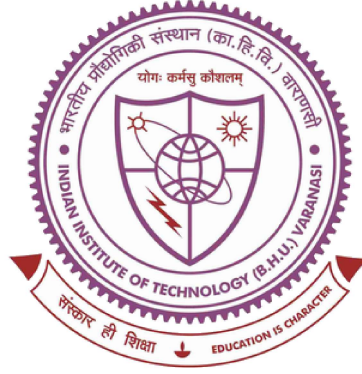


MATHEMATICAL ASPECTS OF FRACTURE PROBLEMS
IN FUNCTIONALLY GRADED MATERIALS



The thesis submitted in partial fulfillment

for the Award of Degree

DOCTOR OF PHILOSOPHY

by

Ritika Singh

DEPARTMENT OF MATHEMATICAL SCIENCES

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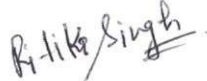
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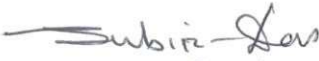
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Ritika Singh

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PREFACE

This dissertation is organized into six chapters. Chapter 1 is an introductory chapter that discusses the historical and fundamental details of fracture mechanics. This also includes a description of functionally graded material and a review of related literature. Furthermore, some mathematical methods and techniques including integral equations, Laplace and Fourier transforms, Jacobi polynomials, and others are also discussed.

Chapter 2 deals with the interactions between a central crack and two symmetrically located collinear Griffith cracks in an infinite functionally graded medium subjected to thermo-mechanical loading. These Griffith cracks have been partially insulated. The Fourier sine and cosine transforms are employed to solve the elasticity and heat conduction equations which are reduced to a system of singular integral equations of the first kind. The equations are numerically solved by using first kind Chebyshev polynomials. Analytically, the expressions of the normalized mode I SIFs and SMFs are found. The primary goal of this chapter is to investigate the effect of the relative sizes of collinear cracks on mode I SIFs as well as the possibilities of crack shielding and amplification. The visual representations of temperature field, thermal crack surface stresses, SIFs, and SMFs under thermal, mechanical, and thermo-mechanical loadings for distinct specific cases are standout features of this chapter.

The interaction between a central and two symmetrically placed collinear Griffith cracks subject to transient response under anti-plane shear impact loading is analyzed in Chapter 3. The cracks are situated in a strip constituted by functionally graded material bonded between two dissimilar elastic strips of equal thickness. The material properties of FGM are assumed to vary exponentially as a function of thickness. Applying integral transforms, the boundary value problem reduces to a system of singular integral equations in the Laplace transformed domain. These equations are solved numerically using the Lobatto-Chebyshev collocation quadrature approach. The inverse Laplace transform is used to find the approximate analytical expressions of dynamic stress intensity factors (DSIFs). The striking feature of the article is the study of phenomenal changes in shielding and amplification through dynamic stress magnification factors (DSMFs) at the tips of

the cracks under the sudden impact loading applied at the upper material surface. The effects of impact load applied at different surfaces, positions of cracks' axis, and the thickness of the strips of the composite material on the possibilities of cracks' arrest are depicted graphically for different particular cases.

Chapter 4 contains two subchapters examining the fracture behaviour of an arbitrary oriented crack in bonded functionally graded strips.

The purpose of sub-chapter 4.1 is to explore the behavior of an arbitrarily oriented crack that crosses the interface of bonded finitely thick functionally graded strips under thermo-mechanical loading. The angled crack is partially insulated. Application of the superposition approach and the Fourier transformation on the governing equations of heat conduction and plane elasticity aids in the reduction of boundary-continuity conditions to Fredholm type singular integral equations. The jump of the temperature and displacement across the crack surface is written as a series of Jacobi polynomials to solve these integral equations. The expressions of SIFs are obtained employing the residue theory and Schmidt technique, which together with the local material parametric function determine SERRs. The analysis of the impact of thermo-mechanical loading on mode I and II crack tip SIFs and SERRs to analyze the behavior of an arbitrary oriented partially insulated crack when it crosses the interface is the novelty of the chapter. The results of the present study are also validated. The graphical representations of crack tip SIFs and SERRs for various values of the crack insulation parameter, crack orientation angle, strip thickness, and non-homogeneity parameters are the main feature of this chapter.

Sub-chapter 4.2 aims to explore the effect of a partially insulated crack on vertical and horizontal components of heat flow. The crack is arbitrarily oriented, with its center at the interface of bonded finitely thick functionally graded strips. The thermoelastic equations are reduced to a set of singular integral equations by using the integral transformation. The temperature jump and displacements across the crack surfaces are the unknown variables in these equations. Representing the unknowns as a series of Jacobi polynomials, the solutions of the singular integral equations are determined by using the Schmidt method, which also aids in determining the analytical forms of crack tip heat flux intensity factors (HFIFs) and thermal stress intensity factors (TSIFs). The findings of the chapter are also validated for a specific case. The crack tip HFIFs and TSIFs as a function of crack angle, crack

insulation parameter, strip width, and non-homogeneity parameter ratio are demonstrated graphically to quantify the strength of heat flow components.

Chapter 5 examines the fracture behavior of a functionally graded magneto-elastic (FGMEE) plane with multiple parallel crack. Under the anti-plane mechanical, in-plane electric, and magnetic loadings, it is assumed that the cracks are either of the magneto-electrically impermeable or permeable types. Here, three distinct crack configurations are taken into consideration. For each of the three crack configuration cases, the boundary collocation and least square methods are used to obtain the semi-analytical expressions of the SIFs at the crack tips. SIFs are used to calculate the SMFs. The novelty of the chapter is the study of shielding and amplification tendencies of cracks under the impact of functionally graded parameter, geometric size, and electric and magnetic loads. The graphical illustrations of SMFs as a function of gradient parameter, the distance between the cracks, and electric and magnetic loadings for three different crack configurations are the key features of the chapter.

Chapter 6 concludes the overall work done and also provides information on possible future work.