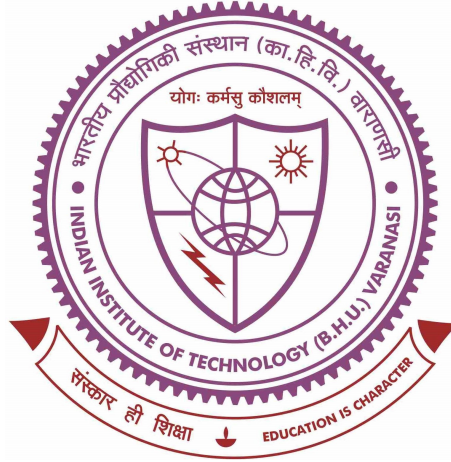


A Study of Fixed Point Theory Using Various Contractions  
in Different Spaces and Some Applications



*The thesis submitted in partial fulfillment*

*for the Award of Degree*

*DOCTOR OF PHILOSOPHY*

*by*

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## CERTIFICATE

It is certified that the work contained in this thesis titled "*A Study of Fixed Point Theory Using Various Contractions in Different Spaces and Some Applications*" by *Jayanta Sarkar* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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I, *Jayanta Sarkar*, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of *Prof. Tanmoy Som* from *January 2021 to August 2024* at the *Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi*. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports, dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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*Dedicated*  
*to*  
*My Beloved Family*

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# Notations and Symbols

$\mathbb{N}$	Collection of all natural numbers
$\mathbb{R}$	Collection of all real numbers
$\mathbb{R}^n$	$n$ -dimensional Euclidean space
$C(X)$	Set of all compact subsets of $X$
$X^m$	Product space “ $X \times X \times \cdots m$ times”
$C(X)^m$	Product space “ $C(X) \times C(X) \times \cdots m$ times”
$\mathbb{N}_N$	First $N$ natural numbers
$Fix(T)$	Set of all fixed points of $T$
$ComFix(S, T)$	Set of all common fixed points of $S$ and $T$
$\square$	End of the proof