74	00.34	00.85	
	Initial	2-2	00.01	-01.89	-00.27	
	Optimized		-02.31	-03.36	-00.38	
	Initial	3-3	01.27	-03.68	-00.32	
	Optimized		00.15	-05.88	-00.41	
	Initial	4-4	00.02	-01.81	-00.26	
	Optimized		-02.22	-03.13	-00.35	
	Initial	5-5	-01.49	-00.18	00.17	
	Optimized		-05.29	00.47	00.81	



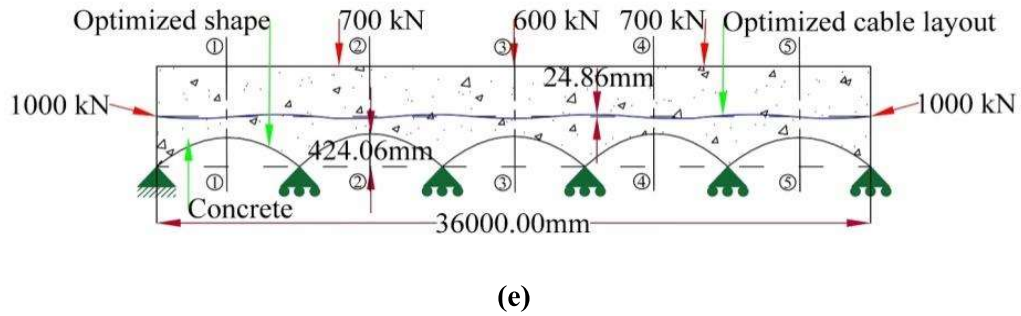


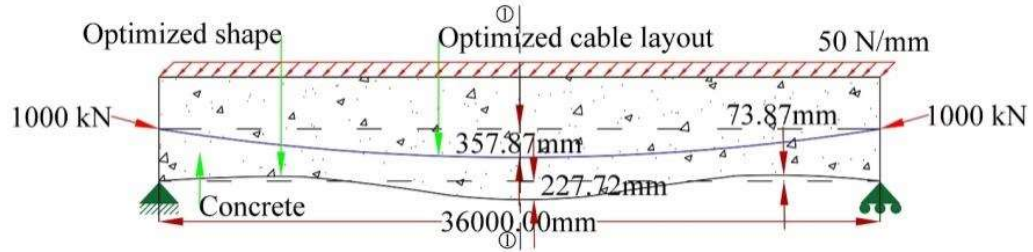
Figure 7.1: Optimized state of pre-stressed concrete beam under 3PL for beam (a) B1, (b) B2, (c) B3, (d) B4, and (e) B5

7.3.2 Load case 2: Uniformly distributed load (UDL)

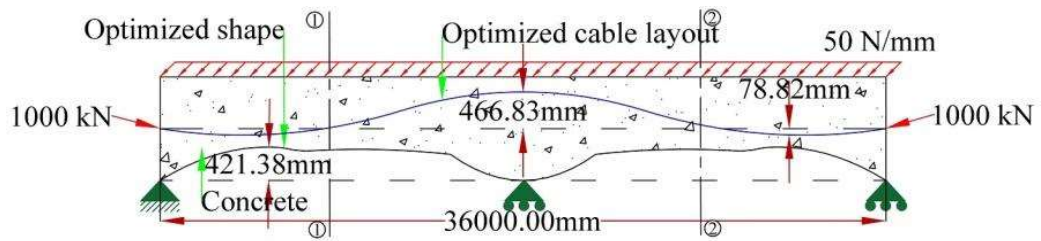
In load case 2, the pre-stressed concrete beam is imposed with a uniformly distributed load of 50 N/mm across the length of the pre-stressed beam acting in the downward (‘-ve Y’) direction. At the jacking end, a 1000 kN pre-stressing force is applied while taking friction loss into account. The pre-stressed beam is then optimized using SSCO, and the results are compared with the beam's original un-optimized condition over the grounds of (i) Bending stress (σ_{bottom} and σ_{top}), (ii) deflection (δ), and (iii) percentage material saved (M_{saved}) and are compiled in Table 7.2. The optimized layout of cable and concrete shape obtained for all the pre-stressed beams B¹⁻⁵ under the UDL loading is shown in Figure 7.2.

Table 7.2: Load case 2: Uniformly distributed load (UDL).

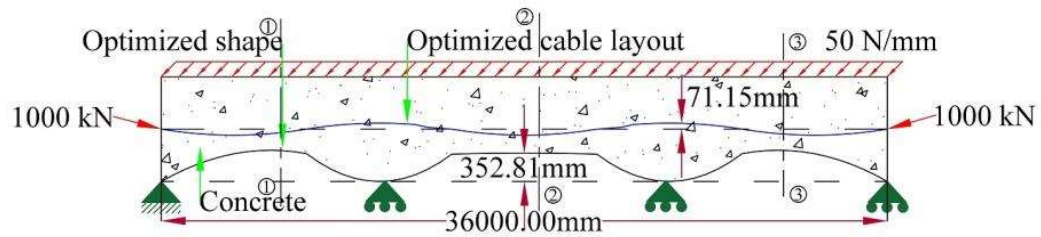
Pre-stressed Beam	State	Section	Bending stress		δ (mm)	M_{saved} (%)
			σ_{bottom}	σ_{top}		
B1	Initial	1-1	34.32	-36.21	-210.3	-3.46
	Optimized		-00.81	-30.76	-069.5	
B2	Initial	1-1	03.37	-05.27	-05.30	20.71
	Optimized		-05.54	-07.92	-05.28	
	Initial	2-2	03.40	-05.25	-05.31	
	Optimized		-05.01	-07.87	-01.15	
B3	Initial	1-1	01.25	-03.13	-01.34	21.31
	Optimized		-00.93	-04.48	-01.55	
	Initial	2-2	-00.40	-01.38	-02.35	
	Optimized		-02.82	-02.05	00.85	
	Initial	3-3	01.35	-03.06	-01.35	
	Optimized		-00.44	-04.51	-01.74	
B4	Initial	1-1	-00.04	-01.84	-00.29	22.76
	Optimized		-03.06	-02.54	00.01	
	Initial	2-2	-00.28	-01.53	-00.19	
	Optimized		-03.16	-02.34	00.09	
	Initial	3-3	-00.28	-01.45	-00.17	
	Optimized		-03.19	-02.05	00.17	
	Initial	4-4	00.06	-01.71	-00.28	
	Optimized		-02.40	-02.48	-00.11	
B5	Initial	1-1	-00.12	-01.77	-00.20	25.39
	Optimized		-04.39	-01.97	00.29	
	Initial	2-2	-00.61	-01.22	-00.05	
	Optimized		-03.45	-02.57	-00.01	
	Initial	3-3	-00.46	-01.33	-00.09	
	Optimized		-03.84	-02.07	00.15	
	Initial	4-4	-00.60	-01.14	-00.05	
	Optimized		-03.41	-02.27	00.03	
	Initial	5-5	-00.02	-01.66	-00.19	
	Optimized		-03.74	-01.95	00.21	



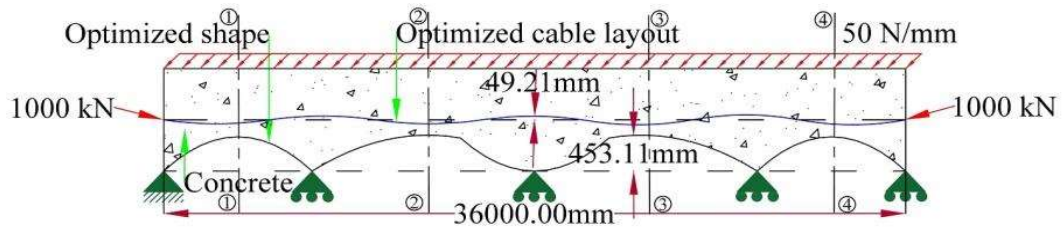
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(b)



(c)



(d)

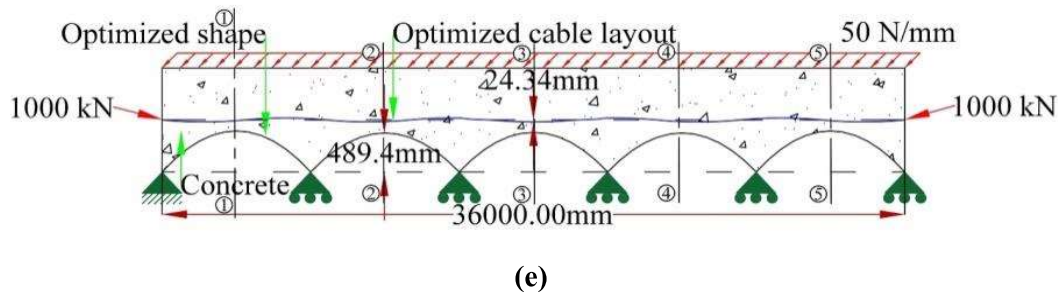


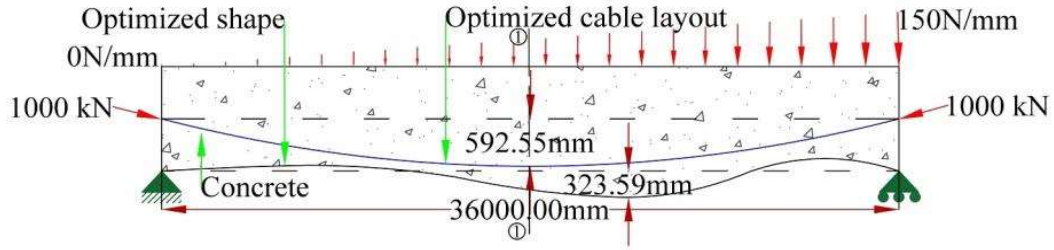
Figure 7.2: Optimized state of pre-stressed concrete beam under UDL for beam (a) B1, (b) B2, (c) B3, (d) B4, and (e) B5

7.3.3 Load case 3: Uniformly varying load (UVL)

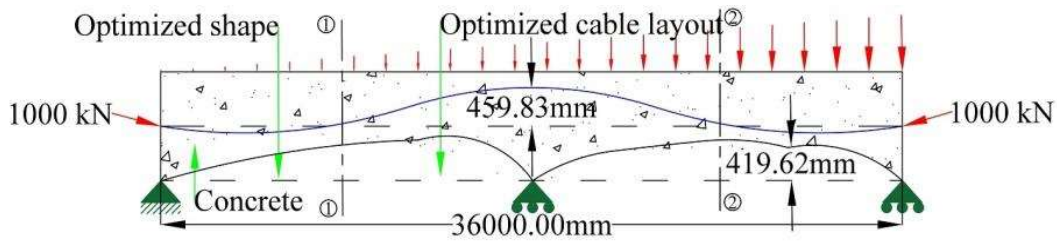
In load case 3, the pre-stressed concrete beam is imposed with a uniformly varying load of 0 N/mm at starting point on the left-hand side and gradually increasing up to 150 N/mm towards the right-hand side, acting in the downward ('-ve Y) direction. At the jacking end, a 1000 kN pre-stressing force is applied while taking friction loss into account. The pre-stressed beam is then optimized using SSCO, and the results are compared with the beam's original un-optimized condition over the grounds of (i) Bending stress (σ_{bottom} and σ_{top}), (ii) deflection (δ), and (iii) percentage material saved (M_{saved}) and are compiled in Table 7.3. The optimized layout of cable and concrete shape obtained for all the pre-stressed beams B¹⁻⁵ under the UVL loading is shown in Figure 7.3.

Table 7.3: Load case 3: Uniformly varying load (UVL).

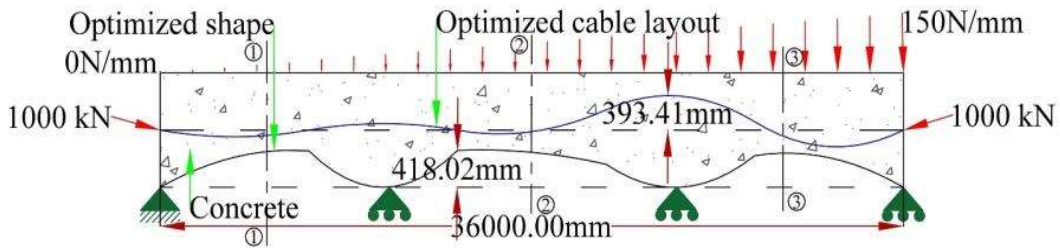
Pre-stressed Beam	State	Section	Bending stress		δ (mm)	M_{saved} (%)
			σ_{bottom}	σ_{top}		
B1	Initial	1-1	52.02	-53.90	-315.6	-4.42
	Optimized		-01.46	-39.45	-081.2	
B2	Initial	1-1	-00.81	-01.08	01.60	19.53
	Optimized		-07.31	-02.32	06.93	
	Initial	2-2	12.01	-13.85	-17.62	
	Optimized		-02.58	-16.55	-16.17	
B3	Initial	1-1	-00.13	-01.74	-00.46	19.74
	Optimized		-11.04	-02.51	02.36	
	Initial	2-2	-00.01	-01.79	-00.11	
	Optimized		-09.24	-03.45	02.26	
	Initial	3-3	05.11	-06.83	-03.64	
	Optimized		-03.13	-08.78	-02.82	
B4	Initial	1-1	-00.90	-00.98	-00.01	19.78
	Optimized		-06.15	-01.75	00.72	
	Initial	2-2	-0.09	-01.72	-00.26	
	Optimized		-05.12	-03.13	00.18	
	Initial	3-3	00.35	-02.09	-00.34	
	Optimized		-04.52	-03.22	00.29	
	Initial	4-4	01.93	-03.58	-00.88	
	Optimized		-00.74	-05.99	-01.14	
B5	Initial	1-1	-00.78	-01.11	-00.04	23.74
	Optimized		-05.07	-00.88	00.55	
	Initial	2-2	-00.58	-01.26	-00.06	
	Optimized		-03.33	-02.57	-00.03	
	Initial	3-3	-00.19	-01.60	-00.14	
	Optimized		-03.20	-02.68	-00.01	
	Initial	4-4	-00.23	-01.51	-00.11	
	Optimized		-03.21	-02.45	00.11	
	Initial	5-5	01.48	-03.17	-00.55	
	Optimized		00.04	-05.17	-00.81	



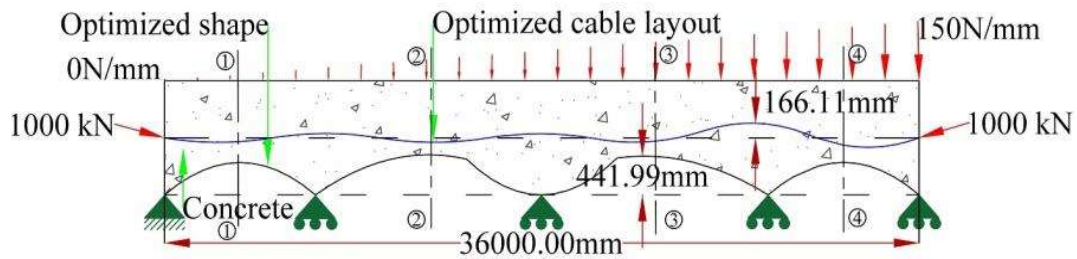
(a)



(b)



(c)



(d)

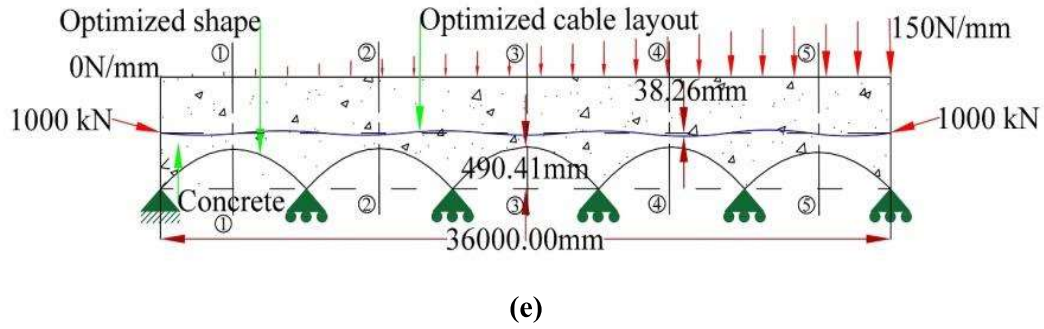


Figure 7.3: Optimized state of the pre-stressed concrete beam under UVL for beam (a) B1, (b) B2, (c) B3, (d) B4, and (e) B5

7.4 Discussion

The δ_{limit} for the beam of span 36000 mm comes to 144 mm. Hence, δ_{max} at any section should not be greater than 144 mm. From the results obtained in section 4, it can be seen that when the pre-stressed beam is loaded in a single span, the deflection in Section 1-1 in all of the load cases is much more than δ_{limit} . But when the pre-stressed beams are optimized for their shape and cable layout using SSCO, the δ_{max} is also controlled and is brought well below the δ_{limit} value for all the load cases.

The percentage reduction in material for the pre-stressed beam optimum state in the 3PL load case is maximum for the five-span beam and lowest for the single-span beam. Similar results are obtained for the UDL and UVL load case, with the maximum percentage of material saving being up to 25%. However, for the single-span pre-stressed beam in UDL and UVL load cases, the optimized state shows an increase in material quantity by approximately 4% compared to the un-optimized state. The maximum change in the reduction of material quantity is obtained when the pre-stressed beam is transformed from a single-span beam to a two-span beam, where it can be observed that around 19% saving in the material is obtained. Further, as the number of spans is increased to three, four and five, the percentage reduction in material increases,

but the change, when compared to two-span beams, is around 1% to 4% which can be considered insignificant for every increase in the number of spans the support needs to be constructed making the structure overall uneconomical.

The final optimized cable layout varies according to the applied load and the number of spans given to the pre-stressed beam. For a single-span beam, the cable shows a concave layout with co-ordinates of cable at mid-reaching near the bottom boundary of the beam. However, in five-span pre-stressed beams, the cable layout hardly shows any significant change from its initial layout. This is because of the high initial tensile stress in single-span pre-stressed beams as compared to the five-span beams. As a result, the cable in order to counter the tensile stress changes its initially straight position to the desired shape as per the loading and number of the span as observed in section 4. Also, it can be deduced here that as the number of spans increases, the chances of having a tensile failure decrease.

7.5 Concluding remarks

The presented study successfully compares the effect of the number of spans in pre-stressed beams on the overall optimized cable layout and concrete shape as obtained using the software SSCO. From this study, it can be concluded that for a given length of the pre-stressed beam, a higher number of spans doesn't ensure high material savings. Instead, there is a certain optimum number of spans which will strike a perfect balance between the amount of material saved and the number of supports needed to be constructed. This optimum number can be found effectively using the software SSCO which will also give the optimized layout of cable and concrete shape for that number span. The pre-stressed beams in an optimized state can also be used to get controlled deflection which might not be possible in its un-optimized state. The present work can

be used by industries in the designing of continuous shape-optimized pre-stressed beams and girders saving a lot of material and cost of construction.