CHAPTER 7

STATISTICAL MODELLING OF TRIBOLOGICAL PARAMETERS USING RESPONSE SURFACE METHODOLOGY

7.1 INTRODUCTION

The tribological characteristics of a material depend upon several factors and to study the influence of each factor in combination of other factors will lead to number of undesirable experiments. Therefore, statistical modelling is a tool, which helps to eliminate the redundancy during experimentation and also optimize the objective function.

With this view, present study utilises Response Surface Methodology (RSM) on Design Expert-13 software for prediction and optimization of wear rate and COF of stir cast as well as CS cast hybrid composites. Response surface methodology (RSM) is a statistical technique to optimize the process parameters. It is widely used, where several independent variables influence the output response [33]. The input factors like applied load, sliding distance, sliding velocity and wt.% of TiB_2 reinforcement were used in present study. RSM is used to establish mathematical relationship between the input variables and output response. The optimization of input variables to optimize wear rate and COF was obtained by desirability function method from the developed model. In order to validate the predicted wear rate and COF, set of confirmation test were performed using the optimised input parameters.

7.2 STATISTICAL MODELLING OF WEAR RATE AND COF OF STIR CAST HYBRID COMPOSITES

7.2.1 Central composite design (CCD)

Central Composite Design (CCD) of experiments on Design Expert-13 software was used to examine the influence of input factors on wear rate and COF. Three factors with three level of variables were used in present investigation as presented in Table 7.1. Table 7.2 presents experimental results with variation of input parameters as per design of experiment.

Table 7.1 Input levels of sliding wear parameters

Sl. No.	**Parameters**	**Level**		
		-1 (Low)	**0 (Medium)**	**+1 (High)**
1	A: Load (N)	20	30	40
2	B: Sliding distance (m)	1000	2000	3000
3	C: Wt.% of TiB_2 (%)	1	3	5

Table 7.2 Design matrix and experimental wear rate and COF

Standard	**Run order for**		**A: Load (N)**	**B: Sliding distance (m)**	**C: TiB_2 (wt.%)**	**Wear rate ($\times 10^{-3}$ mm^3/m)**	**COF**
	Wear rate	**COF**					
1	1	3	20	1000	1	0.952669	0.56
6	2	6	40	1000	5	1.55706	0.51
7	3	12	20	3000	5	0.85739	0.58
3	4	11	20	3000	1	0.98883	0.50
10	5	1	40	2000	3	1.43099	0.51
20	6	2	30	2000	3	1.07553	0.51
8	7	18	40	3000	5	1.20719	0.55
14	8	9	30	2000	5	1.18569	0.51
13	9	8	30	2000	1	1.45873	0.49
15	10	7	30	2000	3	1.09563	0.52
4	11	14	40	3000	1	1.80842	0.52
9	12	13	20	2000	3	0.75754	0.53

2	13	5	40	1000	1	2.0078	0.55
5	14	20	20	1000	5	0.98116	0.57
19	15	10	30	2000	3	1.09843	0.51
11	16	19	30	1000	3	1.14486	0.54
18	17	15	30	2000	3	1.05633	0.51
17	18	17	30	2000	3	1.05273	0.52
16	19	16	30	2000	3	1.05553	0.51
12	20	4	30	3000	3	1.08292	0.54

7.2.2 Quadratic model and analysis of variance

RSM was used to evaluate experimental data in order to construct a statistical model to optimise response. The linear, 2FI and quadratic model were analysed to establish mathematical expression between the input parameters and output results. Quadratic model is significantly suggested by the established model.

Analysis of variance (ANOVA) was applied to check competency of developed model and determination coefficient (R^2) indicating the accuracy and degree of fit of model. When value of R^2 close to 1 and adjusted R^2 and predicted R^2 is very close to each other that suggests developed model fits experimental results.

Table 5 presents the ANOVA observations for the wear rate of quadratic model. The p value is lower than 0.0001, shows that expectedness of developed model for wear rate is at 99% confidence. Experimental wear results fit well with the predicted response. The determination coefficient (R^2) value of 0.9946 indicates that Eqn. 2 is very consistent. The F-value 206.56 and p-value less than 0.0001 specifies that developed model is significant. F-value 1217.10 and p-value less than 0.0001 for wear rate suggest that load is most important variable, which affects the wear rate.

Table 6 shows the ANOVA results of COF of suggested quadratic model. It suggests that probability of developed model for COF is at 99% confidence. Experimental results of COF fit well with the predicted response. The R^2 value of 0.9755 indicates that Eqn. 3 is very consistent. F-value 44.23 and p-value less than 0.0001 indicate that developed model is significant. F-value and p-value suggest that load and wt.% of TiB_2 are the important variables influencing the COF of hybrid composite.

7.2.3 ANOVA for wear rate and COF of quadratic model

Table 7.3 ANOVA for wear rate

Source	Sum of Squares	df	Mean Square	F-value	p-value
Model	1.84	9	0.2048	206.56	< 0.0001
A-Load (N)	1.21	1	1.21	1217.10	< 0.0001
B-Sliding distance (m)	0.0488	1	0.0488	49.25	< 0.0001
C-TiB2 (wt.%)	0.2039	1	0.2039	205.65	< 0.0001
AB	0.0266	1	0.0266	26.87	0.0004
AC	0.1126	1	0.1126	113.54	< 0.0001
BC	0.0120	1	0.0120	12.15	0.0059
A^2	0.0003	1	0.0003	0.3377	0.5740
B^2	0.0002	1	0.0002	0.2047	0.6606
C^2	0.1294	1	0.1294	130.49	< 0.0001
Residual	0.0099	10	0.0010		
Lack of Fit	0.0078	5	0.0016	3.60	0.0932
Pure Error	0.0022	5	0.0004		
Cor Total	1.85	19			

R^2: 0.9946, adjusted R^2: 0.9898 and predicted R^2: 0.9728, Adeq. Precision: 57.06

Table 7.4 ANOVA for COF

Source	Sum of Squares	df	Mean Square	F-value	p-value
Model	0.0109	9	0.0012	46.98	< 0.0001
A-Load (N)	0.0010	1	0.0010	38.73	< 0.0001
B-Sliding Distance (m)	0.0002	1	0.0002	6.20	0.0320
C-Composition (TiB2 wt.%)	0.0010	1	0.0010	38.73	< 0.0001
AB	0.0004	1	0.0004	17.43	0.0019
AC	0.0012	1	0.0012	48.42	< 0.0001
BC	0.0024	1	0.0024	94.89	< 0.0001
A^2	0.0003	1	0.0003	9.71	0.0110
B^2	0.0024	1	0.0024	92.98	< 0.0001
C^2	0.0003	1	0.0003	11.64	0.0066
Residual	0.0003	10	0.0000		
Lack of Fit	0.0001	5	0.0000	0.7212	0.6357
Pure Error	0.0002	5	0.0000		
Cor Total	0.0112	19			

R^2: 0.9769, adjusted R^2: 0.9561 and predicted R^2: 0.9295, Adeq. Precision: 23.0954

7.2.4 Regression equation for wear rate and COF

All the coefficients were examined for its significance using F- test on Design expert software. These coefficients were used to establish a mathematical expression to evaluate wear rate and COF. The obtained regression equations for wear rate (Eqn.2) and COF (Eqn.3) of quadratic model with the input variables A (load), B (sliding distance) and C (wt.% of TiB_2) are presented as-

Wear rate = $-0.11303 + 0.064667\times A + 0.000149\times B - 0.17184\times C - 6\times10^{-6}\times AB - 0.00593\times AC - 1.9\times10^{-5}\times BC - 1.5\times10^{-5}\times A^2 + 0.052864\times C^2$ Eqn. 2

COF = $0.7515 - 0.00605\times A - 0.00017275\times B + 0.024\times C + 7.5e\text{-}07\times AB - 0.00075\times AC + 8.75\times10^{-6}\times BC + 0.0001\times A^2 + 3\times10^{-8}\times B^2 - 0.0025\times C^2$ Eqn. 3

Figure 7.1 displays surface and contour plot for wear rate at various input variables. Figure 7.1 (a) displays surface and contour plot showing influence of load and sliding distance on wear rate of hybrid composite having 3wt.% TiB_2 phase. It demonstrates that wear rate increases as load and sliding distance both increase simultaneously. At lower sliding distance, increase in wear rate is more as load increases compared to higher sliding distance owing to interaction of asperities on the contact surfaces. At higher load, wear rate reduces with increasing sliding distance owing to oxide layer formation and mechanically mixed layer (MML) between contact surfaces.

Figure 7.1 (b) shows the surface and Contour plot demonstrating influence of load and TiB_2 wt.% on wear rate at sliding distance of 2000m. Surface plot displays that initially wear rate reduces as the TiB_2 content increases from 1 to 3 wt.% after that it increases marginally to 5 wt.% TiB_2 content. This effect owing to hard reinforcement particle, which protects matrix from large deformation and delamination during dry sliding. The increase of wear rate beyond 3 wt.% TiB_2 content is owing to the poor bonding with matrix and agglomeration of TiB_2 particle in the composite [135]. Contour plot demonstrates that the maximum wear rate observed for hybrid composite have 1wt.% TiB_2 at 40N applied load. Different colour of contour and 3D surface image shows minimum and maximum wear rate. Blue colour for minimum wear rate, green for average and yellowish red shows maximum wear rate.

Figure 7.1 (c) displays surface and Counter plot showing effect of wt.% of TiB_2 and sliding distance on wear rate at fixed load of 30N. It displays that wear rate reduces with increasing TiB_2 wt.% and sliding distance from 1000m to 3000m, as shown in contour plot. The high wear rate at lower sliding distance owing to interaction of asperities on the contact surfaces. As sliding distance increases, it increases the pin surface temperature resulting in oxide layer development on pin surface that reduces the wear rate. Similar results have been observed by N. Radhika et al. [136].

Figure 7.2 presents 3D surface and contour plot displays influence of the input variables on COF. Fig. 7.2 (a) displays the surface and contour graph exhibiting effect of load and sliding distance on COF of A356-10Mg_2Si-3TiB_2 hybrid composite. It indicates COF reduces initially as the load and sliding distance increases up to certain point and then increases. Contour plot displays that minimum COF obtained at around 30N load and 2000m. Beyond 30N load oxide layer formed over the pin surface start deforming and delamination slightly increases the COF value.

Figure 7.2 (b) presents the surface and contour plot showing the influence of load and wt.% of TiB_2 on COF. It shows that with increase of TiB_2 content, COF increases at lower applied load owing to interaction of asperities and the existence of reinforcement phase, which resist deformation of matrix and increases the COF. As load increases with TiB_2 phase, the COF starts decreasing after certain load owing to the development of oxide layer because of high frictional heating at higher loads. This effect can be observed that with 2D contour plot. Minimum value of COF was observed at around 27N of load and 1 wt.% of TiB_2 content.

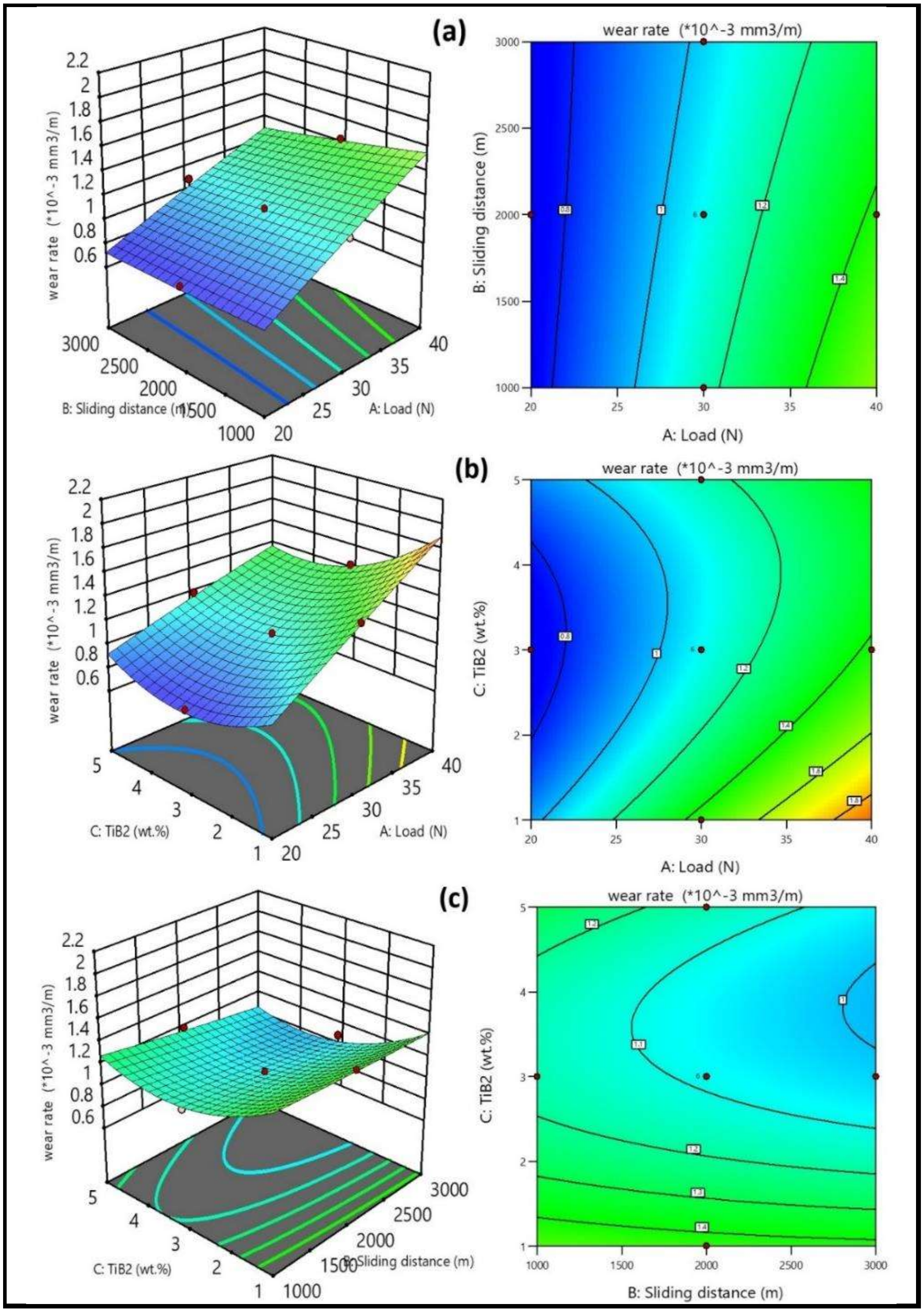


Fig. 7.1 Surface and contour plots for predicting the wear rate (a) applied load vs sliding distance, (b) applied load vs wt.% of TiB_2 and (c) sliding distance vs wt.% of TiB_2

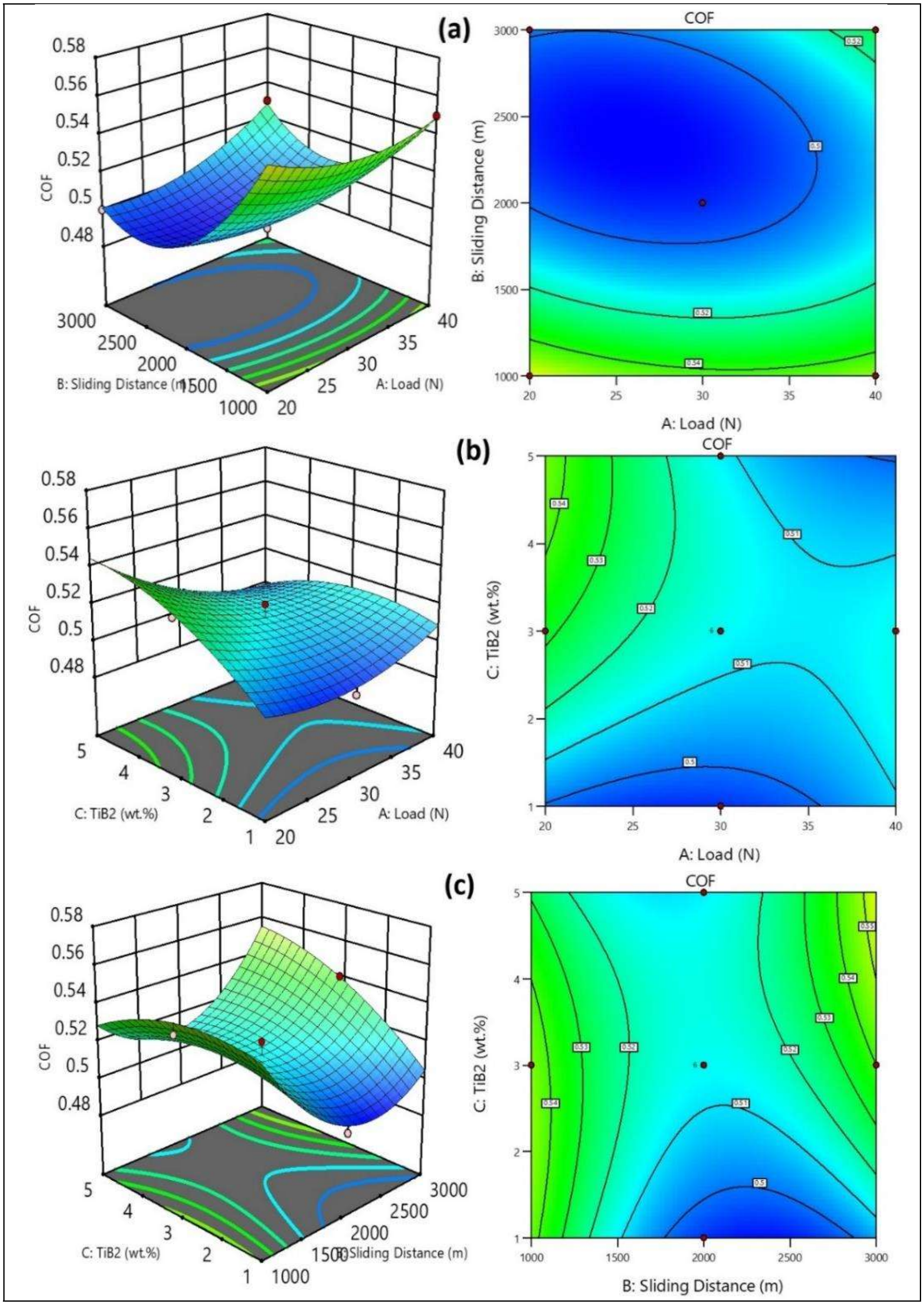


Fig. 7.2 Surface and contour plots for COF (a) load vs sliding distance, (b) load vs wt.% of TiB_2, and (c) sliding distance vs wt.% of TiB_2

Figure 7.2 (c) displays the 3D surface and contour plot revealing the influence of sliding distance and TiB_2 wt.% on COF. 3D surface plot demonstrates that COF reduces initially with increasing the sliding distance and TiB_2 wt.% up to certain point, owing to oxide layers formed on pin surface. After that COF starts increasing as the delamination increases which removes the oxide layer from pin surface. Contour plot presents that the minimum COF was observed with 1wt.% of TiB_2 composite at sliding distance of approximately 2250m.

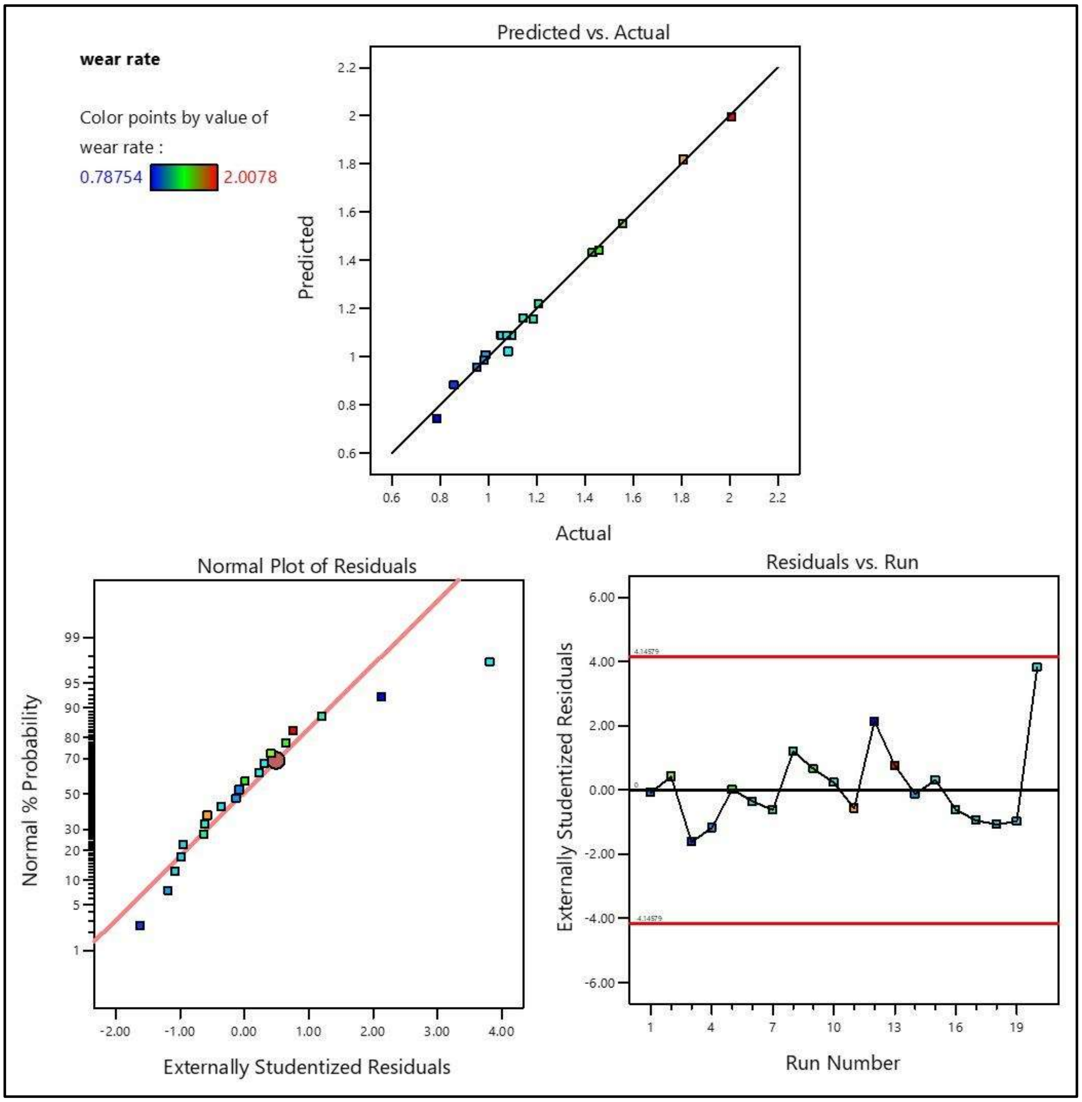


Fig. 7.3 Comparison of experimental and predicted values and normal probability for residuals of wear rate

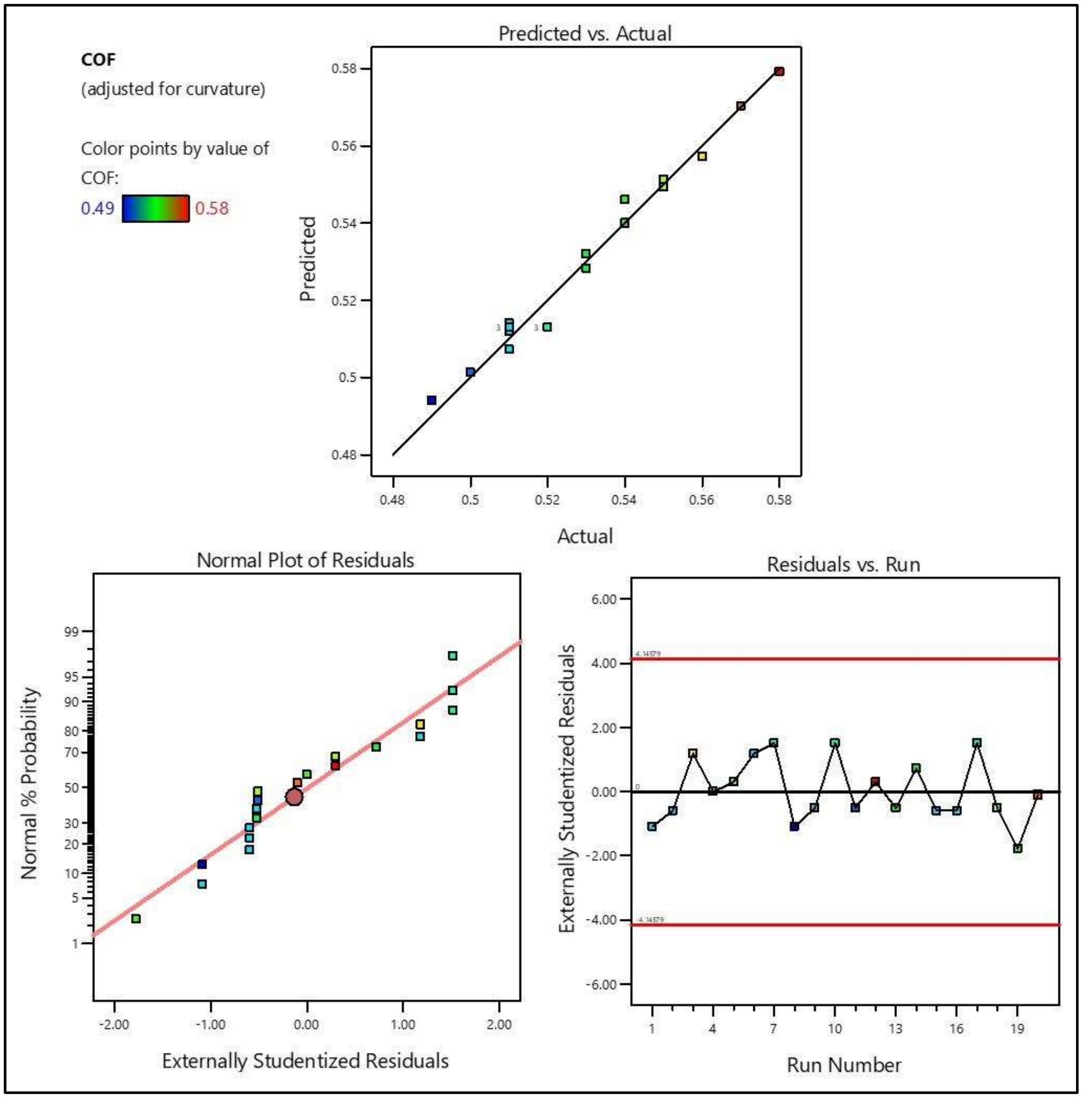


Fig. 7.4 Comparison of experimental and predicted values and normal probability for residuals of COF

Figure 7.3 and 7.4 presents the corelation between the experimental value and predicted value and normal probability for residuals of wear rate and COF, respectively. It indicates that the actual values are very close to inclined lines showing that the established model is suitable for predicting the wear rate and COF of composite system. Figures show the residuals against run order for wear rate and COF of hybrid composite, respectively. It is used to represent the distribution of residuals. It indicates that residuals are properly distributed and show a certain corelation. So, the developed model is adequate.

7.2.5 Optimization of tribological parameters

It is crucial to recognize optimum parameters for a process to reduce number of experiments. The purpose of the optimization is to determine input variables to optimize wear rate and COF. It is difficult to determine the optimum condition when the number of input variables influencing the response. Desirability-based approach is popular method for optimization of parameters. The desirability ranges between 0 to 1, when it is close to 1, it shows nearness of the target[29]. RSM developed a set of variables to optimise the wear rate and COF. From various set of input variables which have highest desirability is selected as the optimum conditions as shown in Fig. 7.5 and 7.6 for wear rate and COF respectively.

Figure 7.5 displays that optimum predicted wear rate is 0.902092 mm^3/m at a load of ~26 N, sliding distance of ~2766 m and TiB_2 content of ~3.36 wt.%. Table 7.5 presents the experiment result of wear rate at optimum parameter is 0.942057mm^3/m of 3 wt.% TiB_2 hybrid composite. Optimum predicted value of COF is 0.4896 at ~27 N load, ~2369 m sliding distance and 1wt.% of TiB_2 as presented in Fig. 7.6. Table 7.6 presents the experiment result of COF at optimum parameter is 0.51 for 1wt.% TiB_2 hybrid composite. Since the predicted value and experimental results of wear rate and COF are very close (error 4.43% for wear rate and 4.1% for COF) suggests that developed model is adequate and significant.

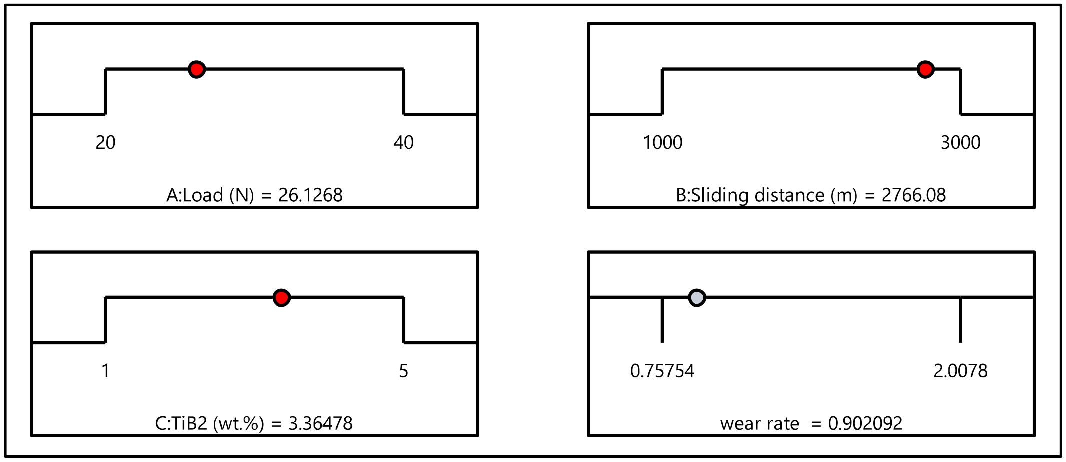


Fig. 7.5 Desirability graph of optimum parameters for wear rate

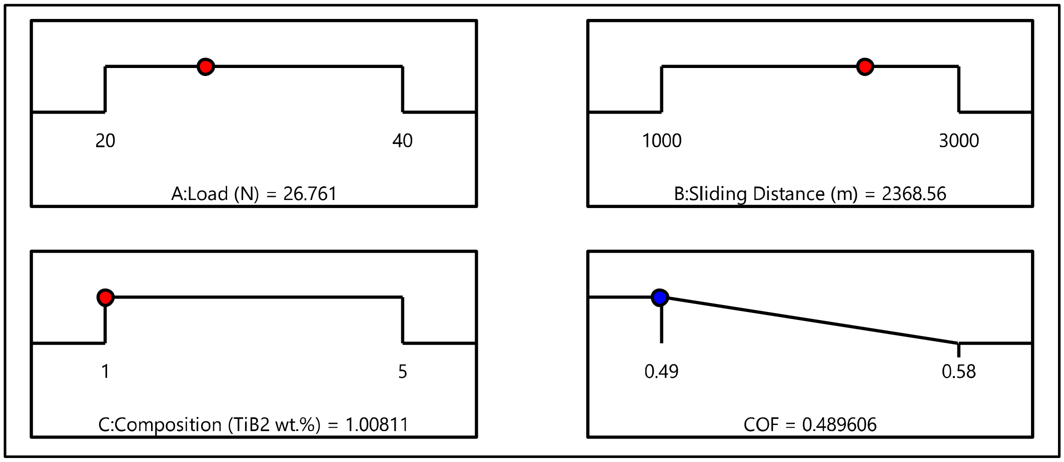


Fig. 7.6 Desirability graph of optimum parameters for COF

Table 7.5 Experimental and Predicted values of the model for optimal wear

Wear rate (mm^3/m)				
Parameters	**Optimum parameter**	**Predicted value**	**Experimental value**	**% Error**
Load (N)	26	0.902092	0.942057	4.43
Sliding distance (m)	2766			
TiB_2 (wt.%)	3.36 (~3)			

Table 7.6 Experimental and Predicted values of the model for optimal COF

COF				
Parameters	**Optimum parameter**	**Predicted value**	**Experimental value**	**% Error**
Load (N)	27	0.4896	0.51	4.1
Sliding distance (m)	2369			
TiB_2 (wt.%)	1			

7.3 STATISTICAL MODELLING OF WEAR RATE AND COF OF COOLING SLOPE (CS) CAST HYBRID COMPOSITES

7.3.1 Central composite design (CCD)

The wear test of composite was carried out considering four parameters including applied load, sliding distance, sliding velocity and wt.% TiB_2 phase as discussed in previous chapter. Table 7.7 shows the four input factors with corresponding three levels used in this investigation.

Table 7.7 Sliding wear parameters with their levels

Sl. No.	Parameters	Level		
		-1 (Low)	0 (Medium)	+1 (High)
1	A: Load (N)	20	30	40
2	B: Sliding distance (m)	1000	2000	3000
3	C: Sliding velocity (m/s)	1.5	2.25	3
4	D: Wt.% of TiB_2 (%)	1	3	5

Table 7.8 presents the design matrix of input variables and output response using central composite design of RSM in Design Expert13 software. The input factors such as applied load, sliding distance, sliding velocity and wt. percentage of TiB_2 phase and experimental results of hybrid composites with 30 runs without repetition are shown in the Table 7.8. RSM was applied to execute the mathematical equations, ANOVA, optimization, and interaction influence of input factors on wear rate and COF.

Table 7.8 Design matrix and experimental results of wear rate and COF

Std	Run	Factor 1 A: Load (N)	Factor 2 B: Sliding distance (m)	Factor 3 C: Sliding Velocity (m/s)	Factor 4 D: TiB2 (wt.%)	Response 1 WR×10^{-3} (mm^3/m)	Response 2 COF
28	1	30	2000	2.25	3	1.62583	0.38
12	2	40	3000	1.5	5	1.20231	0.523
14	3	40	1000	3	5	1.71672	0.399
10	4	40	1000	1.5	5	1.47812	0.48
18	5	40	2000	2.25	3	2.01293	0.382
11	6	20	3000	1.5	5	0.726268	0.512

1	7	20	1000	1.5	1	0.874176	0.482
16	8	40	3000	3	5	1.60556	0.364
8	9	40	3000	3	1	2.35479	0.365
20	10	30	3000	2.25	3	1.31615	0.4
21	11	30	2000	1.5	3	1.08388	0.454
15	12	20	3000	3	5	1.24905	0.369
19	13	30	1000	2.25	3	1.38357	0.402
4	14	40	3000	1.5	1	1.53061	0.521
3	15	20	3000	1.5	1	0.889587	0.55
24	16	30	2000	2.25	5	1.73672	0.415
30	17	30	2000	2.25	3	1.65018	0.39
17	18	20	2000	2.25	3	1.1226	0.389
26	19	30	2000	2.25	3	1.50212	0.39
2	20	40	1000	1.5	1	1.98232	0.453
29	21	30	2000	2.25	3	1.60647	0.39
13	22	20	1000	3	5	0.912545	0.381
5	23	20	1000	3	1	1.55061	0.367
9	24	20	1000	1.5	5	0.76743	0.48
27	25	30	2000	2.25	3	1.65583	0.41
7	26	20	3000	3	1	1.67452	0.385
6	27	40	1000	3	1	2.59102	0.357
22	28	30	2000	3	3	1.41292	0.344
25	29	30	2000	2.25	3	1.70854	0.415
23	30	30	2000	2.25	1	2.61838	0.393

7.3.2 Quadratic model and analysis of variance

The statistics model summary recommended that quadratic model is important for evaluation of wear rate and COF. The R^2 regression coefficient can be used for assessing the accuracy of wear rate (WR) model. The established wear model in this study has an R^2 value of 0.9793 for wear rate and 0.9779 for COF, both of which are close to unity, and the adjusted R^2 and predicted R^2 are very close to each other. Therefore, this model is desirable and well-fitted.

Table 7.9 Statistics model summary for wear rate

Source	**Std. Dev.**	**R^2**	**Adjusted R^2**	**Predicted R^2**	**PRESS**	
Linear	0.2690	0.7296	0.6863	0.6269	2.50	
2FI	0.2686	0.7951	0.6872	0.6540	2.31	
Quadratic	**0.0961**	**0.9793**	**0.9599**	**0.9145**	**0.5717**	**Suggested**
Cubic	0.0661	0.9954	0.9811	0.7560	1.63	Aliased

Table 7.10 Statistics model summary for COF

Source	**Std. Dev.**	**R^2**	**Adjusted R^2**	**Predicted R^2**	**PRESS**	
Linear	0.0280	0.7874	0.7534	0.6756	0.0299	
2FI	0.0272	0.8472	0.7668	0.4326	0.0523	
Quadratic	**0.0117**	**0.9779**	**0.9572**	**0.9314**	**0.0063**	**Suggested**
Cubic	0.0136	0.9859	0.9418	0.4540	0.0503	Aliased

Table 7.11 ANOVA for wear rate

Source	**Sum of Squares**	**d_f**	**Mean Square**	**F-value**	**p-value**	
Model	**6.55**	**14**	**0.4680**	**50.65**	**< 0.0001**	**significant**
A-Load	2.50	1	2.50	270.51	< 0.0001	
B-Sliding distance	0.0278	1	0.0278	3.01	0.1032	
C-Sliding Velocity	1.14	1	1.14	123.54	< 0.0001	
D-TiB_2	1.21	1	1.21	131.19	< 0.0001	
AB	0.1424	1	0.1424	15.41	0.0013	
AC	0.0002	1	0.0002	0.0201	0.8891	
AD	0.0787	1	0.0787	8.52	0.0106	
BC	0.0469	1	0.0469	5.08	0.0397	
BD	0.0131	1	0.0131	1.41	0.2531	
CD	0.1569	1	0.1569	16.98	0.0009	
A^2	0.0106	1	0.0106	1.15	0.3002	
B^2	0.2060	1	0.2060	22.30	0.0003	

C^2	0.3809	1	0.3809	41.22	< 0.0001	
D^2	0.7716	1	0.7716	83.50	< 0.0001	
Residual	0.1386	15	0.0092			
Lack of Fit	**0.1146**	**10**	**0.0115**	**2.39**	**0.1746**	**not significant**
Pure Error	0.0240	5	0.0048			
Cor Total	6.69	29				

Table 7.12 ANOVA for COF

Source	**Sum of Squares**	**d_f**	**Mean Square**	**F-value**	**p-value**	
Model	**0.0901**	**14**	**0.0064**	**47.33**	**< 0.0001**	**significant**
A-Load	0.0003	1	0.0003	2.06	0.1718	
B-Sliding distance	0.0020	1	0.0020	14.44	0.0017	
C-Sliding Velocity	0.0702	1	0.0702	516.02	< 0.0001	
D-TiB$_2$	0.0001	1	0.0001	1.02	0.3283	
AB	0.0000	1	0.0000	0.2224	0.6440	
AC	0.0001	1	0.0001	0.4136	0.5299	
AD	0.0008	1	0.0008	5.76	0.0298	
BC	0.0034	1	0.0034	24.73	0.0002	
BD	0.0011	1	0.0011	8.25	0.0116	
CD	0.0002	1	0.0002	1.15	0.3008	
A^2	8.684E-07	1	8.684E-07	0.0064	0.9374	
B^2	0.0006	1	0.0006	4.24	0.0573	
C^2	0.0004	1	0.0004	3.18	0.0948	
D^2	0.0008	1	0.0008	6.12	0.0258	
Residual	0.0020	15	0.0001			
Lack of Fit	**0.0011**	**10**	**0.0001**	**0.6078**	**0.7650**	**not significant**
Pure Error	0.0009	5	0.0002			
Cor Total	0.0922	29				

Tables 7.11 and 7.12 show the findings of the ANOVA (statistical method) used to assess the most influencing and non-influencing factors affecting the wear rate and COF. The p value can be used to determine which parameters are significant and which are insignificant. All significant input factors have p values less than 0.05, while all non-significant parameters have p values more than 0.10. The F value has also been utilized to assess the influential and non-influential factors. The F value is calculated by dividing the variance of each parameter by the variance of the error term. The value of F may be utilized for estimating the contribution of each parameter. The regression coefficient of R^2 can be employed to determine the reliability of model. The value of F is 50.65 and p-value less than 0.0001 specifies that developed model is significant. F-value 270.51 and p-value less than 0.0001 for wear rate suggests that load is the most important factor followed by TiB_2 wt.%, sliding velocity and sliding distance. While in case of COF, sliding velocity is the dominating factor affecting the COF.

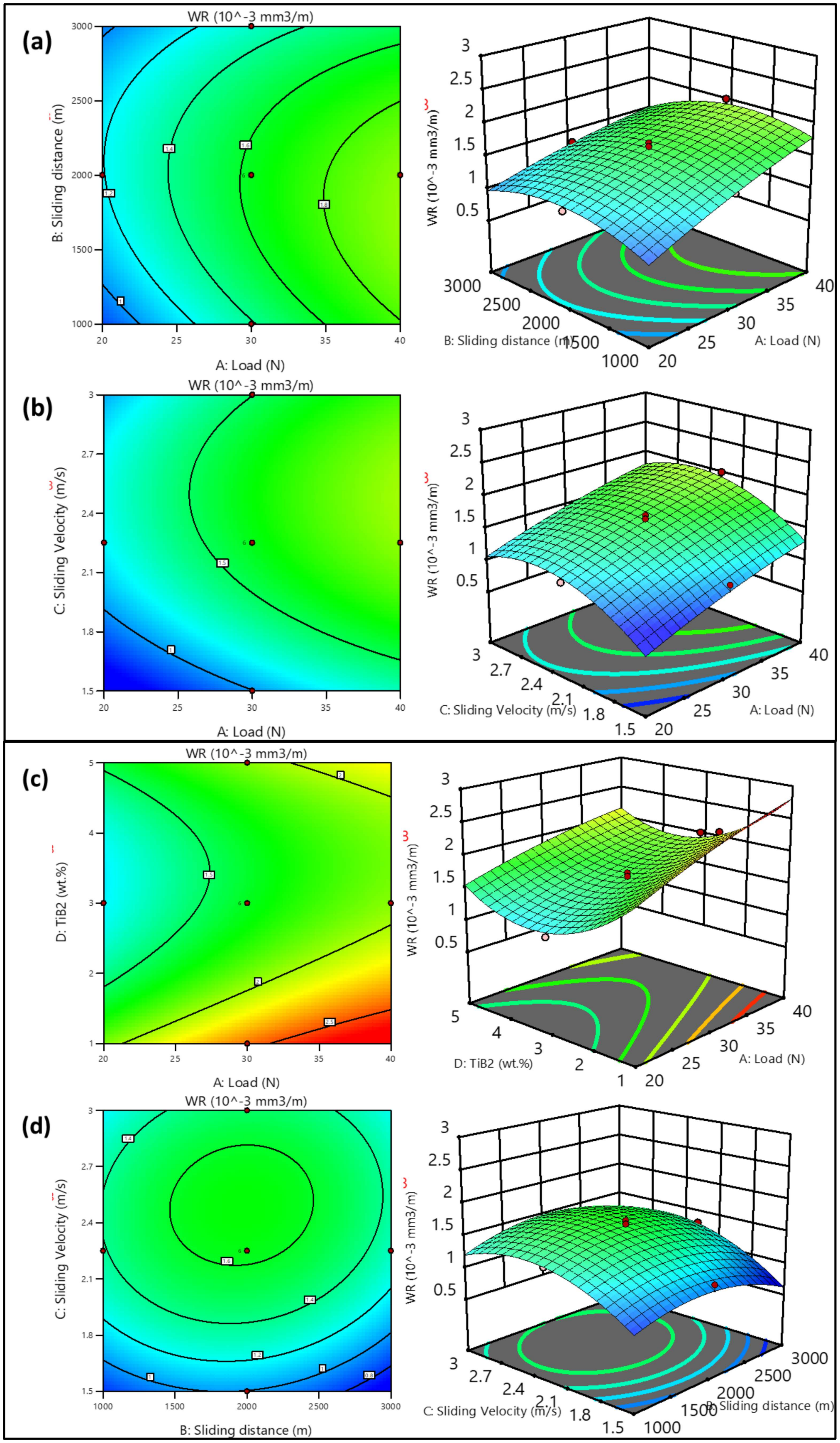
7.3.3 Regression equation for wear rate and COF

The regression model Eqns. (3) and (4) was obtained using the software "*Design Expert 13*" to develop the relationship between the wear test variables including applied load (A), sliding distance (B), sliding velocity (C), and wt.% of TiB_2 (D) and wear rate (WR) and coefficient of friction (COF). The resulting regression equations are as follows:

WR (wear rate) = $-4.64679 + 0.105928\times A + 0.001169\times B + 3.46795\times C - 0.724232\times D - 9.47408\times10^{-6}\times A\times B - 0.000402\times A\times C - 0.003488\times A\times D + 0.000072\times B\times C + 0.000014\times B\times D - 0.065759\times C\times D - 0.000640\times A^2 - 2.81874\times10^{-7}\times B^2 - 0.681473\times C^2 + 0.136455\times D^2$ **Eq. 3**

COF = $0.718215 - 0.001385\times A + 0.000011\times B - 0.161711\times C - 0.032305\times D - 1.37500\times10^{-7}\times A\times B + 0.000250\times A\times C + 0.000350\times A\times D - 0.000019\times B\times C - 4.18750\times10^{-6}\times B\times D + 0.002083\times C\times D - 5.78947\times10^{-6}\times A^2 + 1.49211\times10^{-8}\times B^2 + 0.022971\times C^2 + 0.004480\times D^2$ **Eq. 4**

Figures 7.7 (a-f) demonstrate the variation of 2D contours and 3D surface plots on wear rate as a function of input variables, which may be utilized for predicting wear properties. Figures 7.7 (a) and (b) display the effect and interaction of various input variables (applied load, sliding velocity, and sliding distance) on wear rate regardless of wt.% of TiB_2 phase reinforcement. The figure clearly indicates that the relations of applied load with the sliding velocity have more influence on wear rate. Fig. 7.7 (c) contour and 3D surface plot displays influence of load and wt. percent of TiB_2 on wear rate at fixed sliding distance and velocity. It can be observed that wear rate increases with the load due to ploughing action of asperities. However, wear rate start decreasing with increase in TiB_2 content up to 3 wt.% due to increase of hardness of composite. But in hybrid composite containing 5 wt.% of TiB_2, wear rate increases slightly due to harder particle in the mechanically mixed layer, which increases the abrasive wear. Figure 7.7 (d) displays the contour and surface plot of sliding distance and sliding velocity on wear rate. We can observe that with increase in sliding velocity wear rate increases up to velocity of 2.25 m/s and start decreasing with further increase in sliding velocity. This can be attributed that at lower velocity, abrasive wear increases due to interaction of asperities while at higher sliding velocity, formation of oxide layer due to high frictional heat reduces the wear rate. Fig. 7.7 (e) and (f) present contour and surface plot of wt.% of TiB_2 with sliding distance and sliding velocity on wear rate respectively. We can observe that the wear rate increases gradually as the sliding distance and velocity increase. However, wear rate reduces with increasing TiB_2 content up to 3 wt.% after that it increases. This can be attributed that increasing TiB_2 content in hybrid composite increases hardness of composites. However, with increase of TiB_2 content increases the harder particle in the mechanically mixed layer which increases the abrasive wear.


(a)
WR (10^-3 mm3/m)
B: Sliding distance (m)
3000
2500
2000
1500
1000
A: Load (N)
20
25
30
35
40
WR (10^-3 mm3/m)
3
2.5
2
1.5
1
0.5
(b)
WR (10^-3 mm3/m)
C: Sliding Velocity (m/s)
3
2.7
2.4
2.1
1.8
1.5
A: Load (N)
20
25
30
35
40
(c)
WR (10^-3 mm3/m)
D: TiB2 (wt.%)
5
4
3
2
1
A: Load (N)
20
25
30
35
40
(d)
WR (10^-3 mm3/m)
C: Sliding Velocity (m/s)
3
2.7
2.4
2.1
1.8
1.5
B: Sliding distance (m)
1000
1500
2000
2500
3000

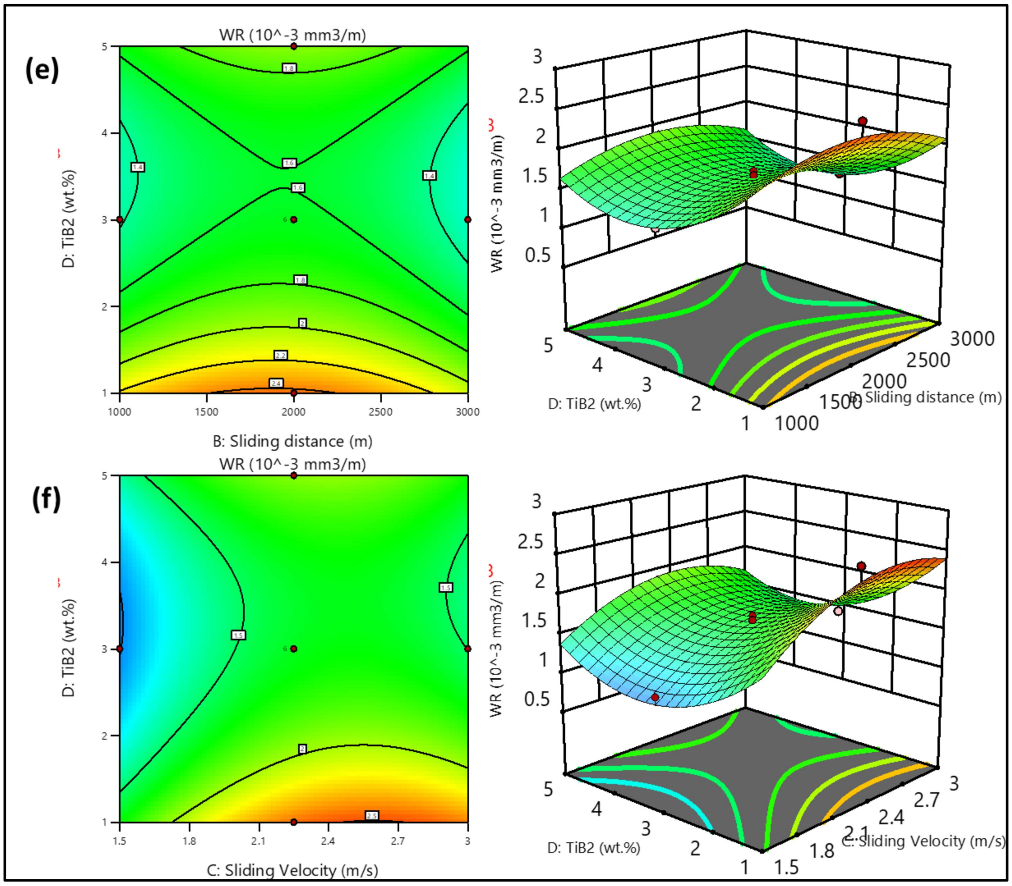
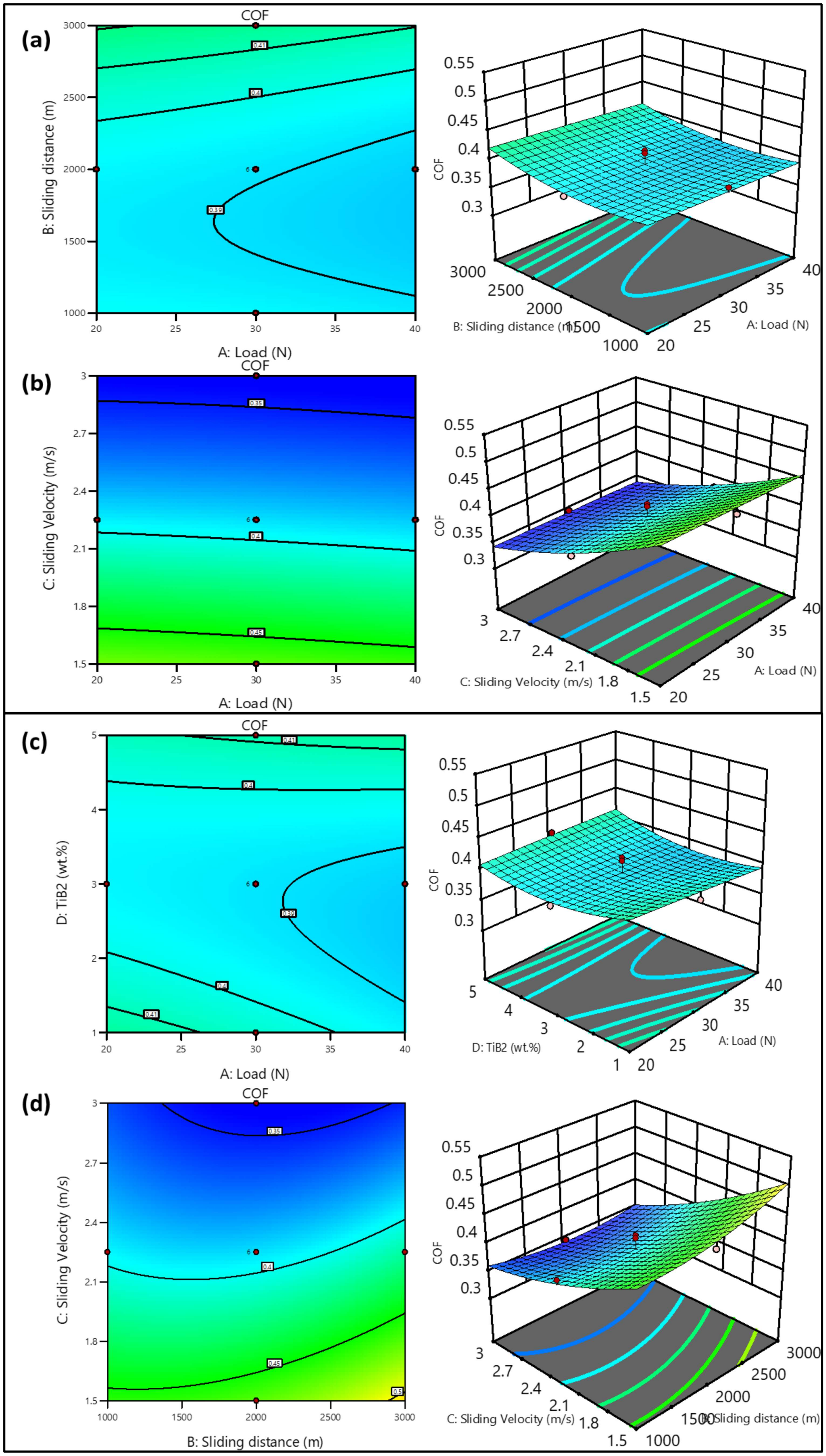


Fig. 7.7 Contour and surface plots for wear rate

To compare the COF of hybrid composites at different input variables such as applied load, sliding distance, sliding velocity, and wt.% of TiB_2 phase, contour and 3D surface plot as displayed in Fig. 7.8 (a-f). Fig. 7.8 (a) presents the influence of load and sliding distance variation on COF. It is found that COF slightly decreases with increasing load. However, with increasing sliding distance, COF slightly decreases up to 2000m after that it starts increasing. Fig. 7.8 (b) shows that the COF decreases with sliding velocity owing to the oxide layer formed over pin surface at higher sliding velocity.


(a)
COF
B: Sliding distance (m)
A: Load (N)
0.41
0.4
0.39
COF
0.55
0.5
0.45
0.4
0.35
0.3
3000
2500
2000
1500
1000
20
25
30
35
40
(b)
COF
C: Sliding Velocity (m/s)
A: Load (N)
0.35
0.4
0.45
3
2.7
2.4
2.1
1.8
1.5
(c)
COF
D: TiB2 (wt.%)
A: Load (N)
0.41
0.4
0.39
5
4
3
2
1
(d)
COF
C: Sliding Velocity (m/s)
B: Sliding distance (m)
0.35
0.4
0.45
0.5

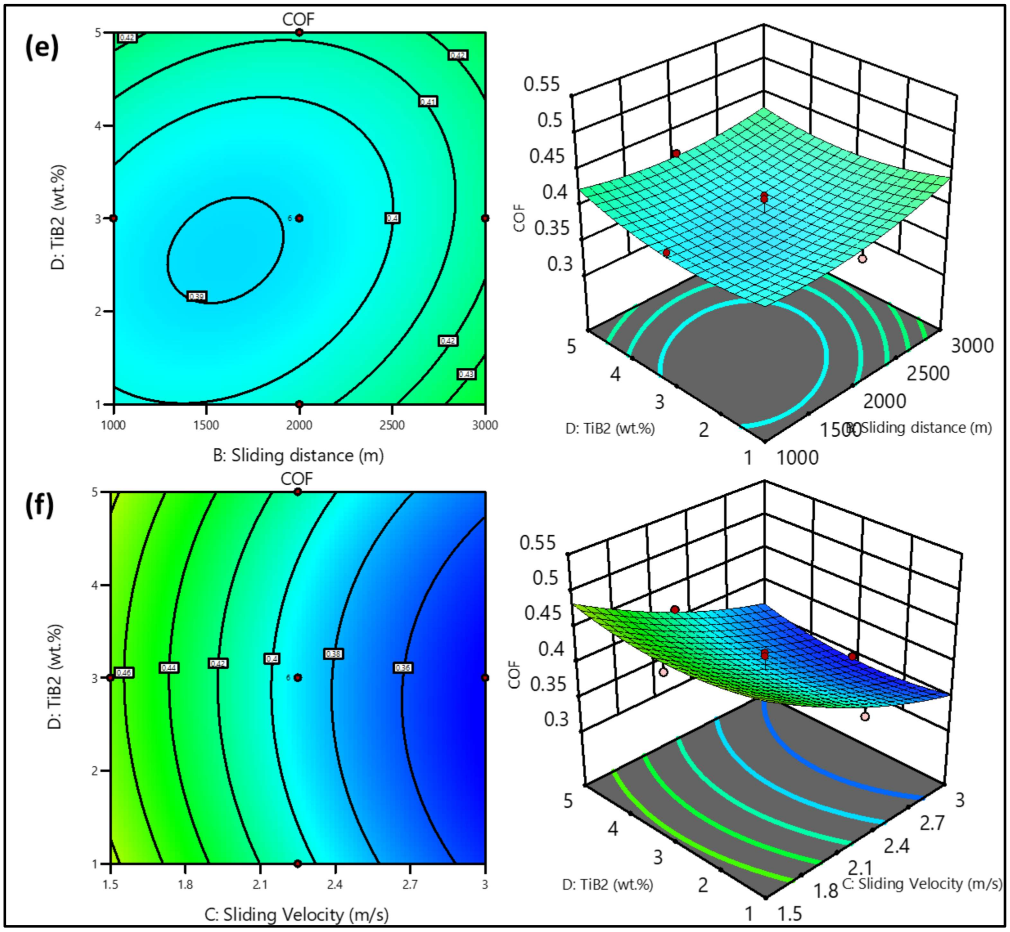


Fig. 7.8 Contour and surface plots for COF

. Fig. 7.8 (c) displays the deviation of COF with load and TiB_2 wt.%. COF decreases with increased load due to smoothening of surface by forming an oxide layer on the surface. However, as the TiB_2 content increases in the hybrid composite, initially COF decreases due to increase of lower contact of surfaces but it starts increasing with further increase of TiB_2 phases in the MML and increases the wear. Fig. 7.8 (d) shows the influence of variation in sliding distance and sliding velocity on COF. COF reduces with increasing sliding velocity from 1.5 to 3 m/s, but increases slightly with increasing sliding distance. Fig. 7.8 (e) displays that the COF reduces initially with the increase of TiB_2 content then starts increasing for 5wt.% TiB_2 hybrid composite at low sliding distance. Fig. 7.8 (f) shows that the COF is higher at low sliding velocity compared to the high sliding velocity at all composition of hybrid composite due to smoothening of surface by frictional heating.

Figure 7.9 shows the predicted versus actual plots, normal probability graph of the residuals and residuals against the run order for wear rate of the hybrid composites. From the figure, it is found that all the data points are close to the inclined line in both predicted vs actual and normal probability graph. These results demonstrate that there is good correlation between predicted wear rate model and experimental results. The residual plot for wear rate reveals that all the run residues fall between the -4 to 4. Therefore, the developed mathematical model by the RSM technique is adequate to predict the wear rate.

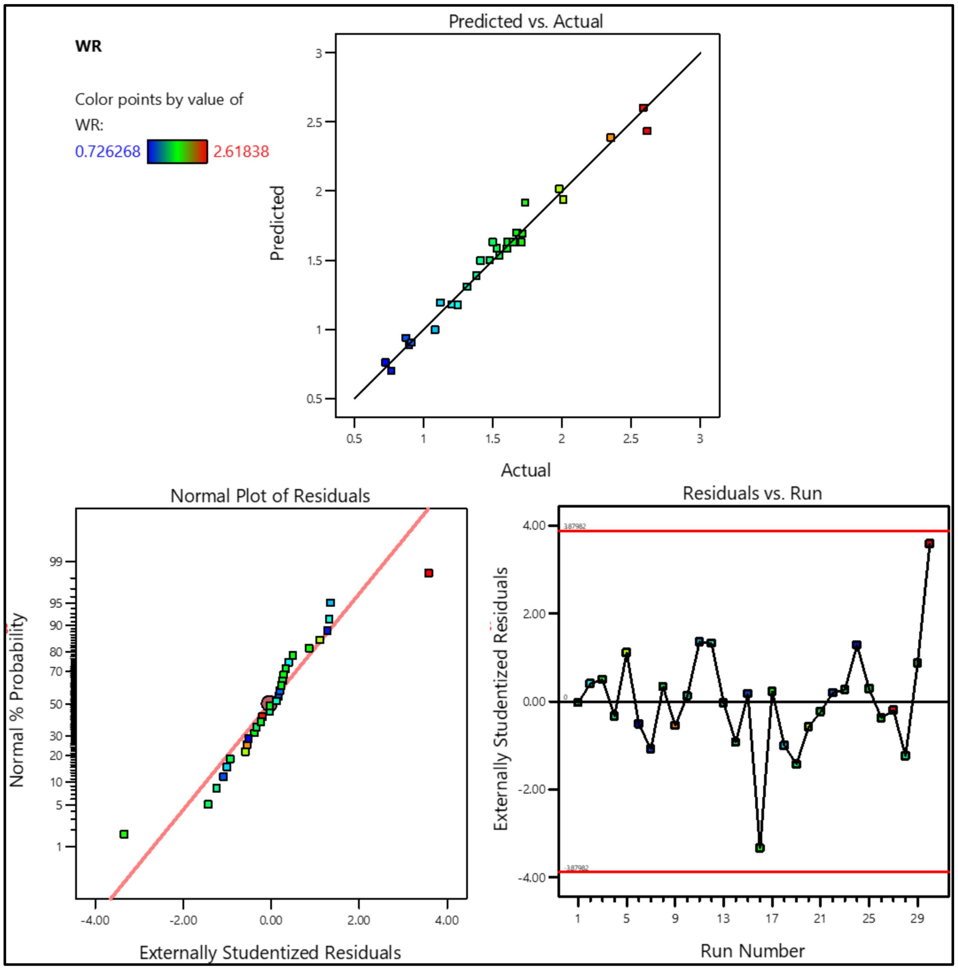


Fig. 7.9 Predicted versus actual plots, normal probability of residual and residuals against the run order for wear rate

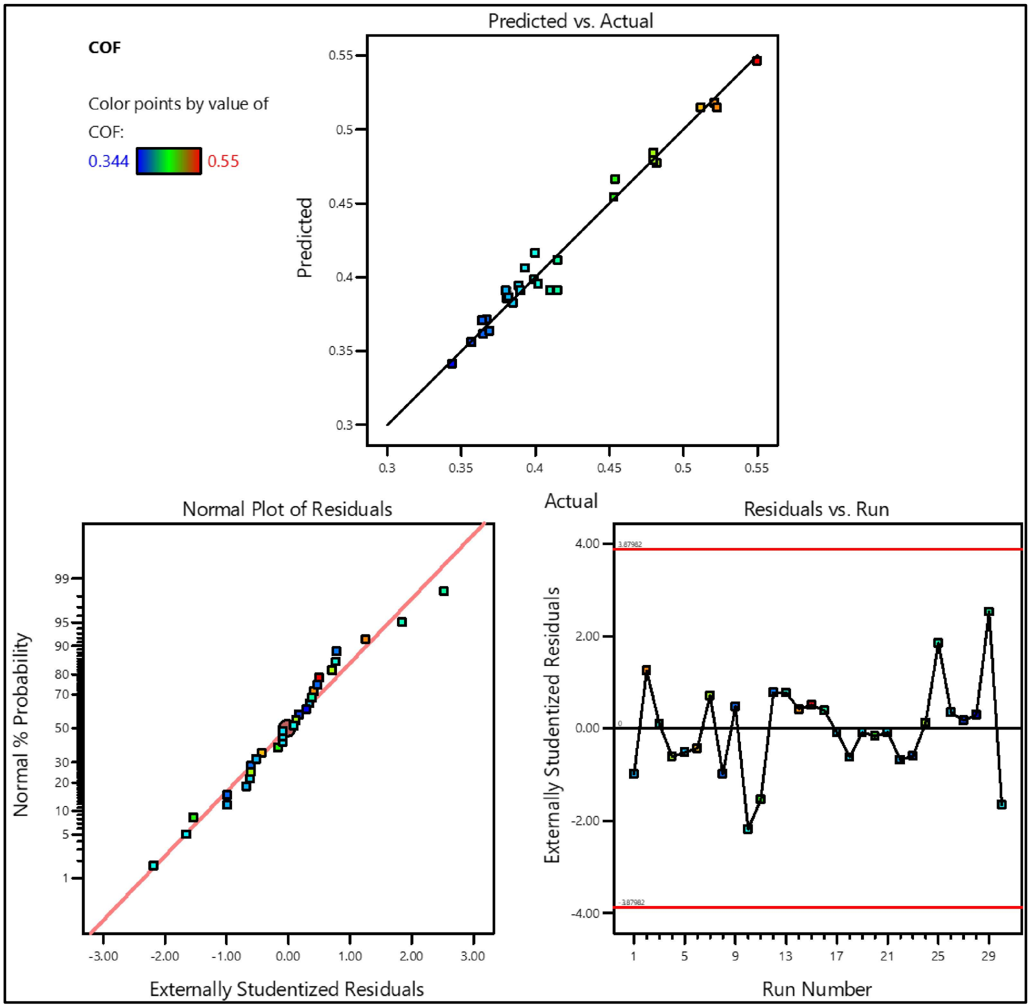


Fig. 7.10 Predicted versus actual plots, normal probability of residual and residuals against the run order for COF

Figure 7.10 presents the predicted versus actual plots, normal probability of residuals and residuals against the run order for COF. It can be observed that all data points fall very close to the fitted line in predicted vs actual plot validating the experimental results. The normal probability graph of residuals shows that residuals are close to the inclined line which means that errors were distributed normally. The residuals against the observed order show the distribution of residuals. It emphasized that a tendency to have runs of positive and negative runs indicates that residuals are properly distributed and demonstrates the existence of a particular correlation.

7.3.4 Optimization of tribological parameters

A collection of RSM data was used to optimise the wear rate and coefficient of friction. The optimal solution involves evaluating the input process parameters in order to reduce the wear rate and coefficient of friction. Fig. 7.11 shows the ramp function graph of Desirability for A356-10Mg_2Si-xTiB_2 hybrid composite. The set of variables with the highest desirability value is selected as the optimal conditions for the responses. The desirability value ranges from 0 to 1. When the desirability value is near to one, it implies that the closeness of response towards target. The following are the input parameters for optimum wear rate and COF: load-20 N; sliding distance-1129.48 m; sliding velocity- 3 m/s; TiB_2- 3.23 wt.%; wear rate- 0.7262×10^{-3} mm^3/m; and COF-0.357.

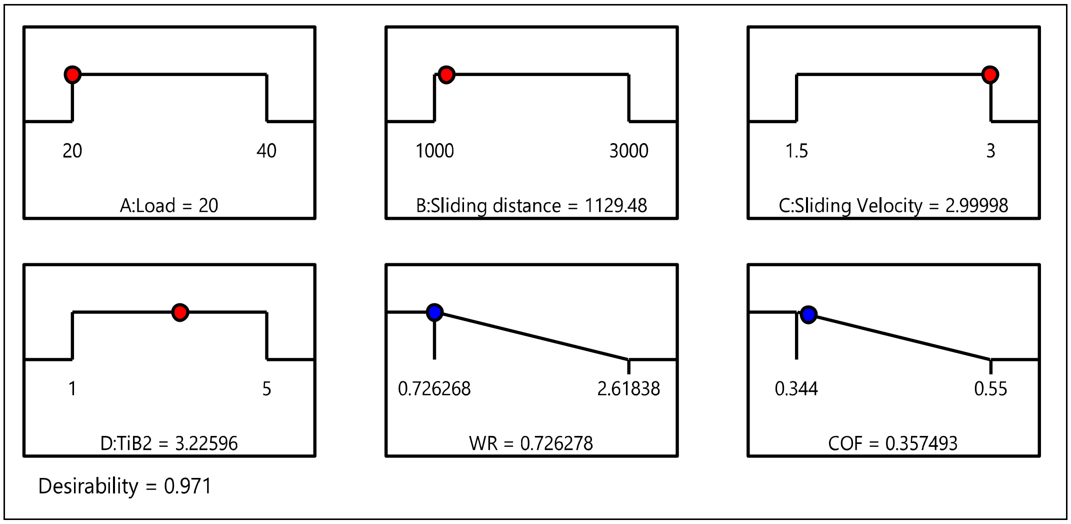


Fig. 7.11 Ramp function graph of Desirability for A356-10Mg_2Si-xTiB_2 hybrid composite

The confirmatory test was carried out at obtained optimum parameters from the ramp function graph of desirability to check the accuracy of the developed model, which is displayed in Table 7.13. Further, the experimental results were compared with the predicted results by the established model. It is found that the obtained confirmatory test result produced an error of 8.58% for wear rate and 3.12% for COF, that indicates the model is reliable. Therefore, the developed model is adequate and producing accurate results.

Table 7.13 Experimental and Predicted values of the model for optimal wear rate and COF

Parameters	**Optimum parameter**	**Predicted value**		**Experimental value**		**% Error**	
		WR	**COF**	**WR**	**COF**	**WR**	**COF**
Load (N)	20	0.726278	0.3575	0.7944	0.369	8.58	3.12
Sliding distance (m)	1130						
Sliding velocity (m/s)	3						
TiB_2 (wt.%)	3.23 (~3)						

7.4 Summary

To reduce the number of experiments, time and resource consumption optimization of wear rate and COF with variable input parameters is essential. Following conclusions can be drawn from this study.

- Central composite design of RSM can be effectively applied to develop a mathematical model to predict wear rate and COF, and reduce the number of experiments.
- According to the ANOVA results, applied load is most prominent factor affecting wear rate and COF of stir cast hybrid composites followed by TiB_2 wt.% and sliding distance.
- In case of CS cast hybrid composites, applied load is the most important factor followed by TiB_2 wt.%, sliding velocity and sliding distance affecting the wear

rate. While in case of COF, sliding velocity is the dominating factor affecting the COF.

- Desirability function method results show that the optimum conditions for least wear rate of stir cast hybrid composites are 26 N load, 2766 m sliding distance and 3.36 wt.% of TiB_2. However, the optimum conditions for minimum COF are 29 N load, 2233 m sliding distance and 1wt.% TiB_2.
- Optimum parameters for least wear rate and COF for CS cast hybrid composites are applied load of 20 N; sliding distance of 1129.48 m; sliding velocity of 3 m/s; and TiB_2 content of 3.23 wt.%.
- Conformity tests at optimum parameters with error value less than ~5% for wear rate and COF in stir cast hybrid composite, and error of 8.58% for wear rate and 3.12% for COF in CS cast hybrid composites signifies that the developed model is adequate for optimization.
- This can be concluded that the Response surface methodology is significant statistical tools to optimize the tribological parameters.