

# Chapter 6

## Conclusions

The Weyl transform is a non-commutative analog of the Euclidean Fourier transform. Various results about the Euclidean Fourier transform have simple analogs for the Weyl transform. For instance, the analog of the Riemann-Lebesgue lemma is that the Weyl transform of an  $L^1$  function is a compact operator. In this thesis, we proved that under appropriate curvature and dimension assumptions, the Weyl transform of a measure is compact, and belongs to some Schatten class.

Chapter 1 is introductory in nature, and contains the appropriate background and history of the subject.

In Chapter 2, we proved that if  $\mu$  is a smooth measure supported on a compact connected smooth hypersurface of positive Gaussian curvature in  $\mathbb{R}^{2n}$ ,  $n \geq 2$ , then the Weyl transform of  $\mu$  is compact and belongs to the  $p$ -Schatten class if  $p > n \geq 6$ . Since the Weyl transform of the normalized surface measure on a sphere in  $\mathbb{R}^{2n}$  belongs to  $S^p(\mathcal{H})$  if and only if  $p > 4n/(2n - 1)$ , with no restriction on  $n$ , we conjectured in [32] that if  $\mu$  is a smooth measure supported on a compact connected

smooth hypersurface in  $\mathbb{R}^{2n}$  with positive Gaussian curvature, then  $W(\mu) \in S^p(\mathcal{H})$  if  $p > 4n/(2n - 1)$ , with no restriction on  $n$ .

In Chapter 3, we proved that the Weyl transform of a smooth measure supported on a finite type real-analytic submanifold of  $\mathbb{R}^{2n}$  is compact. We also proved that when  $n = 1$ , we do not need the additional assumption of real-analyticity, i.e., we proved that the Weyl transform of a smooth measure supported on a finite type smooth submanifold of  $\mathbb{R}^2$  is compact.

In Chapter 4, we proved that the Weyl transform of a compactly supported distribution on  $\mathbb{R}^{2n}$  is  $p$ -th power traceable if and only if the Fourier transform of the distribution is  $p$ -th power integrable. Moreover, we proved that the Weyl transform of a compactly supported distribution on  $\mathbb{R}^{2n}$  is compact if and only if the Fourier transform of the distribution vanishes at infinity. As a consequence of this, we conclude that if  $\mu$  is a smooth measure supported on a compact connected smooth hypersurface in  $\mathbb{R}^{2n}$ , whose Gaussian curvature is nonzero everywhere, then  $W(\mu)$  is a compact operator, and  $W(\mu) \in S^p(\mathcal{H})$  if and only if  $p > 4n/(2n - 1)$ . Moreover, if  $\mu = \psi\sigma$  is a smooth measure supported on a finite type smooth submanifold of  $\mathbb{R}^{2n}$ , then  $W(\mu)$  is a compact operator, and  $W(\mu) \in S^p(\mathcal{H})$  if and only if  $p > 2nk$ , where  $k$  is the type of the submanifold inside the support of  $\psi$ .

Finally, in Chapter 5, we provide applications of these results. We described the conditions under which the quantum translates of a non-zero Schatten class operator are linearly independent. Moreover, we proved an analog of the Fourier restriction theorem for the Fourier-Wigner transform.