
Chapter 1

Introduction: A bibliographic review

Within the framework of Landau's approach to condensed matter physics, phases are linked to different symmetries of the system. These symmetries spontaneously break down as the system goes through a phase transition. For instance, during the transition from the solid state to the liquid state, the continuous translation symmetry of the liquid transforms into the discrete translation symmetry of the solid. In a manner analogous, when a magnetic material goes from being ferromagnetic to being paramagnetic, the continuous rotational symmetry of the magnetic moments that exist within the paramagnetic phase is disrupted when the material transforms into the ferromagnetic phase. This is because the magnetic moments now have to align themselves along a particular direction in space. However, recently, it was found that certain phases of matter can exist without disrupting any of the system's symmetry; instead, these phases of matter merely involve a change in the topology of the underlying electronic structure. Therefore, such categories of phase transitions do not fall inside Landau's domain of phase transitions, and as a result, they make up a new category of phase transitions that is known as topological phase transitions. One example that is both straightforward and historically significant is the well-known quantum Hall state, which may be characterised by the topological order that it contains[1], [2]. The topological order is usually characterized by the existence of certain invariants for the system under the action of various perturbations acting on the system. In the quantum Hall state, the quantized value of the Hall conductance and the



Figure 1.1: Transformation of a coffee mug to a donut through continuous deformation. During the transformation genus number (g) remain fixed. ($g = 1$). (Image is taken from Ref. [3])

number of gapless boundary modes are examples of two fundamental quantities that do not change under gradual shifts in the material parameters that are characteristic of the topological phase.

1.1 Introduction to topology

Topology is a branch of mathematics that deals with the shape and its transformation of an object. In order to characterize the shape for measuring its topological features one can consider the Gaussian curvature of the surface of the object. If we integrate the Gaussian curvature in a closed surface for a particular object, we get a fixed number. This fixed number characterizes the topology of the object. The above theory is called the Gauss-Bonnet theorem[4]. The theorem talks about the conservation of that (genus) number under continuous deformations (without tearing, gluing etc.) of the object. Thus two objects possessing the same genus number would belong to the same topological class. For example, if we integrate the Gaussian curvature (K) in a closed surface of a sphere, we get a genus number (g) equivalent to 0.
$$\left(\frac{1}{2\pi} \oint K \cdot dA = 2 - 2g \right)$$

If we apply the above mathematics on a cube, we get the same genus number. The result indicates that the sphere and the cube are topologically equivalent. Such a result can raise a question. How can two completely different objects become topologically the same? The answer lies only in the shape of the corresponding objects. The transformation from

cube to sphere is simply a geometric phenomena. Theory of topology simply states that they belong to the same topological class. This example explains how the same genus number leads to the same classes of topology. If we apply the above mathematics in a case of coffee mug or donut, we get $g = 1$. The result indicates that the coffee mug and the donut are topologically equivalent and one can change a donut to a coffee mug by continuous (or smooth) deformation (without joining or tearing any parts of the objects) as shown in Fig. 1.1. The genus number, in this case, is equivalent to the number of holes present in the shape as it shown in Fig. 1.2.

The same concept is extended to condensed matter physics in the context of bandstructure of the compound. Let us defined a Bloch wave function as $|u_m(\mathbf{k})\rangle$. when \mathbf{k} is transported around a closed loop, $|u_m(\mathbf{k})\rangle$ acquires a well defined Berry phase given by the line integral of $\mathcal{A}_m = i \langle u_m | \nabla_{\mathbf{k}} | u_m \rangle$. The result can also be expressed as a surface integral of the Berry flux $\mathcal{F}_m = \nabla \times \mathcal{A}_m$. If we integrate the Berry flux (\mathcal{F}_m) over a closed surface in the BZ, we get a special number called the Chern number. The chern number for a Bloch state given by $n = \frac{1}{2\pi} \int d^2\mathbf{k} \mathcal{F}_m$. The total Chern number, summed over all occupied states is invariant. It is a topologically invariant quantity (it does not change with the smooth variation of Hamiltonian). The Chern number, in this case, is the topological invariant and is equivalent to the genus number of a object and hence it is called Chern insulator or topological insulator. The details discussions on Chern insulator are discussed in the subsequent sections.

1.2 Topological nature of Hall conductivity

The topological insulator contains an insulating bulk state whereas the surface (in case of 3D TI) or edge state (in case of 2D TI) are conducting which is protected by time reversal symmetry[6]. Let us start with the integer quantum Hall effect[7]. The integer quantum Hall effect occurred, when a strong magnetic field is applied in a 2D system. In such a case, the motion of the electron is confined to a circular orbit with a cyclotron frequency ω_c (qB/m). The phenomena leads to quantization of energy which are called Landau levels

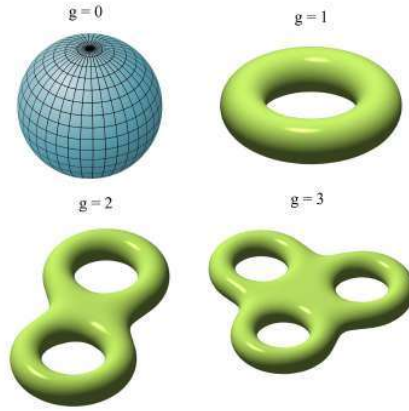


Figure 1.2: The topological classification of different geometries. The genus number is defined as how many numbers of holes the particular object contains. (Figure is recreated from Ref. [5])

$[\epsilon_m = \hbar\omega_c(m + 1/2)]$. If N Landau levels are filled and the rest of it remains empty, then a fixed energy gap separates the occupied state from the empty states just like an insulator. The major difference of above phenomena with an ordinary insulator is the drifting of electron with a circular orbit that leads to the quantized Hall conductivity ($\sigma_{xy} = Ne^2/h$, $N = \text{natural number}$) The quantized Hall conductivity is measured in the precision of 1 part in 10^9 [8]. Such quantization with high precision suggests the topological nature of Hall conductivity. Since TKNN[1] (discussion on TKNN is put in the subsequent section) demonstrated that σ_{xy} and n (Chern number) have the same form when calculated using the Kubo formula, N in the earlier expression is the same as n . The Chern number n is a topological invariant in the sense that it is unaffected by smooth variations in the Hamiltonian. This clarifies why σ_{xy} 's robust quantization exists.

1.3 Effect of magnetic field on Bloch electrons

Because of the lattice's translational symmetry, we know that the Bloch wave function has a periodic component. By applying a magnetic field to the crystal, a new state of circularly quantized electrons is created. The lack of translational symmetry precludes the use of the Bloch wave function to describe such a state. However, if we define a unit cell with area $2\pi\hbar/eB$ enclosing flux quanta, then lattice translation can follow the periodic nature of

wave function. In such a scenario, Bloch's theorem can allow the states to be labeled by 2D crystal momentum (\mathbf{k}). If there is no periodic potential, the energy level becomes \mathbf{k} -independent Landau level ($E_m(\mathbf{k}) = \epsilon_m$). On the contrary, if the periodic potential is on, the energy level disperses with \mathbf{k} . This leads to the compound's band structure similar to an ordinary insulator.

1.4 Nobel prize (2016)

How is the quantum Hall state different from ordinary insulators? The above study suggests that there is a similarity between the quantum Hall state and the ordinary insulator. But, are the two objects same? The answer, greatly explained by Thouless, Kohmoto, Nightingale and den Nijs (1982) (TKNN)[1], is an interesting topic of topology. The band structure in 2D contains a connection from the crystal momentum \mathbf{k} to the Bloch Hamiltonian $\mathcal{H}(\mathbf{k})$. The gapped band structure can be topologically classified by taking the similar classes of $\mathcal{H}(\mathbf{k})$ which can be continuously deformed from one into another without closing the energy gap. These classes are sketched by a topological invariant $n \in \mathbb{Z}$ (\mathbb{Z} denotes the integer) defined as the Chern invariant. Although the concept of Chern invariant originates from the mathematical theory of the fiber bundles[4], the same can be extended to the electronic structure constituted of the Bloch wave function.

David J. Thouless, F. Duncan M. Haldane, and J. Michael Kosterlitz have been given the 2016 Nobel Prize in Physics "for theoretical discoveries of topological phase transitions and topological phases of matter." Kosterlitz and Thouless discovered a novel sort of phase transition in two-dimensional (2D) systems in 1972, and it has since been determined that topological imperfections are a key factor in the so-called Kosterlitz–Thouless phase transition[9], [10]. Understanding fascinating quantum events that occur at extremely low temperatures in specific kinds of magnets, as well as superconducting and superfluid films, depends on this theory. The quantization of the Hall conductance in 2D electron gases was first explained in 1982 by Thouless and coworkers using topology, leading to the formulation of Thouless-Kohmoto-Nightingale-Nijs (TKNN) number.

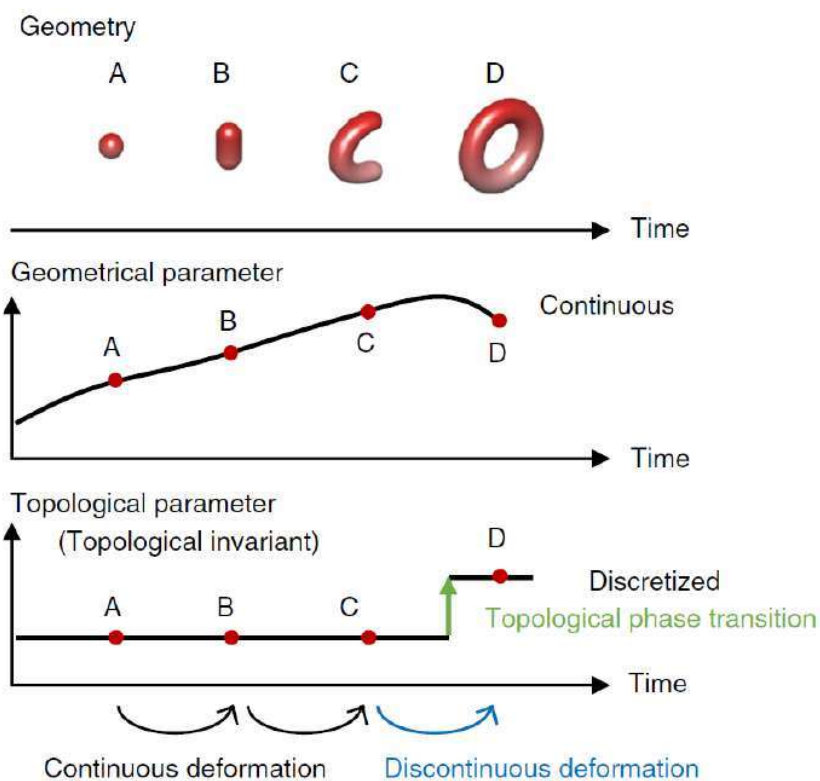


Figure 1.3: Illustration showing the evolution of geometrical and topological parameters. Geometrical characteristics, such as the surface-to-volume ratio, change constantly as a sphere becomes a torus by elongation and end-to-end connection, but a topological parameter known as a topological invariant changes abruptly and discontinuously. A discontinuous deformation is a deformation that modifies the topological invariant as opposed to a continuous deformation, which does not affect the topological invariant. A topological phase transition occurs when two topologically inequivalent states change concomitant to the change of the topological invariant. (Figure is taken from the Ref. [11])

Topologically equivalent items are those that share the same topological invariants. Consider a sphere that changes into a torus over time using two intermediates [Fig. 1.3 (a–d)]. A topological parameter, also known as a topological invariant, is discretized as an integer, unlike geometrical parameters, which vary over time continuously. The topological invariant, which is the quantity of holes is zero for items A through C and one for object D. The topological invariant is preserved in continuous geometrical deformations (excluding operations like cutting open the structure or gluing two parts of the structure together which violate our definition of continuous deformation) from A to B and from B to C. Therefore, regardless of the magnitude of the continuous deformation, objects A, B, and C can be changed into one another. In a broader sense, any two geometrical objects that are topologically equivalent can be transformed into one another by applying suitable continuous deformation, such stretching and compression. In contrast, topological invariants are changed by deformations that involve cutting, ripping, or attaching. These deformations are referred to be discontinuous which is caused by gluing two loose ends of the deformed original sphere causing a topological phase transition. The topological invariant changes from zero to one, from C to D, which is caused by gluing two loose ends of the deformed original sphere causing a topological phase transition. A topological phase transition occur between two topologically different states characterized by different topological invariants. According to the notion of topological equivalence, the systems remains topologically equivalent under any arbitrary transformations applied to it so long as the topological invariant of the system remains the same. (for more illustration, see Fig 1.4).

Such a topological phase transition cannot be explained by the spontaneously symmetry breaking induced phase transitions falling under the Landau's theory and thus constitute a separate branch of phase transitions. Extending the concept into condensed matter physics, there are topological phase transitions in it too but they are now described in terms of the electronic phases of materials. The topology of these phases is known to offer certain protection to the electronic structure of those materials and the protection is robust against any perturbation acting on the system which does not cause a topological phase transition.

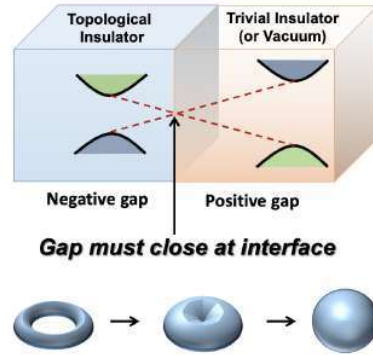


Figure 1.4: The topological phase transition from ‘trivial insulator’ to ‘non-trivial insulator’ is equivalent to the change in genus number. (Image is taken from Ref. [12])

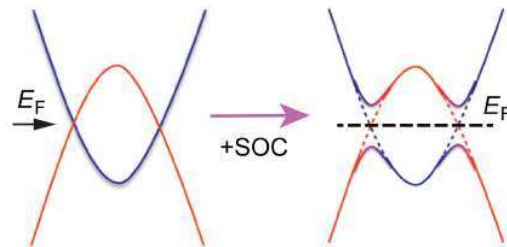


Figure 1.5: The phenomena of band inversion due to the presence of SOC. (Dashed lines) The hybridization of inverted band generates the metallic surface state. (Solid line in the right hand) The non-trivial bulk band. (Figure is taken from Ref. [13])

This protection carries remarkable consequences in the properties of these materials which have great technological appeal. In all cases the topological phases require the admission of a novel paradigm called as a topological order[1] in them analogous to the concept of order characterizing the phases in the Landau’s theory of phase transitions.

1.5 The foundation of Chern Insulator (2D TI)

As we already know, the Landau level formation makes the prime role in Quantum Hall Effect (QHE). But, formation of Landau level requires very high external magnetic field. Alternately, if we find materials possessing strong ‘internal’ magnetic field then that would also lead to similar consequences for the electronic structure leading to the induction of QHE (or an related phenomenon) in the system without the need for an application of external magnetic field. Such an internal magnetic field can be generated by the spin orbit coupling in the electronic structure of materials. In this context, a toy model was theorized

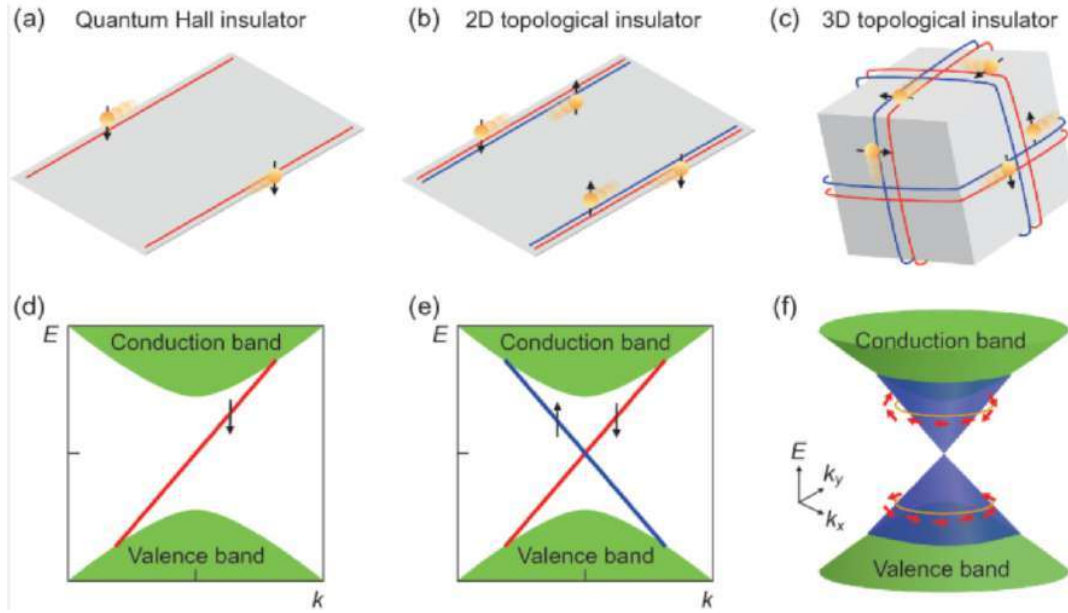


Figure 1.6: The edge states of different classes of compound. (a and d) Quantum Hall state where only one edge state is observed. (b and e) The edge state of 2D TI. The edge state is completely spin-polarized. Two distinct channels are created for up and down spin. (c-f) The surface state of 3D TI. The surface state is completely spin-polarized and hence it shows the spin-momentum locking. (Figure is taken from Ref [14])

by Haldane[15] in 1988, in which he predicted that when the 2D honeycomb lattice system is subjected to a spatially alternating magnetic field (with zero net flux) then the system should show a Quantum Hall effect (QHE). At the time, this model was viewed as a purely theoretical exercise for conceptual exposition without a feasible implementation approach. There were at least two significant challenges for the implementation of the model. One is the challenge of creating the necessary alternating magnetic field within the material, and the other is the widely held belief that any long-range order in 2D materials is unstable and driven by long-wavelength fluctuations[16].

Honeycomb carbon lattice, which has given rise to hope for obtaining the QHE in 2D materials, has provided a perfect platform to study and test Haldane's theoretical model. Graphene[17] is the first material system that may realize the theoretically postulated new topological state, as Kane and Mele pointed out[18], because of its intrinsic spin-orbit coupling (SOC), which may be viewed as an equivalent to alternating magnetic flux introduced in the Haldane model[15]. It was discovered that the SOC changes the system

from a gapless semimetal to a gapped state by opening a small bulk gap at the Dirac point in the electronic band structure of graphene. This SOC-driven state has new gapless edge states that counter-propagate at the edges with opposing spins and an electronic structure with a gapped bulk band (see Fig. 1.6 (b)) and hence called Quantum Spin Hall (QSH). As a result, the QSH conductance is nonzero while the charge Hall conductance is vanishing. With the electronic spin serving as the quantum number, this novel phenomena is known as the QSH effect and is characterised by the Z_2 topological order[19]. Due to the time-reversal symmetry protection, such a spin-helical metallic edge-state within a bulk-gap is resistant to weak disorder. Nonmagnetic impurities are prohibited from scattering any electrons, resulting in dissipationless transport edge channels. Topological insulators (TIs) are such QSH insulators driven by topological phase transitions as explained by the Haldane mechanism.

Graphene is the first predicted 2D TI to exhibit the QSH effect, but due to its weak SOC and relatively narrow bulk energy gap (103 meV)[20], direct experimental detection of this exciting phenomenon has been hampered. Therefore, a significant increase in the SOC effect is essential for the investigation of the fundamentals and prospective applications of graphene-based QSH insulators. The SOC in graphene may be improved by heavy atom doping or by placing it on substrate materials with potent SOC interactions, according to later studies[21]. Exploring alternative honeycomb lattice materials, such as germanene, silicene[22], and stanene[23], that have stronger SOC effects has also contributed significantly to the advancement of the field. A more general method of producing TIs was put forth by Bernevig, Hughes, and Zhang (BHZ) in 2006[24] in parallel with the development of the Kane–Mele model. This method involves an electronic band inversion, in which the normal ordering of the conduction and valence bands with different parities is “inverted” by relativistic effects (or the inert pair effect in chemical terms). The existence of 2D TI states is now possible in a much wider range of materials with strong SOC thanks to this mechanism, which does not require the honeycomb-lattice system and alternating magnetic flux as in the original Haldane model[15]. This new formalism has served as the model for further developments of many 2D TI materials. The earlier inves-

tigation suggests that tuning the width of the quantum well (QW) system (CdTe, HgTe), it undergoes from a topologically trivial to a non-trivial state resulting in a quantum phase transition. So in the above QW system, if thickness lies below some critical value, the system becomes an ordinary insulator. On the other hand, if the thickness becomes larger than a critical value, the valence-band top from the Hg 6s orbit with Γ_6 symmetry (odd parity) is inverted above the conduction band bottom from the p orbit with Γ_6 symmetry (even parity), resulting in a 2D TI state (a schematic of SOC driven band inversion is shown in Fig. 1.5). The topologically protected edge state can be found by solving the BHZ model equation with an open boundary condition, and the critical thickness is estimated to be around 6.3 nm. It has been demonstrated that even though the mechanics of the BHZ model and the Kane-Mele model differ, the end result is the same, namely that all QSH insulators have an energy gap in the 2D bulk that is bridged by metallic edge states, and that on the boundary, two sets of edge states with opposing spin polarisation counter-propagate. Later calculations using continuum models, more realistic first principles, and tight-binding (TB) techniques all of which led to the QSH state and the topological phase transition confirmed the BHZ mechanism. The QSH state and topologically nontrivial gap (10 meV) in HgTe/CdTe QWs are substantial enough for direct experimental verification (15), in contrast to the exceedingly small gap (10^{-3} meV) in graphene[20], which was utilised as the fundamental system in the Kane-Mele model. While the exotic insulating regime for thicker quantum wells ($d > 6.3$ nm) displayed a plateau of residual conductance close to $2e^2/h$, the insulating regime for thin QWs with width $d < 6.3$ nm displayed the typical conventional behaviour of vanishingly tiny conductance at low temperature. This residual conductance is unaffected by the sample width, which suggests that edge states are responsible for the novel phenomena. These findings offer experimental support for the QSH effect's existence. The Kane-Mele Model(8) based on graphene is closely related to the Haldane(5) toy model, which is one of the two methods for establishing 2D TIs. In this model, a bulk band gap is opened by SOC at two inequivalent Dirac points and is generalizable to other honeycomb-lattice structures with two sublattices. In the meantime, the BHZ model[25] suggests inducing a band inversion in a 2D semiconductor

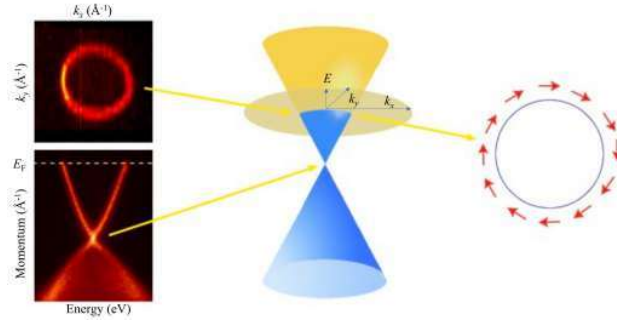


Figure 1.7: The surface band structure of TI. (Left panel) The experimental surface band of Bi_2Se_3 from ARPES study. (Middle panel) The schematic diagram 3D surface Dirac cone which excellently matches with the experimental one. (Right panel) The direction of spin with respect to momentum for Dirac Fermion. (Image is taken from Ref. [26])

using SOC. These many theoretical methods hint to two general methods for achieving 2D TIs, namely by causing a band inversion in narrow-gap semiconductors or by boosting a band gap in 2D Dirac semimetals.

In Fig 1.6 (a), we have shown the quantum Hall state. As there is no SOC only one channel of edge state is observed. The band structure of the edge state is shown in Fig 1.6 (a). In Fig 1.6 (b), we have shown the edge state of 2D TI. Due to presence of SOC, the edge state is split into two separate channels containing spin-up and spin-down electron. The band structure of the edge state is shown in Fig 1.6 (b).

1.6 From 2D TI to 3D TI

Because spin-orbit coupling is present, edge states are partitioned into two distinct channels, each of which can hold either electron with their spin in the up or down position. This is similar to what we covered earlier with regard to the situation of 2D TI. Therefore, if a spin-up electron generates a channel across the edge of the material that rotates in the direction of clockwise, then a spin-down electron must create a channel that rotates in the direction of anticlockwise in order to safeguard the time-reversal symmetry. When the same narrative is transferred from a two-dimensional to a three-dimensional system, the 1D edge state transforms into 2D surface state. The surface state of the 3D TI, just like the 2D system, has two distinct channels of spin-up and spin-down electrons. This

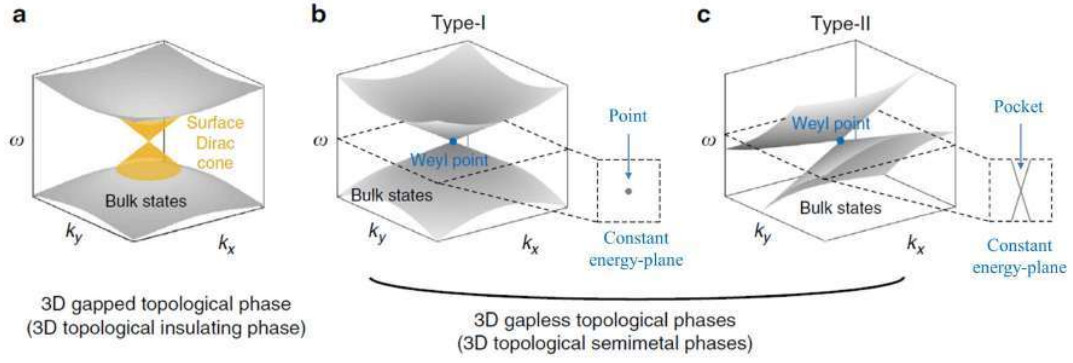


Figure 1.8: (a) The surface Dirac cone while bulk remain in insulating phase in TI. (b) Example of a *type-I* Weyl node where conduction and valence band create a point in the constant energy plane. (c) Example of a *type-II* Weyl node where conduction and valence band create a pocket in the constant energy plane. ((Figure is recreated from the Ref. [11]))

is analogous to the situation in the 2D system. When this occurs, the surface state of the metal becomes entirely spin polarized, whilst the bulk state of the material continues to exist in the insulating phase. Fig. 1.6 (c) and (f) provide a representation of the spin-polarized surface current in the form of a schematic diagram. 3D TI also follows the spin-momentum locking, for example, if the electron with the spin up follows the momentum in the clockwise direction, then the electron with the spin down must follow the momentum in the counterclockwise direction. Fig. 1.7 displays the surface band of topological insulator Bi_2Se_3 . The energy dispersion of the surface state, observed from the ARPES experiment is shown in the left bottom panel of Fig. 1.7. The schematic diagram of the $k_x - k_y$ contour also shown in the middle and right panel of Fig. 1.7. The band inversion phenomena that occur with 3D TI are as same as depicted in the schematic diagram in Fig. 1.5. Both the band inversion phenomena and the hybridization phenomena happen simultaneously.

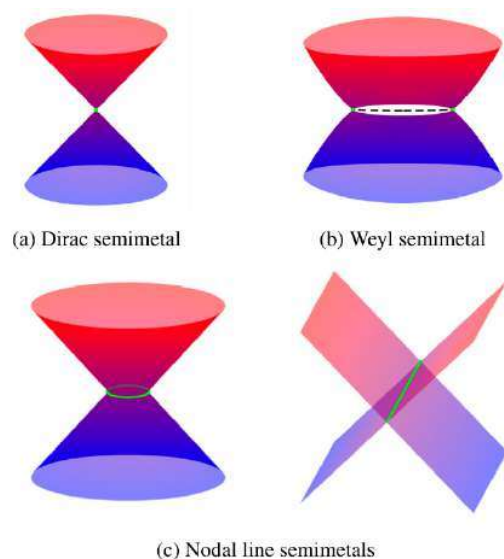


Figure 1.9: The several classes of topological materials. (1) The Dirac semimetal. (b) The Weyl semimetal. (c) The nodal loop semimetal (bottom left), and nodal line semimetal (bottom right). (Figure is taken from Ref. [27])

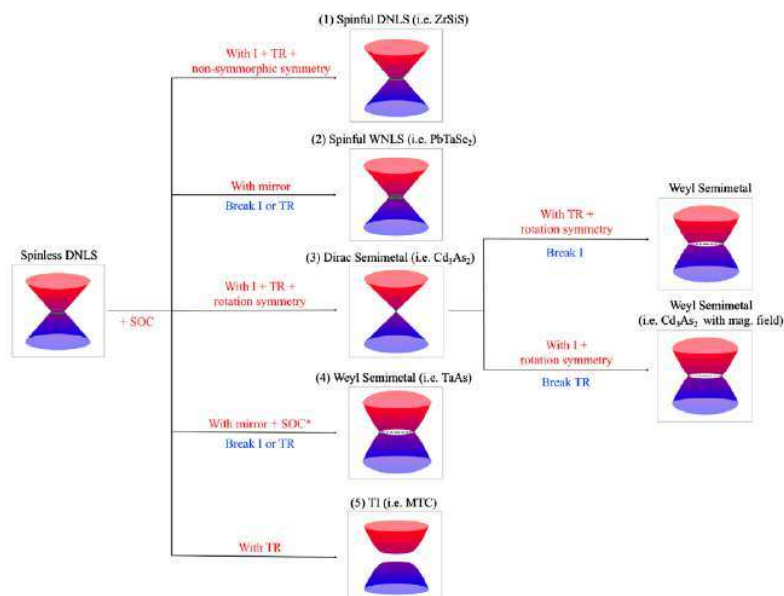


Figure 1.10: The catalog of various classes of topological material that can sustain different classes of symmetries. (I = Inversion symmetry, TR = Time reversal symmetry), (Figure is taken from Ref. [27])

1.7 The Dirac semimetal, Weyl semimetal and nodal line semimetal

As was stated before, the surface Dirac cone of TI is protected by the time-reversal symmetry, but the bulk of the material continues to behave as an insulator. However, in the case of the Dirac semimetal (DSM), the Dirac cone can be seen in the bulk state of the compound. The time-reversal symmetry and crystalline symmetry in the DSM serve to safeguard the novel Dirac cone. Crystalline symmetry can be thought of as a new kind of symmetry because it was not present in the TI. Therefore, in order to be a DSM, the Dirac cone needs to be safeguarded by (1) crystalline symmetry and (2) time-reversal symmetry. In the event that any of these two symmetries is broken, the Dirac cone is further subdivided into two Weyl nodes that have the opposite chirality, and the DSM transforms into a Weyl semimetal (WSM). Therefore, the WSMs always come with a pair of Weyl nodes that are protected in one of two ways: either by time-reversal symmetry (with broken inversion symmetry) or by inversion symmetry (with broken time-reversal symmetry).

The cone (for both DSM and WSM) can be classified into two distinct groups, referred to as *type-I* and *type-II*, respectively. When conduction and valence bands of *type I* touches each other, it creates a point at the constant-energy plane known as the Dirac/Weyl point. When *type II* touches, however, it results in the formation of pockets at the same plane. TI and Dirac (or Weyl) semimetals each have their own unique Dirac cone, which may be seen in Fig. 1.8. The surface Dirac cone and the bulk band for TI are depicted in Fig. 1.8 (a). The Dirac cones of *type I* and *type II* are shown in Fig. 1.8 (b) and (c), respectively. A point-like isoenergy contour and a pocket-like isoenergy contour are also illustrated in the appropriate manner for the cases that correspond to *type I* and *type II*, respectively.

The nodal line semimetals (NLSs) is a higher dimensional extension of the DSM or WSM. In that case, the loci of the Dirac point extends to a line in the 3D BZ. The NLSs, as opposed to DSMs and WSMs, have extended band crossings along unique lines in k -space. This is in contrast to the zero-dimensional band crossings that are present in DSMs and WSMs. It is possible for nodal lines to cross the BZ in the form of a closed ring or a line[28], as

depicted in Fig. 1.9 (c). The 1D topological Fermi arc surface state that is characteristic of WSMs can be analogous to a 2D topological ‘drumhead’ surface state that is characteristic of nodal ring semimetals[29]. The fact that these ‘drumhead’ surface states are located inside the ‘direct gap’ between the conduction and valence bands in the 2D projection of the nodal ring[30], [31]. These edge states are similar to the acoustic vibration that occurs on the surface of a drum, and they give birth to a significant density of states[32]. They are also fairly dispersionless.

The realisation of TSMs relies heavily on the crystallographic symmetries of the material. Dirac Nodal Line Semimetals (DNLS) and Weyl Nodal Line Semimetals (WNLS) are the two general classifications that may be applied to NLS, and each one is named after the symmetry protection that it provides. The DNLS are only discovered in substances that possess both inversion symmetry and time-reversal symmetry. Spin splitting is possible in WNLS because they do not possess either inversion or time-reversal symmetry. As a consequence of this, the otherwise fourfold degenerate nodal lines split into two singly degenerate nodal lines, each of which is safeguarded by an extra symmetry. The different classes of topological phases are shown as a schematic in Fig 1.10

1.8 Importance of single crystal growth

1.8.1 Role of single crystallinity in (a) physical properties and (b) crystal anisotropy

In order to research upon topological semimetals, it is vitally important to have single crystals that have been successfully grown. The single crystalline phase does not have any grain boundaries, and every experiment that measures its physical qualities finds them to be very close to the actual (or ideal) value. It is necessary to have a single crystalline phase in order to conduct research on crystal anisotropy. We are unable to detect crystal anisotropy for the polycrystalline phase of the compound due to the fact that the orientation of the unit cell is not uniform in all directions. When dealing with topological insulators,

obtaining a single crystalline phase is extremely significant since it demonstrates the varied physical properties that are present from the bulk to the surface.

1.8.2 Role of single crystallinity in (a) ARPES (b) Quantum oscillation study

In the field of condensed matter physics, the experimental observation of the Fermi surface is of the utmost importance. Investigating the band structure is one of the most important things to do with topological materials, which is one of the crucial areas of study. ARPES and quantum oscillation studies are examples of prominent experimental approaches that can be used to observe the topology of the Fermi surface. For the ARPES experiment, we require single crystals measuring between one millimeter and one micrometer. In order to successfully complete the ARPES experiment, the crystal must be of high quality and must be cleavable in a specific direction. Millimeter-sized single crystals are required so that we can analyse quantum oscillations caused by transport. The breadth and width of the sample need to be at least three to four millimeters in order for us to be able to investigate the electrical conductivity, as we were required to set up four probes. In order to carry out this experiment, the probe was oriented in a specific crystallographic direction, which had been decided upon in advance. By applying a magnetic field in the desired crystallographic direction and detecting the SdH oscillation, we are able to map the topology of the Fermi surface. In addition to mapping the Fermi surface, these kinds of experiments reveal the anisotropic character of the crystal in terms of its electrical conductivity.

