Mathematical Models for the Treatment of Swallowing Disorders through Oesophageal Catheterisation

Thesis submitted to

Indian Institute of Technology

(Banaras Hindu University) Varanasi

in partial fulfilment of

the requirements for the Award of the Degree of

Doctor of Philosophy

in

Mathematical Sciences

by

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July 2024

Conclusions and Further Scope of Research

Conclusions

This thesis discusses mathematical modeling of the problems associated with biomedical sciences, where various swallowing disorders are addressed using catheters. Catheterisation is a minimally invasive procedure, meaning it can be performed with minimal discomfort and risk compared to open surgical techniques. This makes it a safer diagnostic option, especially for high-risk patients. It allows surgeons to gather critical data without requiring more invasive exploratory surgeries, reducing recovery time and minimizing complications. We tried to present the pre-diagnosis scenarios with an inserted catheter in an oesophagus that may help people in biomedical sciences. Further, we got the answers to the raised queries in the objectives of the thesis, which supports the explanation of how the discussed mathematical models can become the useful clinical real-world treatment of swallowing disorders through oesophageal catheterisation. A detailed discussion of the problems taken up, such as insertion of catheterization, sliding hiatus hernia, achalasia, effects of the elastic nature of the oesophagus, and the inferences drawn chapter-wise are given below:

Chapter 3 deals with the study of a mathematical model of inserting the catheter inside the human oesophagus. The propagation of a sinusoidal wave on the wall's surface is of dilating amplitude. It is concluded that catheter insertion influences various characteristics, such as pressure distribution, axial velocity profile, etc., of the considered flow problem. In this model, the mass conservation in the two layers is duly considered, which was wrongly assumed by earlier researchers. Further, the interface between the two layers has been obtained by solving fourth-order algebraic equations.

Chapter 4 deals with the mathematical model of one of the most occurring swallowing disorders in the human oesophagus, i.e., sliding hiatus hernia. This problem is modeled by a two-layered cylindrical tube. The distal part of this tube is exponentially diverging, which shows the bulging near the lower oesophageal sphincter. It is concluded that less pressure is required at the distal end of the oesophagus, even with the inserted catheter. This helps a patient while feeding. This agrees with biomedical research when a sliding hiatus hernia goes unnoticed in a patient.

Chapter 5 gives a two-layered analysis of a micropolar fluid that helps to study swallowing disorders with the effects of micropolar parameter and coupling number. This study is performed in view of catheter insertion in the core layer. The Homotopy Perturbation Method approach is used to get the approximate solutions. The effect of the dilating amplitude parameter and the broadening of the catheter size is studied.

Chapter 6 of this thesis investigates and provides biological and modeling insights into the study of balloon dilatation with the help of the introduced catheter. This idea helps to treat a patient with dysphagia, which is one of the symptoms of Achalasia. It is concluded that the insertion of a balloon catheter into the oesophagus resulted in a higher difference in pressure distribution throughout the oesophageal length compared to a normal catheter.

The final chapter of this thesis, i.e., Chapter 7, deals with the elastic nature of the oesophagus when an external force is applied in the radial direction to the wall's surface. This study is duly considered in view of non-Newtonian fluid, i.e., the power-law model. We observed the impact of dilating forcing amplitude and applying Gaussian forcing to a solitary wave. We approach this problem through analytical techniques, employing standard methods such as the long wavelength and low Reynolds number approximations. It is concluded that pseudoplastic fluids require less pressure than Newtonian fluids, whereas dilatant fluids necessitate higher pressure to propel boluses into the stomach than Newtonian fluids. Some more chapter-wise results are as follows:

Chapter 3

- The shape of the interface layer is dependent on the viscosity ratio of the peripheral layer.
- For smaller values of the viscosity ratio, the peripheral layer is thicker in the contracted regions when the peristaltic waves propagate and thinner in the relaxed regions for the same catheter.
- The flow rate increases when the peripheral layer is thinner and more viscous. Higher wave amplitude and larger amplitude dilation increase it further.
- Pressure rises with the thickness of the inserted catheter. This increased pressure may cause the patient uneasiness and sometimes disturb the feeding process.
- For similar reasons, no patient should be fed anything directly through the mouth once a catheter has been inserted into the oesophagus through the nose.

Chapter 4

- The main conclusions of the entire discussion in this chapter are associated with examining the impact of catheterisation in a partially diverging tube affected by a sliding hiatus hernia.
- Based on the analysis of this chapter, it can infer for swallowing that even if a catheter is inside the oesophagus, any sliding herniation requires less pressure for flow.
- It is concluded that swallowing will be easier if the divergence of the oesophagus near the cardiac sphincter increases.
- The presence of a catheter requires oesophagus to work much more in order to swallow the same amount of fluid.
- It is inferred that a low flow rate with the increased gradient parameter further enhances the bolus's swallowing.

Chapter 5

- We observed that the presence of a catheter pressure increases with the coupling number and reduces with the micropolar parameter.
- The broadening of a catheter increases pressure exponentially when the micropolar parameter diminishes in magnitude.
- It is concluded that the flow rate increases with the dilating amplitude parameter when pressure rises. This further increases the thickness of the catheter due to impedance, which results in an increase in pressure.
- It is revealed that pressure is higher than a Newtonian fluid once a catheter is introduced with a micropolar fluid.
- Moreover, the micropolar parameter and the coupling number have affected the pressure distribution throughout the oesophageal length with the frictional forces and impedance. Impedence is increasing exponentially with the introduced catheter.

Chapter 6

- In this chapter, we identified the key factors, such as the insertion of a normal and a balloon catheter, that exclude the presence of possible malignancy and may help to restart the flow, respectively, that influences pressure distribution in the oesophagus.
- We observed that the pressure spikes at the places before the affected region where the balloon inflates into the oesophagus.
- Moreover, the pressure at the upper sphincter is released with the insertion of the deflated balloon catheter. This process goes on until it reaches the position where the balloon catheter is finally placed, and the pressure starts increasing.
- At the junction of the inserted catheter, we observed a larger growth in the magnitude of pressure, which shows that a higher pressure is required to get rid of the stricture caused by Achalasia.

- The value of ϵ demonstrates that the pressure gets higher as the balloon inflation takes place, and at the same time, it opposes the existing pressure.

- We observed that on increasing its value in the range $k = 0.0 - 0.02$, the higher pressure we get. In the balloon catheter's inflation region, the pressure gets much higher than it was previously. The cycle of the trends of higher pressure distribution continues with time.

- It is concluded that due to the higher pressure at the inflated region in the oesophagus, the velocity of the peripheral layer increases at the same region. This spiked pressure decreases gradually to overcome the existing pressure as time passes.

Chapter 7

- This investigation is in view of the rheological behavior of a non-Newtonian fluid, typically the power-law model, under the influence of an externally applied force on an elastic oesophageal tube.

- We delve into the impact of dilating forcing amplitude on axial and radial velocity profiles, examining various values of n that correspond to pseudoplastic ($n < 1$), Newtonian ($n = 1$), and dilatant ($n > 1$) behavior of a power- law fluid.

- The analysis has been done using the approach of the regular perturbation method to seek solutions in terms of the dilating forcing amplitude denoted as ϵ.

- We discussed flow behavior indices spanning various flow parameters that correspond to both shear-thinning and thickening behaviors for the different combinations of forcing wave amplitudes.

- We focused on how the pressure distribution responds within the system to varying forcing amplitudes.

- We observe that as the bolus enters the oesophagus, less pressure is required due to the relaxation of circular and longitudinal muscles. As we increase the dilating forcing amplitudes, these muscles contract, propelling the bolus's tail forward into the stomach.

- The externally applied force alters the pressure profile, making it sharper and intensifying the propulsion process.
- We observe a gradual restoration of velocity over time to its original form, especially for lower dilating forcing amplitudes, indicating the absence of slippage at the tube wall. Slippage at the tube wall occurs when the fluid interface moves relative to the wall surface instead of adhering tightly, which may distort or misrepresent measurements in real-time imaging studies. In the absence of slippage, imaging techniques like ultrasound, CT, and fMRI can capture oesophageal profiles with high precision, which may validate the obtained results.

Further Scope of Research

Modeling a real-world problem mathematically with applications in biomedical sciences is very complex. The rich literature examines various biological flows in the human oesophagus. A prior diagnosis of a patient with various swallowing disorders is needed. These investigations sometimes reveal and discard the malignancy present in the oesophagus. Doctors often avoid surgical operations due to the future complications a patient can have. They further suggested a pre-operative method called catheterization in the oesophagus. In all the chapters except Chapter 7, we dealt with catheters for various oesophageal complications, mainly Sliding Hiatus Hernia, Achalasia, etc. Uneven and consistently changing structures add to the woe.

Dealing with the mathematics involved in the Navier-Stokes equation, which governs the various flows, such as Newtonian and non-Newtonian, was difficult. So, we use the standard techniques of long wavelength and low Reynolds number approximations to get the solution done. Further, we use a regular perturbation technique, i.e., the Homotopy Perturbation Method, to get an approximate solution. Non-linear forms of the governing equations are still a huge hurdle in arriving at solutions. Various approaches are available to validate these analytical solutions obtained in the thesis, such as a finite difference scheme using grid independence study and time step convergence. Moreover, computing the relative and absolute errors between the analytical (or approximate) solution and the numerical results can be done for critical quantities like velocity profiles, pressure distributions, and shear stresses. If the relative error is within a tolerable range (e.g., less than 5%), the simulation is considered to give an acceptable estimate of the analytical solution. Further, one can compare the simulation results with well-established numerical codes or solvers (e.g., ANSYS Fluent, OpenFOAM) specifically designed to solve the Navier-Stokes equations. If the simulation matches these established tools under similar conditions, it provides confidence in the results.

In the thesis, as mentioned, we used standard techniques, such as long wavelength and low Reynolds number approximations, and the Homotopy perturbation method to get the desired solutions. The limitations of these techniques are the following:

- They make the approximation inaccurate for flows where inertial effects are significant.
- This approximation is valid primarily in the regimes of slow-moving, viscous-dominated (e.g., creeping flows), limiting its applicability in turbulent flows.
- This approximation works well when the wavelength of a disturbance is significantly larger than the characteristic length scale of the system. It fails when the wavelength is short or comparable to the length scale.
- For complex systems with significant curvature or sudden changes in geometry, the long wavelength approximation may fail to capture important details.
- The Homotopy perturbation method assumes a series solution that converges to the true solution. However, this convergence is not always guaranteed, especially for highly nonlinear problems or problems with strong singularities.
- The construction of the Homotopy is not unique, and different choices of the Homotopy parameter or initial guess can lead to different solutions or poor convergence.
- While it is effective for weakly nonlinear systems, its application to strongly nonlinear systems may be limited, requiring more sophisticated techniques.

Need for Future Directions are the following:

- Hybrid Approaches: Combining low Reynolds number or long wavelength approximations with computational fluid dynamics or machine learning techniques can improve accuracy in regimes where approximations fail.
- Multi-scale Methods: Future work could focus on developing methods that bridge the gap between low Reynolds number approximations and higher Reynolds number scenarios, creating more universal models.
- Improved Convergence Control in Homotopy perturbation method: Developing techniques to ensure the convergence of homotopy perturbation solutions for strongly nonlinear problems is crucial.
- Nonlinear System Extension: Extending the homotopy perturbation method to handle strongly nonlinear, non-perturbative regimes is another promising area for future work.
- Incorporation of Complex Geometries: Developing methods that can extend the accuracy of long wavelength approximations to capture more complex geometries would increase the scope of applications.

Alternatives to these Methods are the following:

- Numerical Methods: Finite Element Method and Finite Difference Method provide numerical solutions to problems without requiring low Reynolds number or long wavelength assumptions.
- Computational Fluid Dynamics: Accurate for solving fluid flow problems in various regimes, including high Reynolds number turbulent flows.
- Perturbation Methods: The Variational Iteration Method can be considered an alternative to the Homotopy perturbation method for dealing with nonlinear differential equations.
- Adomian Decomposition Method: Another effective alternative for solving nonlinear equations without assuming small perturbations.

By addressing these limitations and assumptions, future work could enhance the robustness of these methods across more diverse physical systems.

Experimental and observational data supports model validation, yet managing the sensitive human oesophagus and its swallowing issues remains challenging. Advanced engineering tools and systems are needed to address these complex ailments. Even in small increments, continued research will undoubtedly lead to new discoveries. Some of the problems proposed are as follows:

In a para-hiatus hernia, the protruded portion of the abdomen is stuck above the hiatus, which is the cardiac sphincter. This may be because the abdominal muscles are weak, not the cardiac sphincter. So the investigation required may be "What makes the abdominal muscles weak?"

If not, there may be some anomaly with the sphincter or some sort of pressure on it from the outside. In such a case, the outside pressure needs to be investigated.

In achalasia, the oesophagus ceases to be active and hence cannot swallow any fluid. The longitudinal muscles are responsible for radial expansion/relaxation. Any prosthetic support may be helpful, but it needs to be investigated so that its function is compatible with the oesophagus.

In addition to these specific scopes of further research, the following also fall within the scope:

- Numerical techniques and simulations must be done to compare the results of the analytical solution obtained in the thesis.
- Mathematical models for the multi-layer are needed to better enrich the mucous, sub-mucous, and other layers of the oesophageal wall, where viscosity may vary as a function of axial parameters.
- The Time-dependent model can be considered, which provides better knowledge for real-time pressure distribution and other profiles of flow dynamics.
- Convergence of the wall's surface geometry for the model of sliding hiatus hernia will be a fruitful discussion.

- A study of low Reynolds number flow's effects on particle dispersion may enhance the knowledge of the interaction between fluid and particle with the introduced catheter.
- The presented work of the thesis can lead to help in the development of a mathematical model for the heart valves.

Artificial Intelligence (AI) and Machine Learning (ML) have been increasingly used in treating and managing swallowing disorders in the oesophagus by integrating mathematical models with data-driven approaches. These methods help diagnose, predict, and provide personalized treatments, enhancing the precision and efficiency of care. Here's how AI and ML contribute through mathematical models:

- AI and ML can refine the mathematical models by analyzing large patient datasets and extracting patterns that improve model accuracy.
- AI techniques, such as support vector machines and decision trees, can be trained on medical imaging (e.g., video-fluoroscopic swallowing studies, endoscopic images) to identify patterns that indicate swallowing disorders. These techniques classify different types of dysphagia (e.g., oro-pharyngeal) and determine the severity.
- Predictive models rely on time-series analysis, differential equations, and ML methods like random forests or gradient-boosting algorithms to accurately predict patient outcomes.
- AI-driven 3D modeling of the pharyngeal and oesophageal regions provides detailed insights into the dynamics of swallowing disorders.

Future Directions and Challenges are the following:

- Interdisciplinary Research: Future research should focus on combining AI/ML with advanced biomechanical models, sensor technology, and data from clinical studies to improve the diagnosis and treatment of swallowing disorders.
- Improved Data Integration: There's a need for better integration of multimodal data (imaging, sensor data, clinical records) using AI techniques, leading to more comprehensive patient profiles.

- Personalized AI Models: Developing AI systems that can personalize treatment based on individual patient data remains a key challenge but holds great promise for improving treatment outcomes.
- Robustness and Generalization: Ensuring AI models are robust and can generalize across different populations and conditions will be critical for broader clinical use.

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