A Study of Fixed Point Theory Using Various Contractions in Different Spaces and Some Applications

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DOCTOR OF PHILOSOPHY

by

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Chapter 8

Conclusions and Future Remarks

The thesis is focused on fixed point theory for various kinds of contraction in different spaces. This chapter concludes the thesis and presents some possible future work in the direction of work done in this thesis.

8.1 Conclusions

This section serves as the conclusion of the entire thesis, summarizing the important findings and concluding remarks from the overall investigation.

Chapter 1 has introduced the fundamental concepts and literature related to the work presented in this thesis.

Chapter 2 has presented some common fixed point results using a pair of Ćirić quasi-contraction operators. After that, with the help of graphical contraction and β-quasi contraction, we have explored the sufficient condition for the common fixed point problem to be well-posed and to have Ulam–Hyers stability and Ostrowski property.

Chapter 3 has proposed the concept of enriching contractive type map using the Krasnoselskii-Mann iteration for Ćirić quasi-contraction in the Banach space and convex metric space. Furthermore, we have introduced the concept of enriched cyclic Ćirić quasi-contraction in the Banach space and established their convergence properties.

Chapter 4 has discussed the approximating fixed point of the strictly pseudo-contractive map in the context of the Banach space. We conclude from the numerical experiments presented here that for approximating the fixed points of some pseudo-contractive mappings, the Krasnoselskii-Mann iteration is more convenient than the modified Krasnoselskii-Mann iteration. Also, we have investigated the class of pseudo-contractive mappings is an unsaturated class of mappings in the Banach space.

Chapter 5 has initiated the study of enriched Ćirić-Reich-Rus contraction and enriched Kannan contraction in the quasi-Banach space and obtained their approximating fixed point results and unifying error estimations, which are the actual generalization of the approximating fixed point results in the Banach space. In addition, we have obtained the Maia-type fixed point results for enriched Ćirić-Reich-Rus contraction and enriched Kannan contraction mappings in the quasi-Banach space.

Chapter 6 has discussed the generalization of Proinov contraction in the non-triangular metric space from the metric space. Further, as an application, we find the existence and uniqueness of a solution of the homogeneous Fredholm integral equation in non-triangular metric space using Proinov contraction.

Chapter 7 has proposed the existence of strong m-tuple fixed point using the ideas of cyclic mapping and Φ-tupling. Also, as an application of this strong m-tuple fixed results, a strong m-tuple fractal through the Φ-iterated tupling system is generated.

8.2 Some Future Directions

In spite of continuous research in the field of fixed point theory, significant issues exist in this area. The study of fixed point theory is a burgeoning topic due to its applicability in different areas such as approximation theory, variational inequality problems, fractal analysis, etc. Based on the research presented in this thesis, there are various opportunities for further expansion. The following are some potential future problems one can address.

- In Chapter 2, one can deduce a similar type of common fixed point theory in the complete metric space (X, d) for a pair of self-operators satisfying the generalized quasi-contraction [62].
- In Chapter 3, one can generate the enriched fractal using cyclic Ćirić quasi-contraction and the classical Hutchinson Barnsley theory of IFS.

- In Chapter 5, one can define enriched Ćirić contraction and enriched Hardy-Rogers contraction in the quasi-Banach space and generalize the approximating fixed point results for these two contractions in the quasi-Banach space from the Banach space.
- In Chapter 6, one may try to replace the contractivity condition (6.1) with a generalized contractive type condition (6.9).
- In Chapter 7, in theorems (7.14, 7.15, 7.16) the corresponding mapping $F : X^m \to X$ is assumed to be continuous. It will be interesting to see if the continuity assumption on F could be weakened or removed, and yet one can generate m-tuple fractals.

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