I would like to dedicate this thesis to my loving parents and amazing friends, whose support and motivation kept me going through the tough times.

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It is certified that the work contained in the thesis titled **"Quantum dynamics of Nitrogen-Vacancy center spin coupled with nonlinear nanomechanical resonator"** by **Mr. Abhishek Kumar Singh**, Roll Number **17171007**, has been carried out under my/our supervision and that this work has not been submitted elsewhere for a degree.

Signature: **Supervisor : Dr. Sunil Kumar Mishra**
(Indian Institute of Technology (BHU), Varanasi)

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I, **Abhishek Kumar Singh**, certify that the work embodied in this thesis is my own bona-fide work and carried out by me under the supervision of **Dr. Sunil Kumar Mishra** from July 2017 to June 2022 at the **Department of Physics**, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers whenever and wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any others' work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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Abstract

In this thesis we mainly discuss about a Nitrogen Vacancy (NV) center hosted in a diamond nanocrystal which is positioned at one end of a nonlinear nanomechanical oscillator through a magnetic tip. This hybrid system couples the degrees of freedom of two different systems, one is the nonlinear nanomechanical oscillator and other is a single quantum spin object which is Nitrogen Vacancy center. The distance and the coupling strength between the magnetic tip and the NV center spin can be modulated through the magnetostriction effect.

Firstly, we study quantum dynamics of a NV center spin coupled to a nonlinear periodically driven mechanical oscillator. The continuous periodic driving term explicitly depends on the position of the oscillator. Coupling strength between the NV spin center and the oscillator also depends on the position of the oscillator. Using action-angle variables, we can transform cantilever to a mathematical pendulum. The mechanical motion of the mathematical pendulum follows Mathieu Schrödinger equation whose solutions are Mathieu elliptic functions. The solutions of Mathieu Schrödinger equation are further used to study the dynamics of the quantum spin system including environmental effects and calculation of the purity and the von Neumann entropy of the NV center spin as measure of mixedness and entanglement. We comprehensively analyse decoherence effects in the NV center spin due to Markovian and non-Markovian environments. We also explore unitary generation of coherence. The coherence is generated only for a system which is initially prepared in a maximally mixed or a thermal state. Production of coherence is efficient when the system initially is prepared in the region of the separatrix (*i.e.*, the region where classical systems exhibit dynamical chaos). From the theory of dynamical chaos, we know that phase trajectories of the system passing through the homoclinic tangle have limited memory, and therefore, the information about the initial conditions is lost. We proved that

quantum chaos and diminishing of information about the mixed initial state favors the generation of quantum coherence through the unitary evolution. We introduced quantum distance from the homoclinic tangle and proved that for the initial states permitting efficient generation of coherence, this distance is minimal.

Subsequently, we explore an exactly solvable model of a hybrid quantum-classical system of a NV center spin (quantum spin) coupled to a nanocantilever (classical) and analyze the enforcement of the regular or chaotic classical dynamics onto the quantum spin dynamics. The main problem we focus in this direction is whether the classical dynamical chaos may induce chaotic effects in the quantum spin dynamics or not. We explore several characteristic criteria of the quantum chaos, such as quantum Poincaré recurrences, generation of coherence and energy level distribution and observe interesting chaotic effects in the spin dynamics. Dynamical chaos imposed in the cantilever dynamics through the kicking pulses induces stochastic dynamics on quantum subsystem. We consider a quantum system of two and three levels and show that in a two-level case, type of stochasticity is not conforming all the characteristic features of the quantum chaos and is distinct from it. We also explore the effect of quantum feedback on dynamics of the cantilever and the entire system.

Later, we study out-of-time ordered correlator (OTOC) in a system of two nanomechanical cantilevers coupled with two NV center spins where the oscillators are coupled to each other directly while NV spins are not. OTOC is considered as a quantitative measure of the entanglement spreading process in the system. Particular interest concerns the propagation of quantum correlations in the lattice systems, *e.g.*, spin chains. The radius of the OTOC defines the front line reached by the spread of entanglement. Beyond this radius operators commute. Therefore, the correlation between the NV spins may arise only through the quantum feedback exerted from the first NV spin to the first oscillator and transferred from the first oscillator to the second oscillator via the direct coupling. Thus nonzero OTOC between NV spins may quantify the strength of the quantum feedback. We show that NV spins cannot exert quantum feedback on classical nonlinear oscillators. We also discuss inherently quantum case with a linear quantum harmonic oscillator indirectly coupling the two spins and verify that in the classical limit of the oscillator, the OTOC vanishes.

Throughout this thesis we discus theoretical understanding about the effects of driving and coupling of a nonlinear nanomechanical oscillator to a quantum NV center spin. We explored decoherence due to environment and explained several criteria of quantum chaos and discussed about OTOC in such systems.

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