Chapter 5

Stability analysis, control of Simple chaotic system and its hybrid projective synchronization with fractional Lu system

5.1 Introduction

In the last few years, nonlinear dynamics have gained immense attention by several researchers. Nonlinear systems are those dynamical systems which show drastically changes corresponding to small change in initial conditions. In chaotic systems small changes in initial conditions yield large difference in trajectory. In recent time chaos theory has wide range of applications in meteorology, physics, environmental science, computer science, engineering and economics. After the invention of the chaotic attractor in the year 1963 by the eminent scientist E. N. Lorenz (1963), several researchers have started to developed new chaotic attractors.

Since the integer order system (Jafari et al. (2012), Jafari and Gejji (2006), Das and Yadav (2016), Yadav et al. (2016)) is a particular case of fractional order system for taking the order one, so fractional order derivative have been studied by several researchers in different fields of science. Nowadays, fractional differential operator is used in viscoelastic systems, electrode–electrolyte polarization (Koeller (1984), Sun et al. (1984), Ichise et al. (1971)), dielectric polarization and so on. Fractional differential operator is widely used in control theory and communication. Synchronization of chaotic systems has also applied in ecology (Blasius et al. (1999)), physics (Lakshmanan and Murali (1996)), chemistry (Han et al. (1995)), security of communications (Murali and Lakshmanan (2003)) etc.

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As stated in earlier chapters that in last few years different kinds of synchronization have studied. Among which the hybrid projective synchronization is one which consists both projective anti- synchronization and projective synchronization. This property of hybrid projective synchronization is used to secure signals in communication and encryption schemes. First time, Pecora and Carroll (1990) discussed the synchronization of chaotic systems. Ramirez et al. (2003) analyzed the hybrid chaos synchronization of Chua, Duffing and Chen systems. The concept of hybrid chaos synchronization in information processing has been applied by Xie et al. (2002). Sudheer and Sabir (2009) visualized this type of synchronization between two hyperchaotic Lu systems. In 2013, Das et al. (2013) developed a mathematical model to study identical and non-identical chaotic systems with the help of hybrid phase synchronization. Si et al. (2012) analyzed the projective synchronization in different chaotic systems with unequal fractional orders derivative. Several techniques and many different methods such as adaptive control, predictive control, optimal control, time delay feedback control, backstepping design, tracking control, active control and observer-based control method, nonlinear feedback control method have been used to study the synchronization of chaotic dynamical systems in last few years.

Jia and Wang (2011) analyzed hybrid projective synchronization in four-dimensional chaotic system by adaptive feedback control method. They developed four-dimensional system by adding some necessary variable in a three-dimensional chaotic system proposed by Tigon and Opris. Tripathi et al. (2018) given a glimpse of hybrid projective synchronization in hyper chaotic Rabinovich-Fabrikant system and used tracking control method. They also visualized stability of this chaotic system. Boroujeni et al. (2012) proposed a new idea to control fractional Lu chaotic system. They introduced two approaches, locally and global asymptotically stability around the equilibrium point and used Lyapunov technique to show the global asymptotically stability. Zhou et al. (2012) investigated hybrid projective synchronization (HPS) between Chen chaotic system with fractional order. Zhang and Liu (2016) also studied hybrid projective synchronization. During investigation they considered chaotic system and hyperchaotic system in fractional order with unknown parameters. They also involved compensator and a nonlinear controller in his studied. During performance of the results fractional Chen and fractional Lu system have been considered as chaotic systems and fractional Lorenz system as hyperchaotic system. Authors have implemented Adams-Bashforth-Moulton method to study numerical simulation. Zayernouri and Matzavinos et al. (2016) discussed Fractional Adams–Bashforth-Moulton methods and applied this method in fractional Keller–Segel system. Keller–Segel model is first chemotaxis model developed by Evelyn Keller and Lee Segel during study of dynamics of Dictyostelium discoideum.

In this chapter, basic of the fractional simple system and existence of chaotic attractors of this system have been discussed. Nonlinear feedback control method is applied on the fractional simple system to control chaos. Results based on numerical simulation have presented graphically. Results depict that the nonlinear active control method is more reliable and effective.

The chapter is summarized as follows: In section 5.2, preliminaries, definition and lemma are introduced. Section 5.3 includes the systems' description and stability condition. In section 5.4, application of nonlinear control method in control of chaos is introduced. Sections 5.5 contain hybrid projective synchronization between fractional order simple and Lu systems. Results obtained using numerical simulation have been analyzed in section 6. Section 5.7 consists conclusion of overall research contribution.

5.2 Systems' description and stability condition

5.2.1 Fractional order Simple system

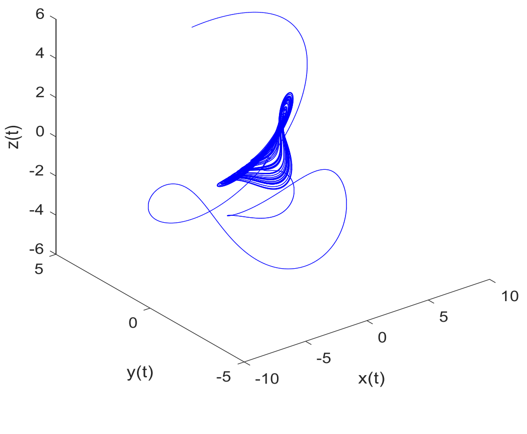
The Simple system (Novikov and Pavlov (2000)) is describe by

$$\frac{dx}{dt} = 1 + \mu zy\,,$$

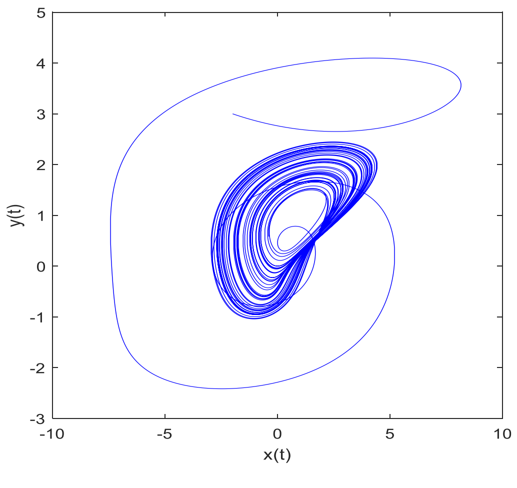
$$\frac{dy}{dt} = x - y\,, \tag{5.1}$$

$$\frac{dz}{dt} = 1 - xy\,,$$

The system (5.1) shows chaotic behavior for the parameter $\mu = 2.1$. The phase portraits of simple chaotic system in three-dimensional space and different planes are depicted through the Figs. 5.1(a)-(d).



(a)



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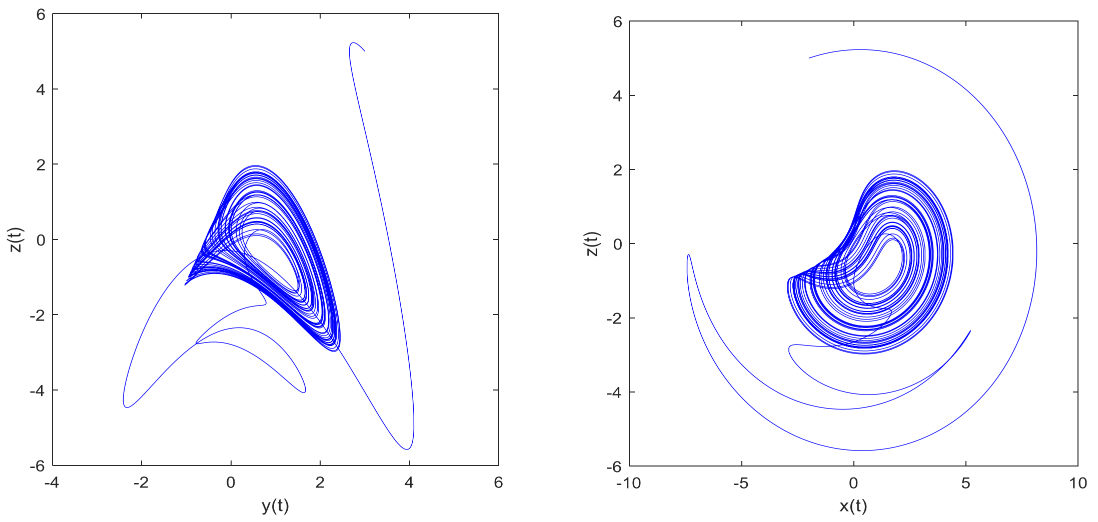


(c) (d)

Fig. 5.1 Phase portraits of Simple system in x-y-z space and x-y, y-z, x-z planes.

The fractional Simple system is defined as

$$\frac{d^{\alpha}x}{dt^{\alpha}} = 1 + \mu zy,$$

$$\frac{d^{\alpha}y}{dt^{\alpha}} = x - y, \qquad (5.2)$$

$$\frac{d^{\alpha}z}{dt^{\alpha}} = 1 - xy,$$

For the parameter $\mu = 2.1$, the system (5.2) is chaotic for order of the derivative $\alpha = 0.99$. The phase portraits of the chaotic simple system (5.2) in three-dimensional space and different planes are visualized in the Figs. 5.2(a)-(d).

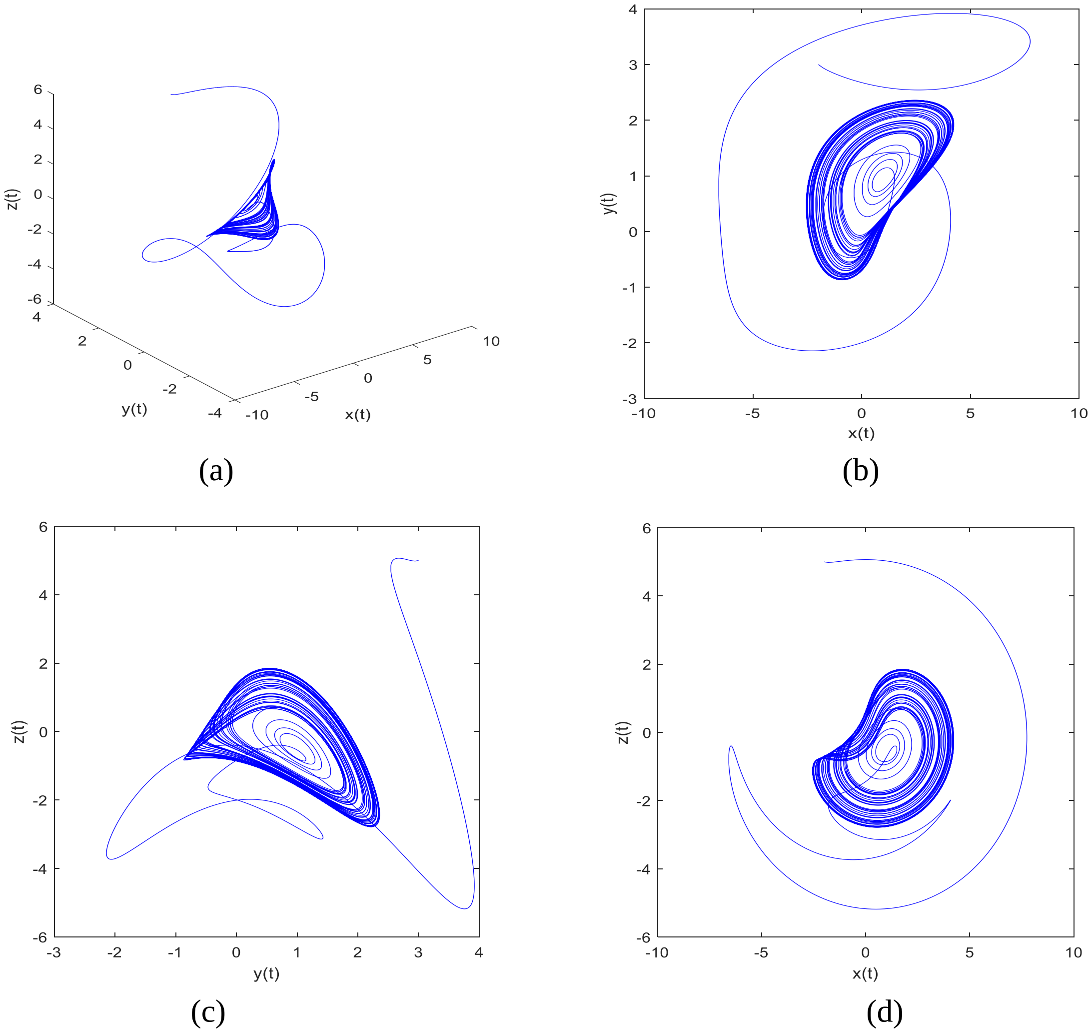


Fig. 5.2 Phase portraits of fractional order Simple chaotic system at $\alpha = 0.99$ in x-y-z space and x-y, y-z, x-z planes.

5.2.1.1 Stability analysis

Now we consider the equation as

$$
\begin{aligned}
&1+\mu y z = 0, \\
&x - y = 0, \qquad (5.3) \\
&1 - xy = 0,
\end{aligned}
$$

After solving equation (5.3) for $\mu = 2.1$, we get two equilibrium points as $E_1(1,1,-0.4762)$ and $E_2(-1,-1,0.4762)$.

At the point $\overline{E}(\overline{x},\overline{y},\overline{z})$ for system (5.2), the Jacobian matrix is given as

$$J(\overline{E})=\begin{pmatrix} 0 & \mu\overline{z} & \mu\overline{y} \\ 1 & -1 & 0 \\ -\overline{y} & -\overline{x} & 0 \end{pmatrix}. \tag{5.4}$$

The characteristic polynomial of the matrix (5.4) for $\mu = 2.1$ is given by

$$P(\lambda)=\lambda^3+\lambda^2+2.1(y^2-z)\lambda+2.1xy+2.1y^2. \tag{5.5}$$

Lemma 5.2: The equilibrium point $E_1(1,1,-0.4762)$ is stable for $\alpha < 0.95855$.

Proof: Substituting the value of E_1 in equation (5.5), we obtain

$$P(\lambda)=\lambda^3+\lambda^2+3.1\lambda+4.2. \tag{5.6}$$

The eigenvalues of the equation (5.6) are $\lambda_1=-1.2375$, $\lambda_{2,3}=0.1188\pm1.8384\,i$. Here absolute value of $\arg(\lambda_{2,3})$ is 1.5063. So $E_1(1,1,-0.4762)$ is stable for $\alpha < 0.95855$.

Lemma 5.3: The equilibrium point $E_2(-1,-1,0.4762)$ is stable for $\alpha < 0.844836$.

Proof: Substituting the value of E_2 in the equation (6.5), we obtain

$$P(\lambda)=\lambda^3+\lambda^2+1.1\lambda+4.2. \tag{5.7}$$

The eigenvalues of the equation (5.7) are $\lambda_1=-1.7468$, $\lambda_{2,3}=0.3734\pm1.5050\,i$. Here absolute value of $\arg(\lambda_{2,3})$ is 1.3276. So $E_2(-1,-1,0.4762)$ is stable for $\alpha < 0.844836$.

5.2.2 Fractional order Lu chaotic system

The fractional order Lu chaotic system (Lu (2006)) is taken as

$$\frac{d^\alpha x}{dt^\alpha}=a(y-x),$$

$$\frac{d^{\alpha}y}{dt^{\alpha}} = c\,y - x\,z, \tag{5.8}$$

$$\frac{d^{\alpha}z}{dt^{\alpha}} = x\,y - bz, \qquad 0 < \alpha < 1.$$

The phase portrait of the system (5.8) at $\alpha = 0.98$ in $x - y - z$ space is depicted through Fig. 5.3 for the values of the parameters $a = 36$, $b = 3$ and $c = 20$ and initial condition $(x(0), y(0), z(0)) = (0.2, 0.5, 0.3)$.

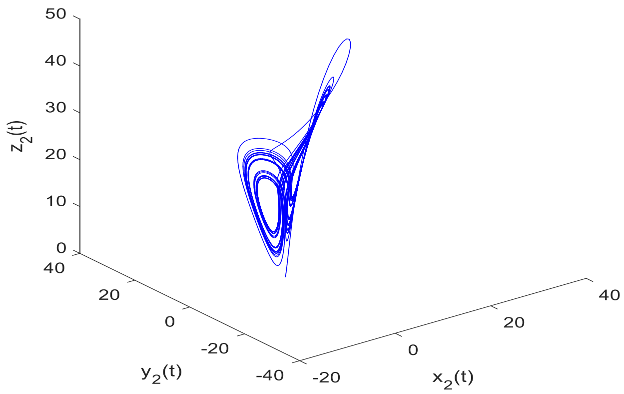


Fig. 5.3 Phase portrait of the fractional Lu system for $\alpha = 0.98$.

5.3 Chaos control of Simple system using nonlinear control method

Let the Simple system (5.2) is considered as a controlled system and control functions are $u_1(t)$, $u_2(t)$ and $u_3(t)$. Now Simple system with fractional order is given as

$$\frac{d^{\alpha}x}{dt^{\alpha}} = 1 + \mu\,zy + u_1(t),$$

$$\frac{d^{\alpha}y}{dt^{\alpha}} = x - y + u_2(t), \tag{5.9}$$

$$\frac{d^{\alpha}z}{dt^{\alpha}} = 1 - xy + u_3(t),$$

Let $(\bar{x},\ \bar{y},\ \bar{z})$ be the solution of equation (5.2), then

$$\frac{d^{\alpha}\bar{x}}{dt^{\alpha}} = 1 + \mu\,\bar{y}\,\bar{z}\,,$$

$$\frac{d^{\alpha}\bar{y}}{dt^{\alpha}} = \bar{x} - \bar{y}\,, \tag{5.10}$$

$$\frac{d^{\alpha}\bar{z}}{dt^{\alpha}} = 1 - \bar{x}\,\bar{y}\,.$$

Now error functions are given as $e_1 = x - \bar{x},\ e_2 = y - \bar{y}$ and $e_3 = z - \bar{z}$. Now we can define error system as

$$\frac{d^{\alpha}e_1}{dt^{\alpha}} = \mu\,y\,z - \mu\,\bar{y}\,\bar{z} + u_1(t)\,,$$

$$\frac{d^{\alpha}e_2}{dt^{\alpha}} = e_1 - e_2 + u_2(t)\,, \tag{5.11}$$

$$\frac{d^{\alpha}e_3}{dt^{\alpha}} = -x\,y + \bar{x}\,\bar{y} + u_3(t)\,.$$

Now the Lyapunov function is given as

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right),$$

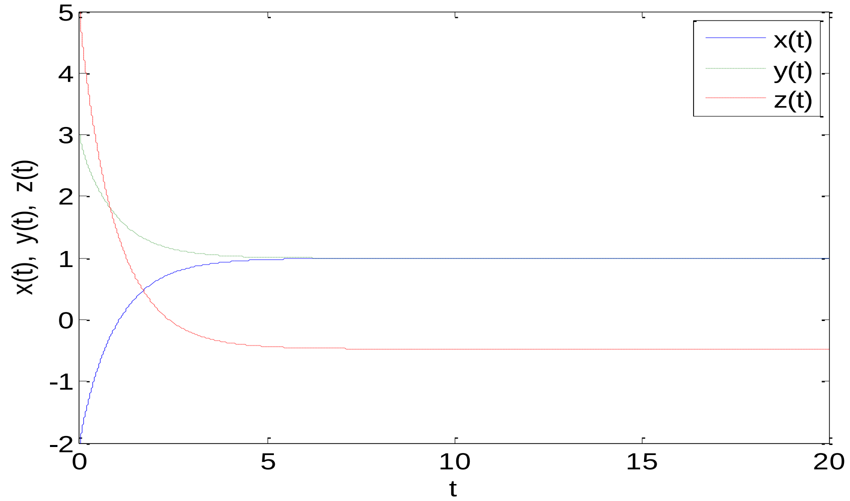
the $\alpha-\mathrm{th}$ order fractional derivative of above expression is given as

$$\frac{d^{\alpha}V}{dt^{\alpha}} = \frac{1}{2}\left(\frac{d^{\alpha}e_1^2}{dt^{\alpha}} + \frac{d^{\alpha}e_2^2}{dt^{\alpha}} + \frac{d^{\alpha}e_3^2}{dt^{\alpha}}\right),$$

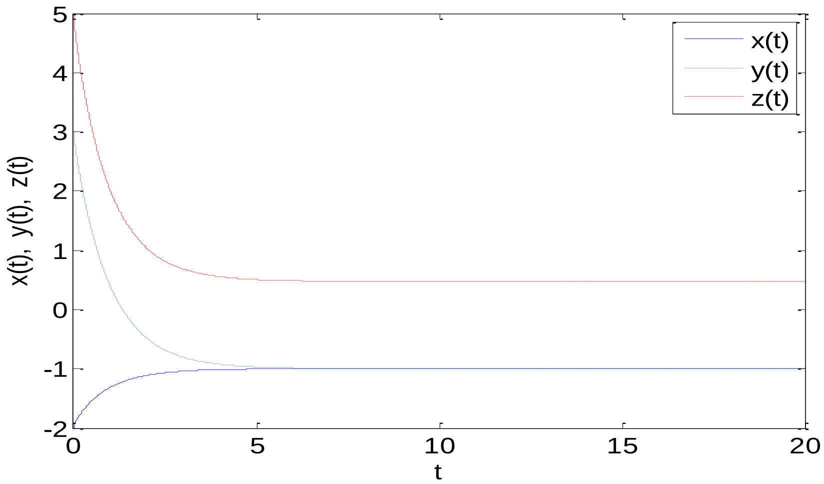
$$\leq e_1\frac{d^{\alpha}e_1}{dt^{\alpha}} + e_2\frac{d^{\alpha}e_2}{dt^{\alpha}} + e_3\frac{d^{\alpha}e_3}{dt^{\alpha}}\,, \qquad \text{(from Lemma 1.7)}$$

i.e., $\leq e_1[\mu\,y\,z - \mu\,\bar{y}\,\bar{z} + u_1(t)] + e_2[e_1 - e_2 + u_2(t)] + e_3[-x\,y + \bar{x}\,\bar{y} + u_3(t)]$.

If we take $u_1(t) = -e_1 - \mu\, y\, z + \mu\, \bar{y}\, \bar{z}$, $u_2(t) = -e_1$ and $u_3(t) = -e_3 + xy - \bar{x}\bar{y}$, it becomes $\frac{d^\alpha V}{dt^\alpha} \le -e_1^2 - e_2^2 - e_3^2 < 0$. This depicts the convergence of the trajectory $(x(t),\ y(t),\ z(t))$ to the equilibrium point $(\bar{x},\ \bar{y},\ \bar{z})$.



(a)



(b)

Fig. 5.4 Plots of trajectories of $x(t)$, $y(t)$, $z(t)$: (a) at equilibrium point E_1; (b) at the equilibrium point E_2.

5.3.1 Results based on numerical simulation for the points E_1 and E_2

In this section, the parameter of the Simple system is $\mu = 2.1$. Initial value and time step size are $(-2, 3, 5)$ and 0.005 respectively. It is clear from Figs. 5.4(a) and 5.4(b) that at

$E_1(1,1,-0.4762)$, the system (5.2) is stable for $0<\alpha<0.95855$. Similarly for $E_2(-1,-1,0.4762)$, the system (5.2) is also stable for $0<\alpha<0.844836$.

5.4 Hybrid projective synchronization between fractional order Simple and Lu systems using nonlinear control method

The fractional order simple chaotic system (5.2) is taken as drive (master) system and fractional order Lu chaotic system is considered as response (slave) system and defined as

$$\frac{d^\alpha x_2}{dt^\alpha}=a(y_2-x_2)+u_1(t),$$

$$\frac{d^\alpha y_2}{dt^\alpha}=cy_2-x_2z_2+u_2(t), \tag{5.12}$$

$$\frac{d^\alpha z_2}{dt^\alpha}=-bz_2+x_2y_2+u_3(t),$$

where $u_1(t)$, $u_2(t)$ and $u_3(t)$ are the control functions. Defining the error functions as $e_1=x_2-k_1x_1$, $e_2=y_2-k_2y_1$, $e_3=z_2-k_3z_1$, where k_1, k_2, k_3 are the scaling factors. The error systems are expressed as

$$\frac{d^\alpha e_1}{dt^\alpha}=a(e_2-e_1)+ak_2y_1-k_1(1+ax_1+\mu y_1z_1)+u_1(t),$$

$$\frac{d^\alpha e_2}{dt^\alpha}=ce_2-x_2z_2-k_2(x_1-y_1-cy_1)+u_2(t), \tag{5.13}$$

$$\frac{d^\alpha e_3}{dt^\alpha}=-be_3+x_2y_2-k_3(1+bz_1-x_1y_1)+u_3(t).$$

Now we define Lyapunov function for stabilize error systems as

$$V(e)=\frac{1}{2}(e_1^2+e_2^2+e_3^2). \tag{5.14}$$

The $\alpha-th$ order fractional derivative of equation (5.14) is

$$\frac{d^{\alpha}V(e)}{dt^{\alpha}} = \frac{1}{2}\left(\frac{d^{\alpha}e_1^2}{dt^{\alpha}} + \frac{d^{\alpha}e_2^2}{dt^{\alpha}} + \frac{d^{\alpha}e_3^2}{dt^{\alpha}}\right)$$

$$\leq \left(e_1\frac{d^{\alpha}e_1}{dt^{\alpha}} + e_2\frac{d^{\alpha}e_2}{dt^{\alpha}} + e_3\frac{d^{\alpha}e_3}{dt^{\alpha}}\right). \qquad \text{(from Lemma 1.7)} \qquad (5.15)$$

Substituting the values of $\frac{d^{\alpha}e_1}{dt^{\alpha}}$, $\frac{d^{\alpha}e_2}{dt^{\alpha}}$ and $\frac{d^{\alpha}e_3}{dt^{\alpha}}$ from equation (5.13) in equation (5.15) and choosing the control function as

$$u_1(t) = -e_1 - a(e_2 - e_1) - ak_2y_1 + k_1(1 + ax_1 + \mu y_1z_1),$$

$$u_2(t) = -e_2 - ce_2 + x_2z_2 + k_2(x_1 - y_1 - cy_1),$$

$$u_3(t) == -e_3 + be_3 - x_2y_2 + k_3(1 + bz_1 - x_1y_1).$$

The $\alpha - th$ order fractional derivative of $V(e)$ becomes

$$\frac{d^{\alpha}V(e)}{dt^{\alpha}} \leq -e_1^2 - e_2^2 - e_3^2 < 0.$$

It can be concluded that $\lim_{t\to\infty}\|e_i(t)\| = 0,\ i = 1,2,3$, and hence the hybrid projective synchronization between drive and response systems is achieved.

Remark 5.1: If the scaling factors $k_1 = k_2 = k_3 = 0$, the hybrid projective synchronization problem will be changed into a chaos problem.

Remark 5.2: If scaling factors are taken as $k_1 = 1,\ k_2 = -1,$ and $k_3 = 1$, then hybrid projective synchronization is reduced to hybrid synchronization.

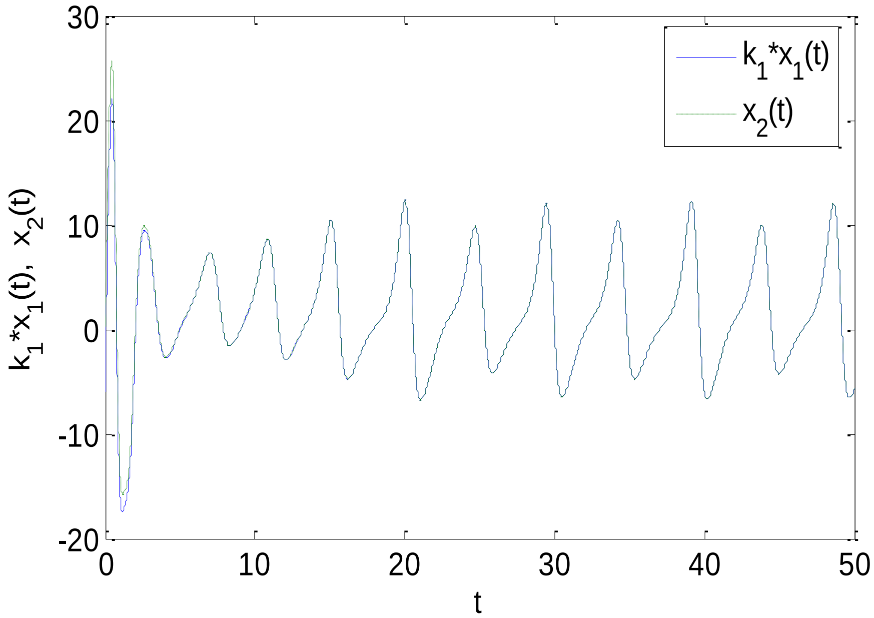
Remark 5.3: If we take scaling factors $k_1 = k_2 = k_3 = 2$ and $k_1 = k_2 = k_3 = -2$, the problem becomes projective synchronization and projective anti-synchronization respectively.

Remark 5.4: If scaling factors are taken as $k_1 = k_2 = k_3 = 1$ and $k_1 = k_2 = k_3 = -1$, the problem is complete synchronization and anti-synchronization respectively**.**

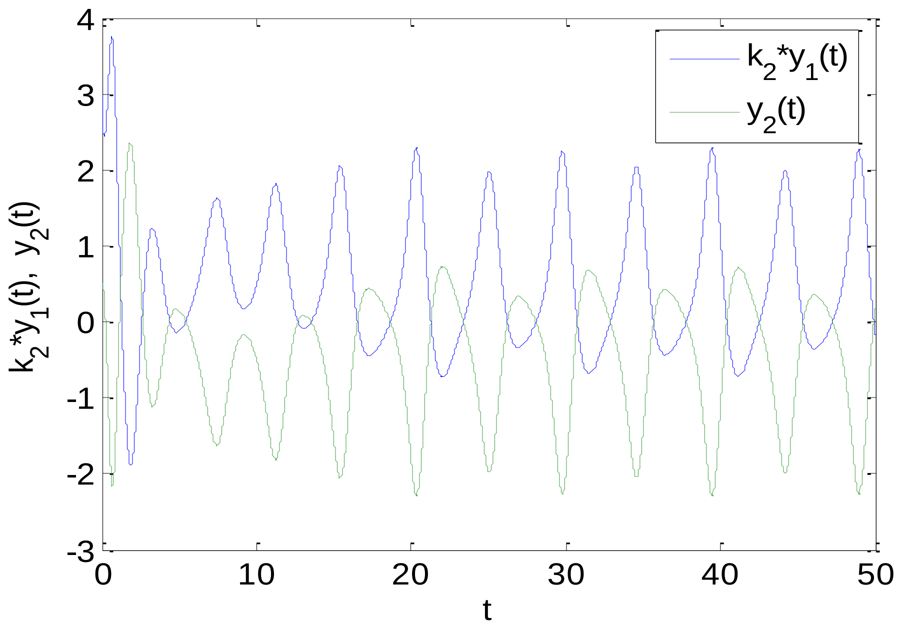
5.5 Numerical results and discussion

In this section, the parameters' values of fractional Simple chaotic and Lu chaotic systems are considered as earlier. For drive and response systems the initial conditions are taken as $(x_1(0), y_1(0), z_1(0)) = (-2, 3, 5)$ and $(x_2(0), y_2(0), z_2(0)) = (0.2, 0.5, 0.3)$. During hybrid projective synchronization the scaling factors are chosen as $k_1 = 3$, $k_2 = -1$ and $k_3 = 2$. Thus the initial condition of error system will be $(e_1(0), e_2(0), e_3(0)) = (6.2, 3.5, -9.7)$. The time step size for Adams-Bashforth Moulton method (Diethelm et al. (2004), Diethelm and Ford (2004)) is 0.005. The Figs. 5.5(a)-(c) representing the hybrid projective synchronization between the fractional Simple and fractional Lu systems. Fig. 5.5(d) shows that when time becomes large error states converge to zero . For $k_1 = 1$, $k_2 = -1$ and $k_3 = 1$, time variation of the states are depicted in Figs. 5.6(a)-(c) and the corresponding hybrid synchronization error is shown in Fig. 5.6(d). For $k_1 = 2$, $k_2 = 2$ and $k_3 = 2$, projective synchronization of systems (5.2) and (5.12) are shown in Fig. 5.7, Figs. 5.7(a)-(c) explain that the state vectors of two chaotic systems (5.2) and (5.9) are almost proportionally synchronized. The Fig. 5.7(d) depicts that the behaviors of trajectories of the error system are tending to zero. For $k_1 = -2$, $k_2 = -2$ and $k_3 = -2$, the nature of state variables of the fractional simple chaotic and Lu chaotic systems is to be proportionally synchronized in the opposite direction with the important configuration that the error system is converging at zero which is given in Figs. 5.8(a)-(d). Similarly, for $k_1 = 1$, $k_2 = 1$ and $k_3 = 1$, complete synchronization of systems

(5.2) and (5.12) are shown in Figs. 5.9(a)-(c) and the corresponding error system (5.13) is shown in Fig 5.9(d). For $k_1 = -1$, $k_2 = -1$ and $k_3 = -1$, state response and the anti synchronization error system converge to zero as shown in Figs. 5.10(a)-(d).



(a)



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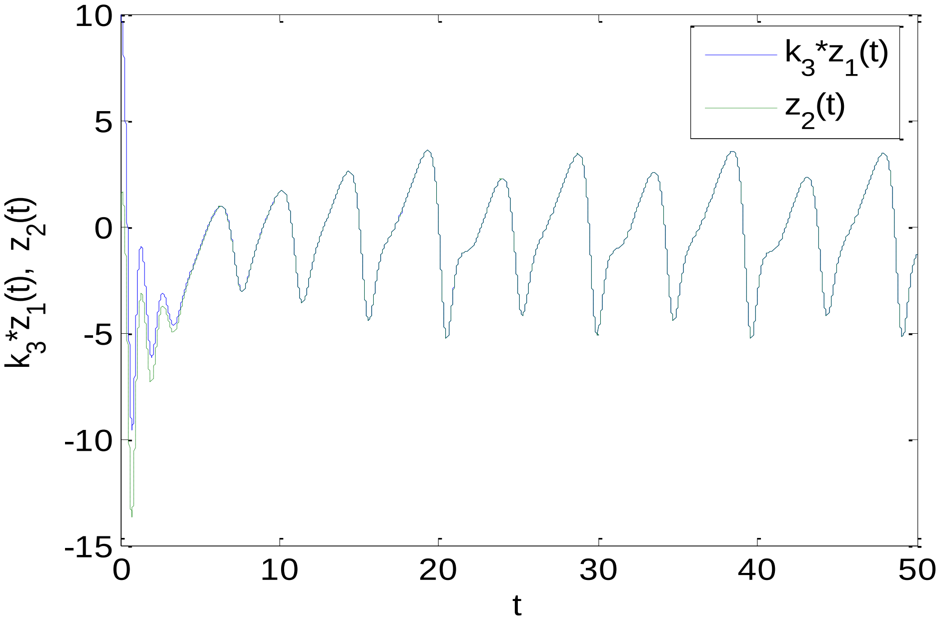
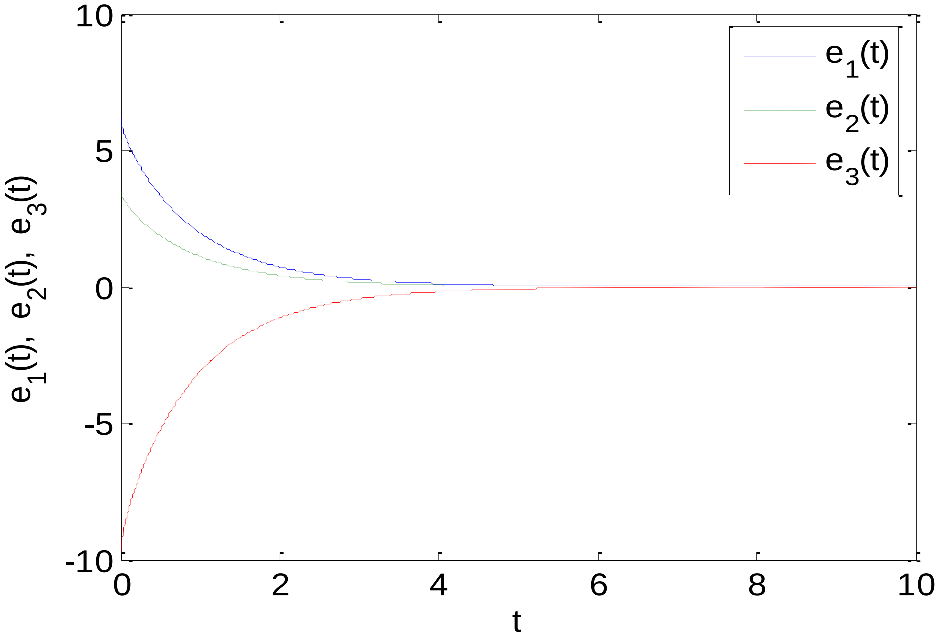
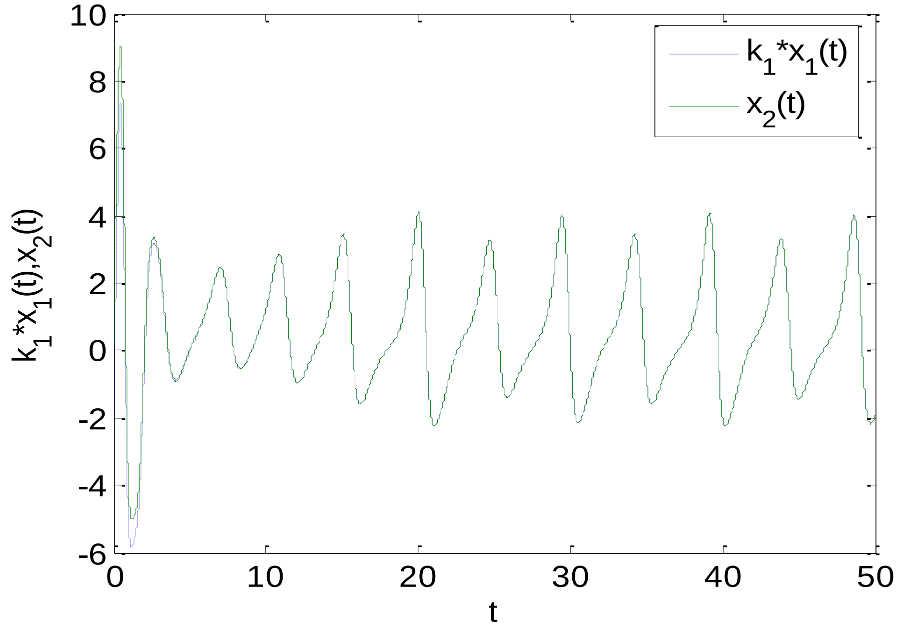


(c)

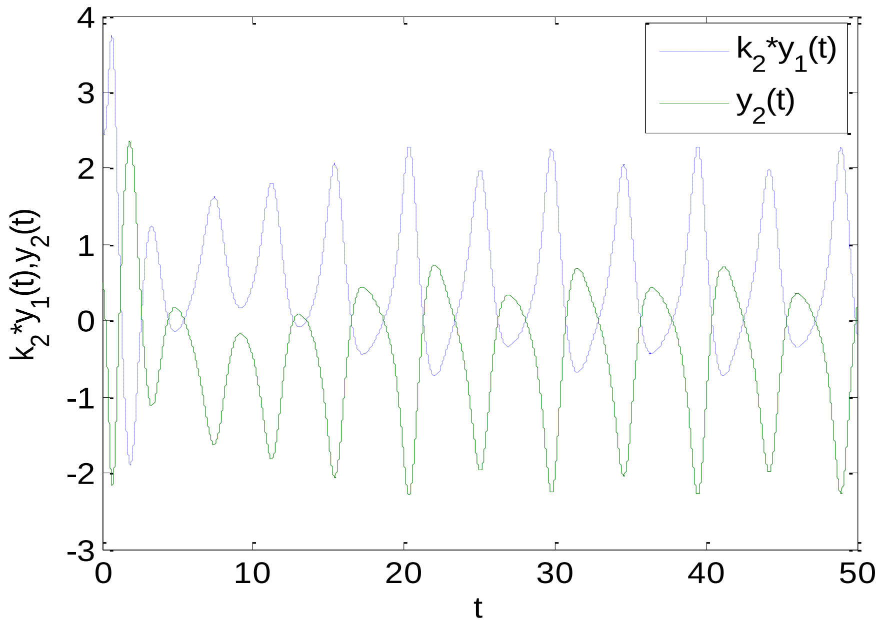


(d)

Fig. 5.5 Trajectories of state vectors of systems (5.2) and (5.12) at $k_1 = k_2 = k_3 = 0$ for order $\alpha = 0.98$: (a) between $k_1 * x_1(t)$ and $x_2(t)$; (b) between $k_2 * y_1(t)$ and $y_2(t)$; (c) between $k_3 * z_1(t)$ and $z_2(t)$; (d) The evolution of $e_1(t), e_2(t)$ and $e_3(t)$.


k1*x1(t)
x2(t)
k1*x1(t),x2(t)
t
10
8
6
4
2
0
-2
-4
-6
0
10
20
30
40
50

(a)


k2*y1(t)
y2(t)
k2*y1(t),y2(t)
t
4
3
2
1
0
-1
-2
-3
0
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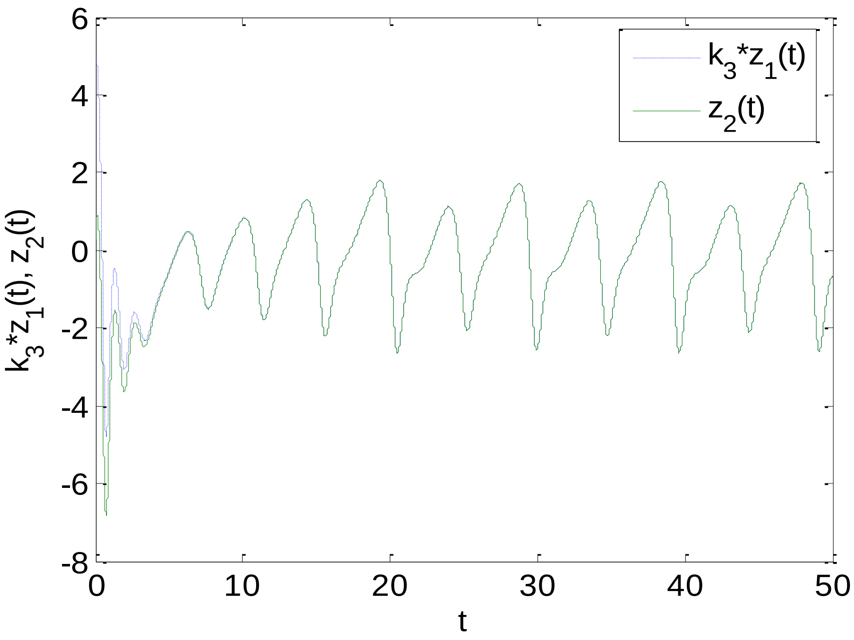
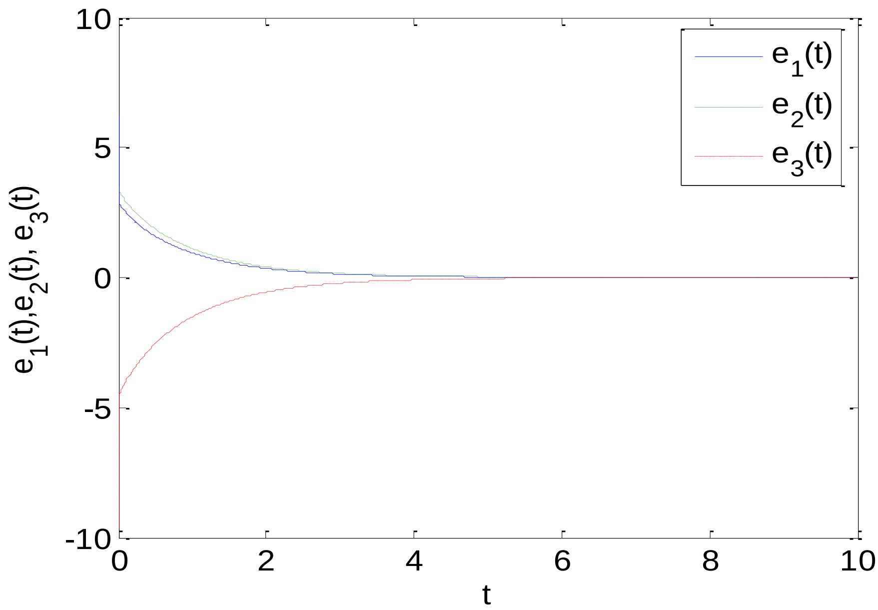
40
50

(b)

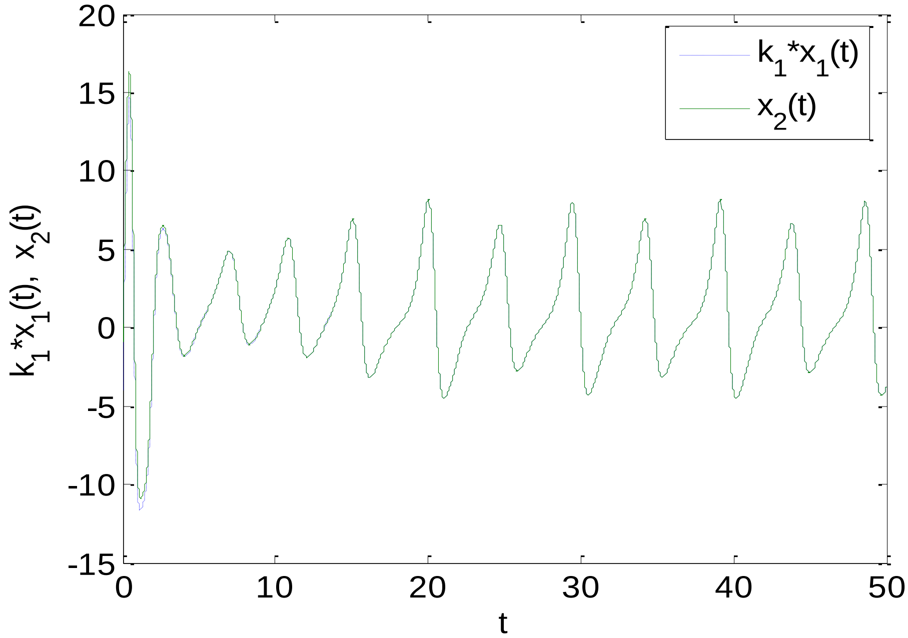


(c)



(d)

Fig. 5.6 Trajectories of state vectors of systems (5.2) and (5.12) at $k_1 = 1,\ k_2 = -1,$ and $k_3 = 1$ for order $\alpha = 0.98$: (a) between $k_1 * x_1(t)$ and $x_2(t)$; (b) between $k_2 * y_1(t)$ and $y_2(t)$; (c) between $k_3 * z_1(t)$ and $z_2(t)$; (d) The evolution of $e_1(t), e_2(t)$ and $e_3(t)$.


20
15
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5
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-5
-10
-15
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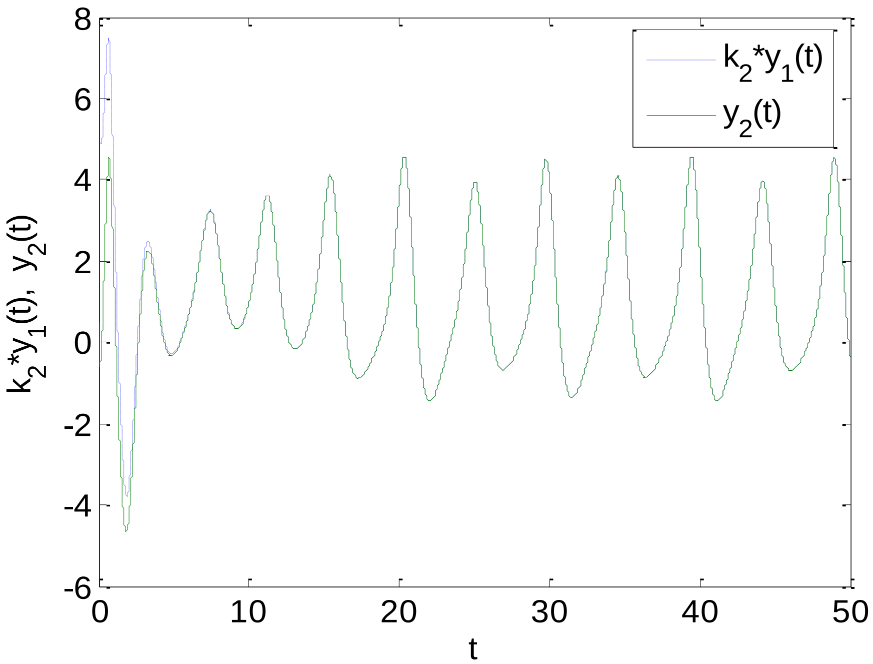
50
t
k1*x1(t), x2(t)
k1*x1(t)
x2(t)

(a)


8
6
4
2
0
-2
-4
-6
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10
20
30
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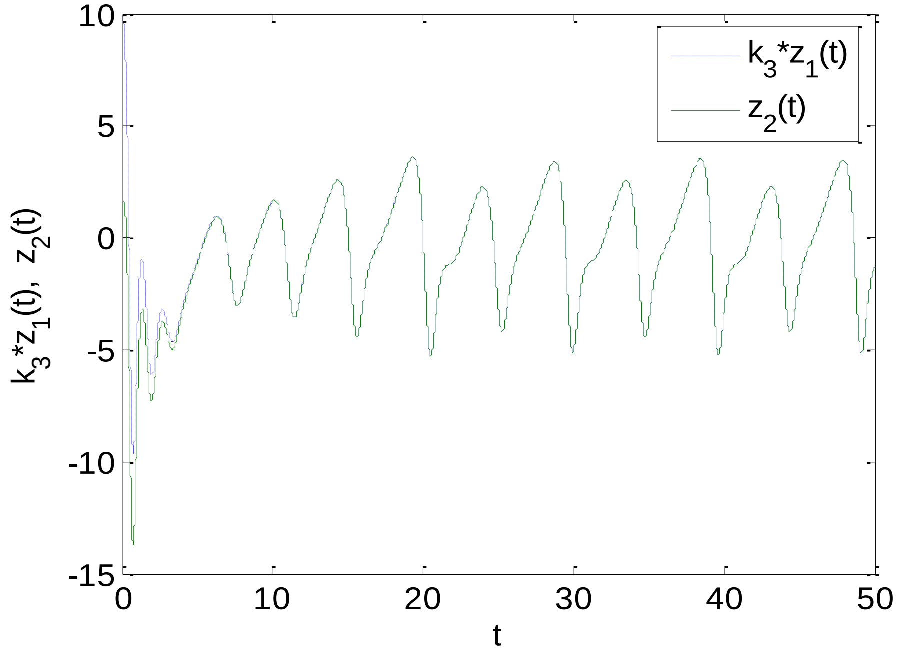
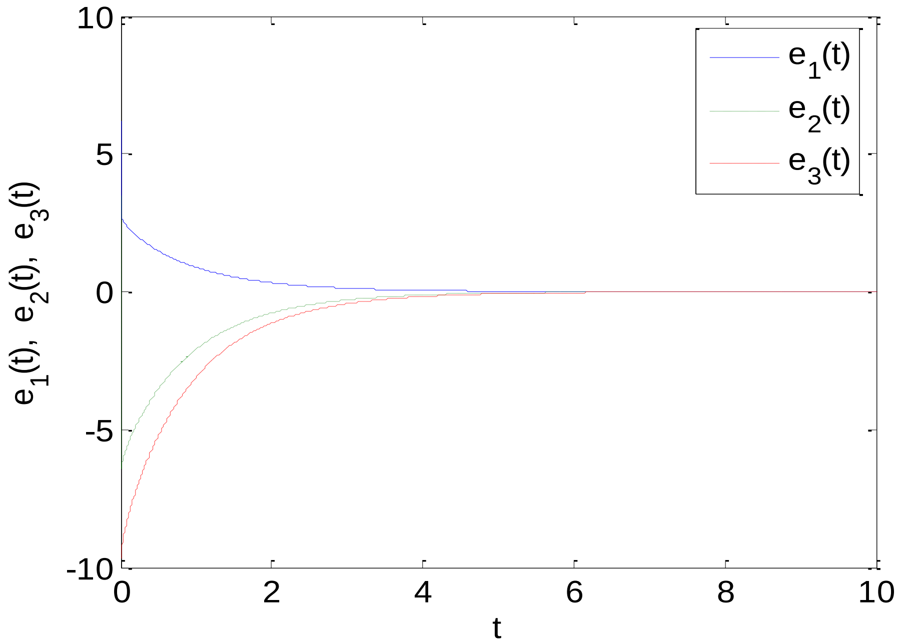
50
t
k2*y1(t), y2(t)
k2*y1(t)
y2(t)

(b)

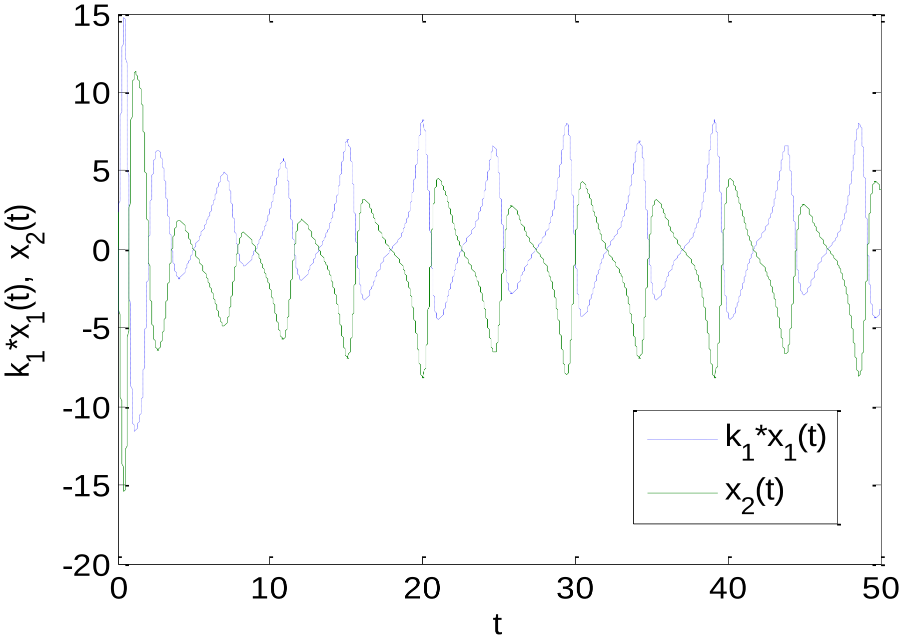


(c)

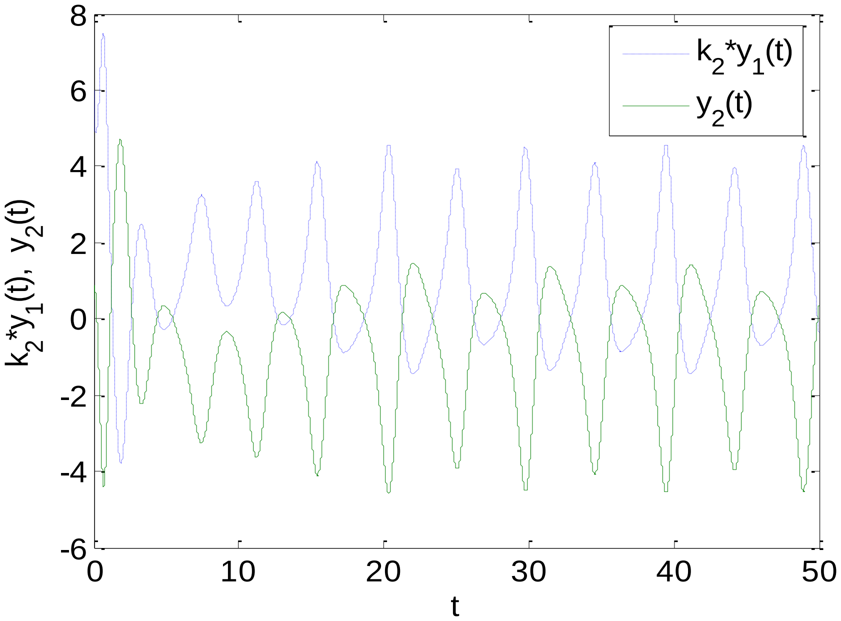


(d)

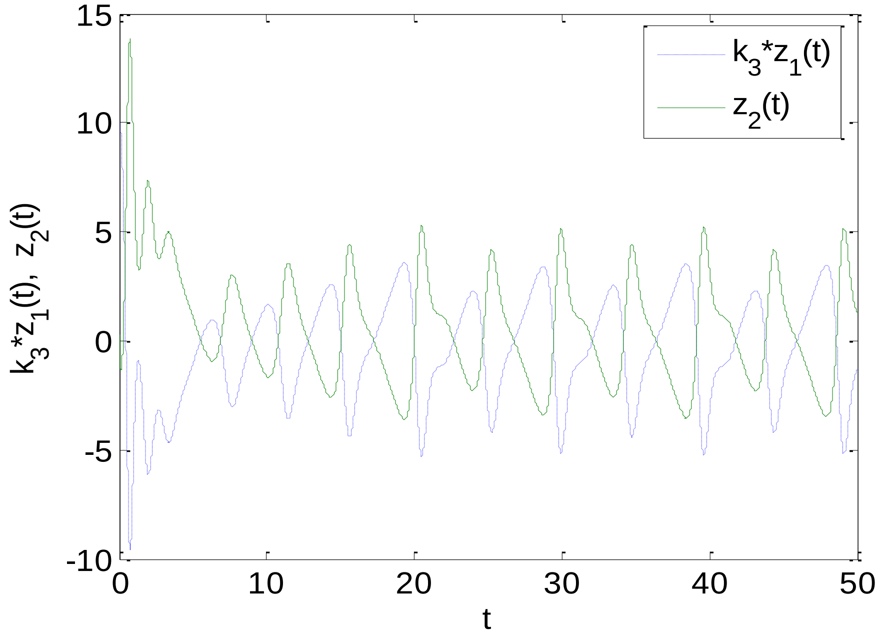
Fig. 5.7 Trajectories of state vectors of systems (5.2) and (5.12) at $k_1 = k_2 = k_3 = 2$ for order $\alpha = 0.98$: (a) between $k_1 * x_1(t)$ and $x_2(t)$; (b) between $k_2 * y_1(t)$ and $y_2(t)$; (c) between $k_3 * z_1(t)$ and $z_2(t)$; (d) The evolution of the $e_1(t), e_2(t)$ and $e_3(t)$.


15
10
5
0
-5
-10
-15
-20
0
10
20
30
40
50
t
k1*x1(t), x2(t)
k1*x1(t)
x2(t)

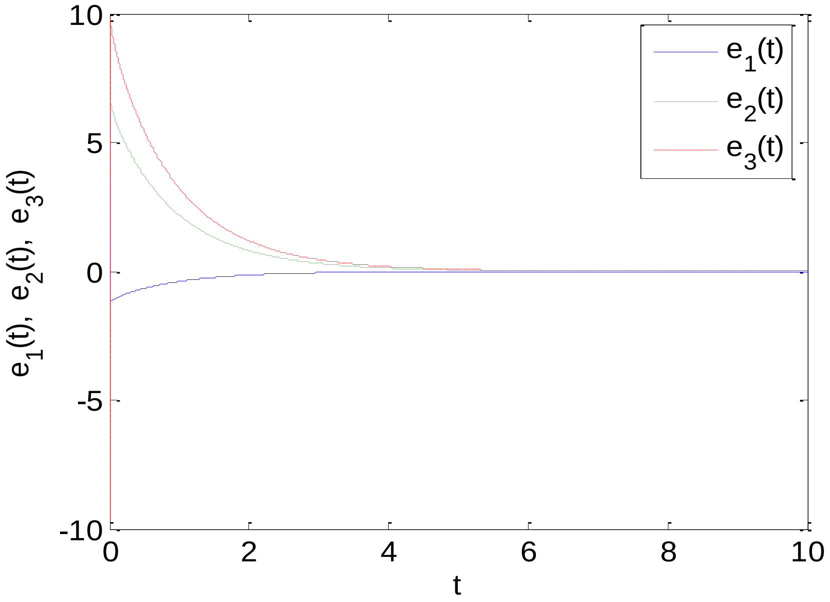
(a)


8
6
4
2
0
-2
-4
-6
0
10
20
30
40
50
t
k2*y1(t), y2(t)
k2*y1(t)
y2(t)

(b)

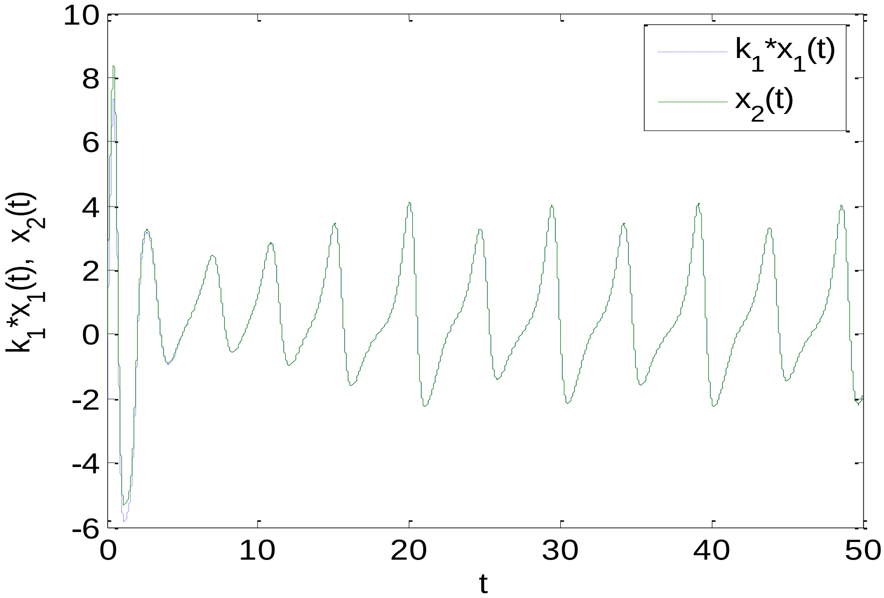


(c)

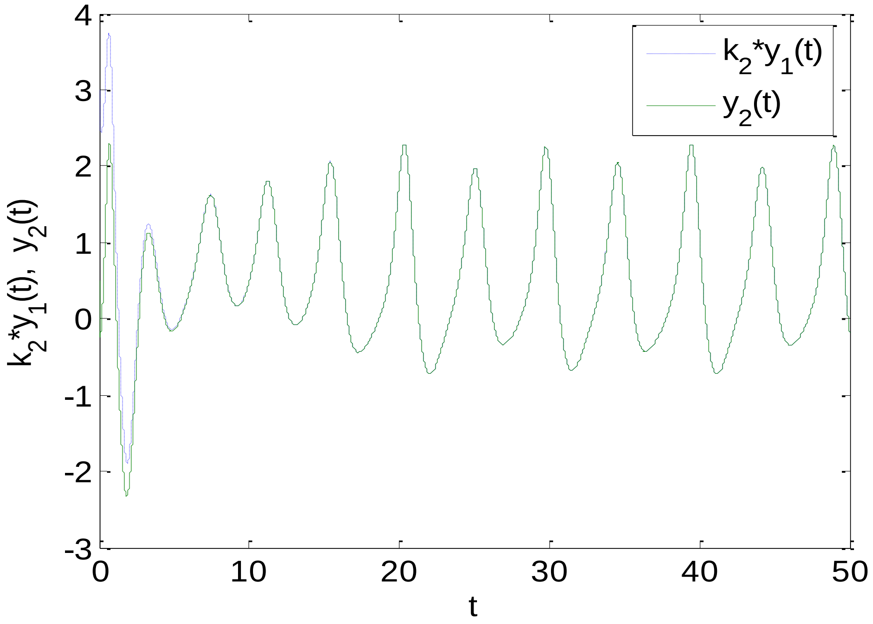


(d)

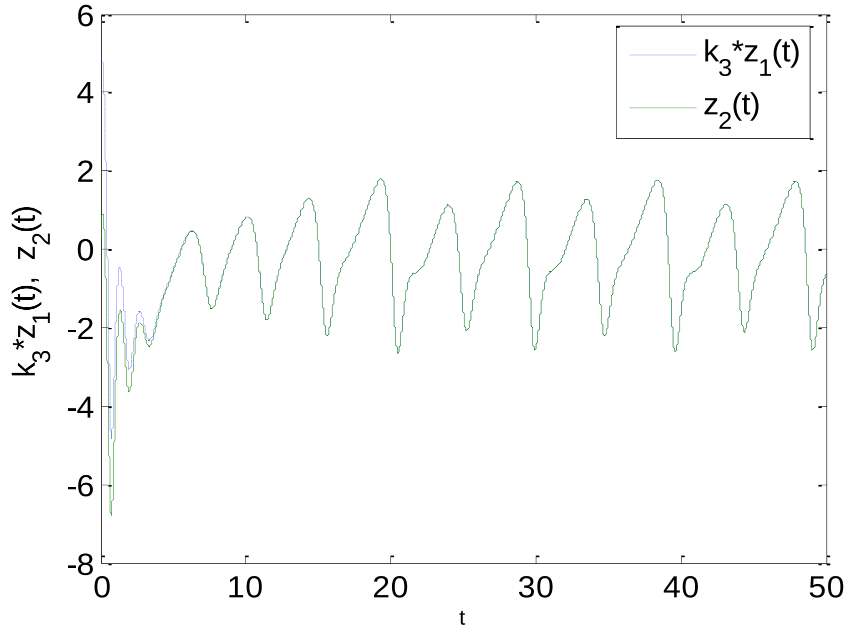
Fig. 5.8 Trajectories of state vectors of systems (5.2) and (5.12) at $k_1 = k_2 = k_3 = -2$ for order $\alpha = 0.98$: (a) between $k_1 * x_1(t)$ and $x_2(t)$; (b) between $k_2 * y_1(t)$ and $y_2(t)$; (c) between $k_3 * z_1(t)$ and $z_2(t)$; (d) The evolution of $e_1(t), e_2(t)$ and $e_3(t)$.


k1*x1(t)
x2(t)
k1*x1(t), x2(t)
t
10
8
6
4
2
0
-2
-4
-6
0
10
20
30
40
50

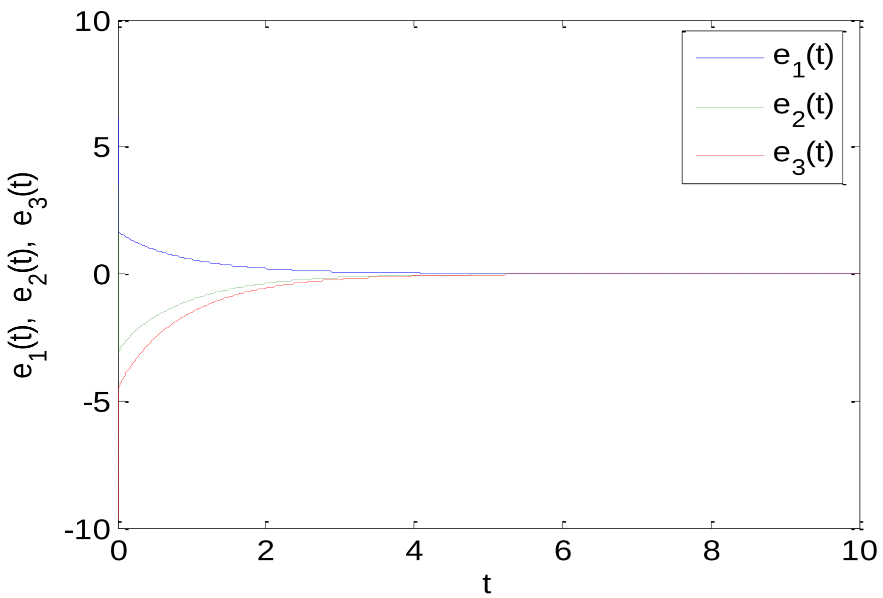
(a)


k2*y1(t)
y2(t)
k2*y1(t), y2(t)
t
4
3
2
1
0
-1
-2
-3
0
10
20
30
40
50

(b)

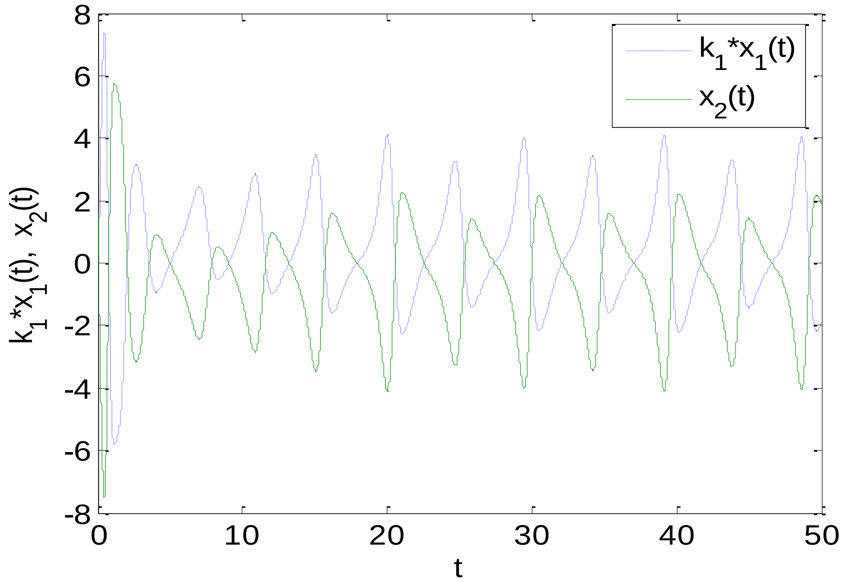


(c)

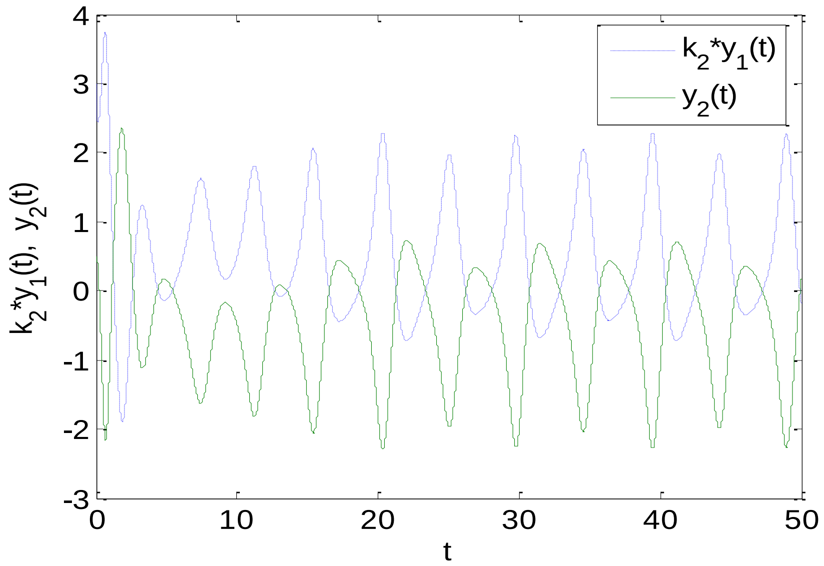


(d)

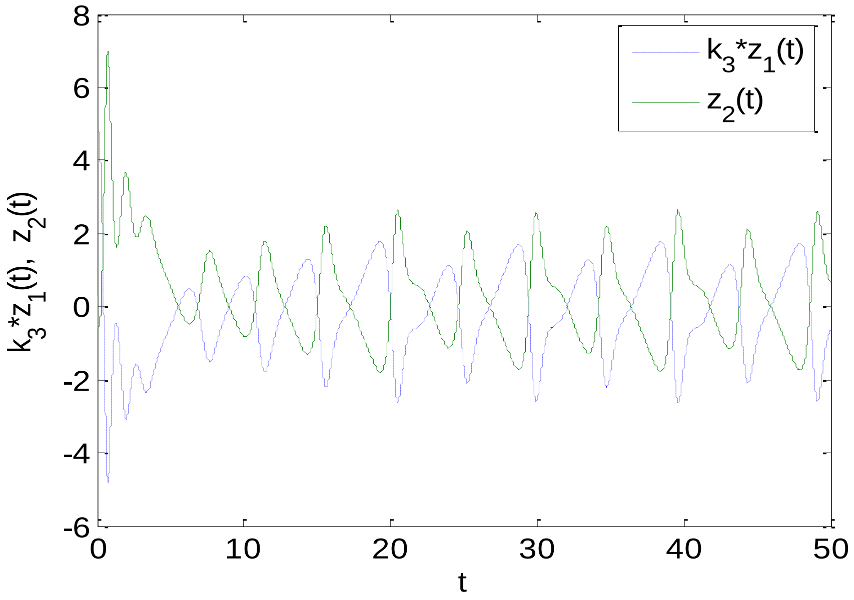
Fig. 5.9 Trajectories of state vectors of systems (5.2) and (5.12) at $k_1 = k_2 = k_3 = 1$ for order $\alpha = 0.98$: (a) between $k_1 * x_1(t)$ and $x_2(t)$; (b) between $k_2 * y_1(t)$ and $y_2(t)$; (c) between $k_3 * z_1(t)$ and $z_2(t)$; (d) The evolution $e_1(t), e_2(t)$ and $e_3(t)$.


k1*x1(t)
x2(t)
k1*x1(t), x2(t)
t
8
6
4
2
0
-2
-4
-6
-8
0
10
20
30
40
50

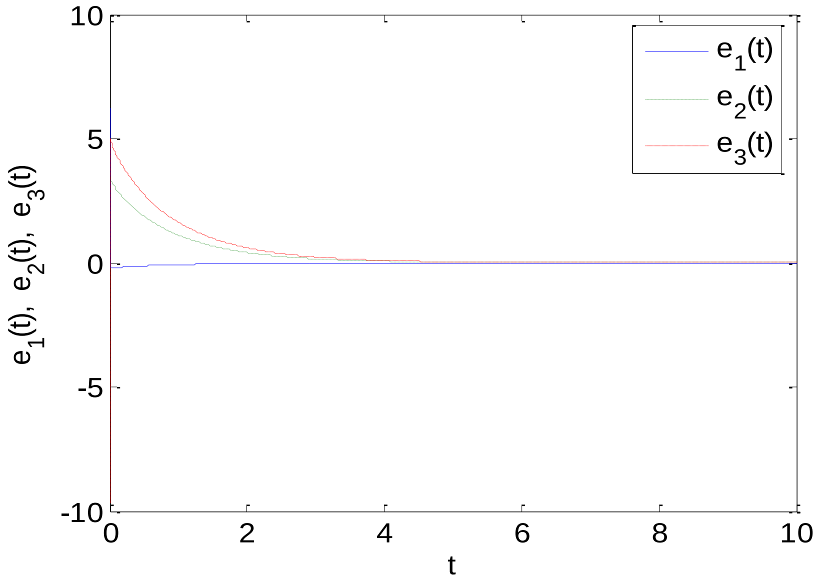
(a)


k2*y1(t)
y2(t)
k2*y1(t), y2(t)
t
4
3
2
1
0
-1
-2
-3
0
10
20
30
40
50

(b)



(c)



(d)

Fig. 5.10 Trajectories of state vectors of systems (5.2) and (5.12) at $k_1 = k_2 = k_3 = -1$ for order $\alpha = 0.98$: (a) between $k_1 * x_1(t)$ and $x_2(t)$, (b) between $k_2 * y_1(t)$ and $y_2(t)$, (c) between $k_3 * z_1(t)$ and $z_2(t)$ and (d) The evolution of $e_1(t), e_2(t)$ and $e_3(t)$ with time t.

5.6 Conclusion

In this chapter, three important results have been achieved by the author in which first goal is the analysis of local stability of the Simple chaotic system. Second one is the application of the control function of fractional order Simple system at different equilibrium points. Nonlinear feedback control method has employed to discuss sufficient conditions for control of the fractional order system. It is noticed that one can easily control Simple system. Third one is the investigation of the hybrid projective synchronization between non-identical fractional orders chaotic system using nonlinear control method. This proves the reliability of the process of implementation and potential application of the nonlinear control method. The results obtained by numerical simulation justify the strength of the proposed method.

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