

Chapter 1

Introduction and Literature review

1.1 Introduction

This chapter presents an overview of energy management scheme (EMS) for DC-Microgrid (DC μ G), serving as a foundation for the research reported in this thesis. It begins with a discussion on the background of the study, followed by a literature review to identify existing research gaps, outlines the motivations driving this work, and describes the thesis organisation.

1.2 Background

Increasing power requirements and climate degradation have stimulated the growth of alternative energy generation and storage options to reduce fuel consumption, operating cost (OC), and greenhouse gas emission [1]. To address these challenges, utilisation of small generation sources close to the customer site at the distribution level, known as distributed generation (DG) [2], has become an attractive option. DG encompasses a range of prime mover technologies, including dispatchable (e.g., renewable-based like solar photovoltaic and wind turbine) and non-dispatchable sources (e.g., diesel generator, microturbine (MT), fuel cell (FC), biomass, gas turbine (GT), etc). While DGs offer numerous advantages for distribution networks, their increased penetration, particularly from intermittent renewable sources, introduces various technical, operational, and safety challenges [3]. To address the challenges posed by the uncoordinated integration of individual DGs into the main grid, a systematic approach has emerged, culminating in the

concept of the microgrid (μG) [4]. While various definitions of μG exist in the literature, μG can broadly be described as follows. “A *microgrid is a group of interconnected loads and distributed energy resources within clearly defined electrical boundaries that act as a single controllable entity concerning the grid. A microgrid can connect and disconnect from the grid to enable it to operate in both grid-connected and island mode*” [5, 6]. μGs offer operational flexibility by functioning in grid-connected, islanded, or transitional modes.

μG can be categorised into AC μG , DC μG , and hybrid AC/DC μG . AC μG is the most common type, leveraging conventional AC power systems and offering easy integration with existing grid infrastructure. DC μG , on the other hand, is gaining traction due to its higher efficiency in systems with predominantly DC-based sources (e.g., FCs, batteries) and loads (e.g., LED lighting, electronic devices). By eliminating the need for repeated AC/DC conversions, DC μG reduces energy losses and simplifies control schemes. The DC μG architecture has inherent advantages over its AC counterpart, like the absence of frequency synchronisation and reactive power management requirements. Therefore, the DC μG architecture is becoming a preferred solution for various applications like electrifying remote rural areas, community buildings, data centres, etc. [7]. Hybrid AC/DC μG combines the strengths of both AC and DC systems, featuring bidirectional converters to manage energy flow between the AC and DC sides [7].

The focus of this thesis is on developing EMS for a DC μG . The goal of a well-designed EMS for a DC μG is to ensure economic competitiveness of electricity generation and address various technical challenges associated with the DC μG operation. Apart from economic considerations, the EMS of a DC μG should address the operating challenge of intermittent renewable energy sources (RES) generation and limited controllable generation capacity. The operational uncertainty associated with RES arises due to dependence on variable climatic conditions [8]. Moreover, load demand and wholesale market electricity prices are also subject to fluctuations and cannot be considered deterministic inputs. Consequently, managing the dispatch strategy becomes increasingly complex, particularly in a DC μG where the balance between demand and generation may not be adequately maintained, especially with DG of small ratings. High RES generation or peak load demands also risk breaching critical bus voltage and line rating constraints, amplifying the likelihood of cascading failures. Therefore, proper uncertainty modelling of input

uncertainties should be incorporated, along with correlation, into the EMS of a DC μ G to mitigate the risk of the dispatch strategy [9]. The RES generation volatility can be mitigated by leveraging flexible distributed energy resource (DER) (dispatchable DG and energy storage system (ESS)) or by consumer-side measures like demand response (DR) implementation [10]. However, designing a DR implementation framework, coordinating it with the DC μ G operator (DC μ GO) scheduling strategy, and balancing the interests of the DC μ GO and the flexible consumers, considering all operating constraints, poses another challenge.

Moreover, the challenge of designing the DC μ G EMS becomes more complex if islanding capability is to be incorporated so that the DC μ G transitions from grid-connected to islanded mode with minimum load curtailment after a fault in the upstream grid without violating network and equipment-level constraints. The operational challenge intensifies during emergencies and natural disasters like windstorms, hurricanes, etc., requiring grid and renewable unit disconnection to prevent widespread damage. It is necessary to maintain an uninterrupted electricity supply to critical loads (hospitals, military facilities, data centres, technology centres, transport stations, etc.) and minimise curtailment of non-critical loads after islanding due to natural disasters. The capability of a power system to survive and continue stable operation after a high-impact and low-probability event is called resilience [11, 12, 13]. An EMS designed only from an economic perspective struggles to supply enough power to critical facilities [14]. Therefore, apart from economic considerations, it is crucial to incorporate resilience in the EMS to ensure the survival of critical loads during emergencies.

1.3 Literature review

A review of relevant literature is presented below:

1.3.1 Studies on EMS of DC μ G in deterministic framework

Yin et al. proposed an EMS to maintain load generation balance and DC bus voltage stabilisation for a multi-source islanded DC μ G consisting of a diesel generator, a BESS, a supercapacitor, and a solar photovoltaic generator (SPG) feeding a load [15]. The EMS proposed by the authors was rule-based, where the diesel generator was used to meet load

demand during the off-grid mode. The supercapacitor, having a fast dynamic response and high power density, was added to the system to compensate for the slow dynamic behaviour of the diesel generator during the system start-up phase. A study by Wang and Xu considered the optimal planning problem of sizing DG and BESS units in a DC μ G to minimise the total investment and OCs using particle swarm optimization (PSO) [16]. Maulik and Das reported a small-signal stability-constrained economic dispatch algorithm for an islanded DC μ G [17]. The droop parameters of the controllable DG units were taken as decision variables (DVs) to realise the objectives. The optimisation was carried out using PSO, and a fuzzy max-min approach was used to solve the multi-objective (MO) problem with two objectives. Chauhan [18] considered a residential DC μ G system powered by multiple sources like SPG, upstream grid, BESS units, and vehicle to grid (V2G) operation of plug-in hybrid vehicles (PHEV)s. The authors in the above paper proposed a load scheduling algorithm for the home DC μ G system to optimise energy consumption and electricity bill, leveraging the flexibility offered by the BESS unit and PHEV loads. Dissanayake and Ekneligoda [19] reported a MO approach to determine the droop coefficient of an islanded DC μ G. The objectives were to improve voltage regulation, reduce system losses and improve current sharing among different DG units. The MO problem was solved using the “elitist nondominated sorting Genetic algorithm (GA)” approach. A state feedback linearisation approach for implementing the control logic was also reported in the paper. Aluisio et al. [20] proposed an optimal operation strategy for a DC μ G comprising SPG, BESS and PHEV charging stations for minimising the net OC. The problem was formulated in a deterministic “mixed-integer linear programming” framework considering the degradation costs of the BESS units and losses in the network and the converters. Dicorato et al. proposed an optimal scheduling strategy of PHEV fleets to minimise the OC and “expected energy not supplied” in a DC μ G [21].

Authors in [22] examined the impact of the battery ageing model on the OC of a SPG-BESS grid-connected DC μ G using the “Combined Arrhenius-Peukert-NREL” model. The objective of the EMS was to minimise the OC. The non-convex optimisation problem was solved using PSO. The authors did not consider a detailed network model and operating constraints like bus voltage limit and feeder current limit of the DC μ G in the work. Grisales-Norena et al. minimised the energy purchase cost of a DC μ G by optimally selecting the power setpoints of controllable DG units and BESS charging/discharging

schedule [23]. The DC μ G network was modelled by a set of non-convex algebraic power flow equations, and the non-convex optimisation problem was solved using parallel PSO. Ullah et al. reported a distributed Lagrange multiplier-based consensus algorithm to minimise the OC of a DC μ G comprising controllable units, ESS, and loads [24]. However, the formulation did not include bus voltage and line loading constraints [24].

Singh and Lather [25] proposed a power management strategy to maintain the DC bus voltage in an islanded DC μ G comprising diesel generators, SPG, wind power generator (WPG), BESS and supercapacitors using optimal current and voltage controllers. The work also included optimal power sharing among the BESS and supercapacitor units and minimised the stress on the BESS unit. Xie et al. considered a DC μ G network for transportation electrification and proposed a “distributed optimal algorithm” for “fair load allocation” and loss minimisation in the presence of continuous and random load variations [26]. The investigation revealed that the step size in the distributed optimisation algorithm is an important design parameter, and a trade-off exists between the effect of noise on the global optimum and the load-tracking capability of the algorithm. The authors also proposed a method for optimal step size selection to achieve a fair trade-off. Jiang et al. proposed an adaptive droop control approach to minimise the network and converter loss in a DC μ G [27]. The work employed a hierarchical control strategy comprising three control levels, where the tertiary controller used a Lagrange multiplier-based approach to provide a voltage reference shift signal to DG converters for relaxing the objective. Dorahaki et al. proposed a centralised peer-to-peer (P2P) energy trading framework based on behavioural risk attitudes [28]. The objective was to maximise the sum of prospective values of end-users. Using modified prospect theory, researchers modelled end-users’ behavioural attitudes in P2P energy trading. The researchers concluded that the proposed methodology effectively enhanced end-user satisfaction with minimal impact on expected profit. Khazaei formulated an optimal power flow problem for a medium voltage DC shipboard power system comprising dispatchable and non-dispatchable sources and a hybrid energy storage system (BESS and supercapacitor) to minimise the OC [29]. A virtual resistive droop was used with dispatchable and BESS units, while a virtual capacitive loop was used with the supercapacitor. The non-convex problem was converted to a convex model using second-order cone programming. Kumar et al. reported a study on the optimal sizing of components in an islanded

DC μ G along with virtual energy storage-based EMS using the thermal inertia of thermostatically controlled load (TCL) to minimise the energy cost and load curtailment [30]. A quasi-consensus-based distributed control strategy was proposed to minimise the power loss in an islanded DC nanogrid [31].

Zia et al. [32] proposed a two-stage EMS for a DC μ G comprising curtailable loads, SPG, WPG, tidal power and BESS to reduce the OC. The first stage of the optimisation was framed in an mixed-integer linear programming (MILP) framework based on day-ahead forecasts. The second stage involved adjusting the setpoints based on the mismatch between the forecasted and actual values. The authors had considered the degradation cost of the BESS and load shedding cost in the formulation. Alam et al. [33] considered a DC μ G comprising a BESS, supercapacitor, DG, constant power and resistive loads and proposed a distributed rule-based EMS to maintain the DC bus voltage. A fractional order voltage shifting term was used to immediately restore the DC bus voltage following a disturbance. The controller parameters were tuned using the PSO algorithm to minimise the integral square of the error term. The supercapacitor supplied/absorbed energy during a disturbance, but its state of charge (SOC) was restored after the DC bus voltage reached the nominal value. Casagrande et al. proposed a “distributed optimisation” based “model predictive control (MPC)” algorithm to ensure maximum RES generation utilisation, minimise load curtailment, and minimise degradation of BESS even in the face of line, distribution grid, and communication network faults [34]. Jiang et al. proposed a Lagrange multiplier-based adaptive droop control strategy to minimise the line and converter loss in a DC μ G with multiple DG converters connected in parallel [35]. The objective was modelled as a convex function, and Karush-Kuhn-Tucker (KKT) conditions were employed to ascertain the optimal Lagrange multipliers. The proposed control was implemented at the tertiary level to change the voltage reference setpoint of the secondary controller. Carvalho et al. suggested a distributed price-based EMS for direct current (DC) nanogrids [36]. The scheme proposed by the authors addressed the challenges posed by droop controls and state machine strategies. It utilised DC bus signalling and price theory to regulate and maintain the DC bus voltage.

Li et al. developed an EMS for a data centre DC μ G, optimising supply and demand coordination with an ϵ -constraint approach considering multiple criteria [37]. Also, the authors used the “Balanced Decision-Making” method to determine the optimal schedul-

ing solution. Grisales-Noreña et al. proposed a framework for optimal operation of BESS for both standalone and grid-connected DC μ G [38]. The authors considered three objectives: minimising losses, OC, and emissions, considering SPG and BESS. The optimisation problem was solved using parallel metaheuristic optimisation techniques, i.e., parallel PSO, parallel vortex search algorithm (PVSA), and parallel ant lion optimizer (PALO). Bhattar and Chaudhari proposed a centralised EMS for a solar-BESS DC μ G for optimal utilisation of SPG, BESS and minimisation of load curtailment using a linear programming technique [39]. A MO-EMS of a grid-connected DC μ G comprising SPG and BESS feeding non-interruptible and interruptible loads was solved using a combination of non-dominated sorting genetic algorithm-II (NSGA-II) and linear search method for simultaneous minimisation of OC and converter loss and maximisation of load expectation ratio [40]. Alam et al. proposed a fractional-order voltage compensation controller for an islanded DC μ G. The authors considered BESS, supercapacitor, DG, along with constant power load and a resistive load to reduce the voltage deviation [33]. Shen, Wang, and Zhu proposed a discrete time-based control methodology to obtain a trade-off solution between cost and loss minimisation along with voltage stabilisation in a DC μ G [41]. The authors used an adaptive step size method to accelerate the algorithm's convergence.

Further, power electronic (PE)-based devices such as soft open point (SOP) have gained traction in recent years and can be used for targeted improvements in power system operation. SOP is generally connected between two end nodes of feeders and can control active and reactive power in the AC power system. Many researchers have reported works on SOP for AC systems. The concept of DC soft open points (DCSOP) has only recently been proposed. Mudaliyar and Mishra [42] proposed a real-time-based control strategy using DCSOP for a DC distribution network comprising renewable sources and PHEVs. Ramadan and Kazakos [43] proposed using DCSOP in a DC distribution system. The objective was to improve the system reliability and voltage profile.

1.3.2 Studies on EMS of DC μ G considering input uncertainties

Although the above-mentioned papers dealt with different facets of DC μ G EMS, uncertainties and randomness of RES generation, load demand, and upstream power price were not considered in EMS formulations. However, uncertainties associated with RES generation and load demand should be incorporated in the day-ahead/hour-ahead scheduling

to hedge the risk of dispatch scheme in the face of RES generation and load demand randomness. Some studies dealing with AC systems, integrated energy system (IES), and energy communities have considered input uncertainties in the EMS. For instance, the researchers in [44] considered the optimal scheduling problem of an IES comprising RES, thermal power plant, combined heat and power units, BESS, and heat exchangers feeding electrical and heating load. The researchers coordinated thermal and electrical DR and used a “Stackelberg game strategy” with chance constrained programming (CCP) to solve the problem. Li et al. [45] considered a community-integrated energy system with electric vehicles (EV) charging stations and proposed a two-stage dynamic pricing scheme to optimise the OC for the energy system and EV charging stations. To achieve optimal performance, the authors coordinated the flexibility of shiftable and interruptible electrical loads, heating DR, and EVs. Uncertainties were modelled in the probabilistic domain, and a CCP approach was used. Gholami, Karimi, and Rastgou proposed a fuzzy risk-based EMS to optimise the scheduling of a SPG-rich interconnected μ G comprising SPG, BESS, and diesel generators [46]. The objectives were to reduce the OC and improve the voltage stability of the network. The DVs were the charge/discharge schedule of the BESS, electricity generation by the diesel generators, and electricity procured from the upstream grid. The problem was solved using the heuristic Teaching Learning-based optimisation algorithm. Uncertainty scenarios were reduced using the K-means approach. The authors also explored the impact of “risk-seeker”, “risk-neutral”, and “risk-averse” strategies on the OC. The “risk-averse” strategy has the highest OC but a more stable system. On the other hand, the “risk-seeker” strategy has lower OC but a comparatively less stable system. The “risk-neutral” strategy lies between the “risk-averse” and “risk-seeker” strategies. Rastgou and Hemati formulated an integrated planning methodology for medium and low-voltage networks as a bi-level optimisation problem in [47]. The authors optimised the distribution transformer’s location and size in the upper-level problem and conveyed the information to the lower-level problem. The feasibility of satisfying all constraints was checked in the lower-level problem. If the constraints were not satisfied, the upper-level problem was re-optimised until a consensus was reached between the upper and lower-level problems. The authors employed the “GA” to solve the complex bi-level problem, considering uncertainties of RES generation and load demand. Marasciuolo et al. proposed a chance-constrained approach to provide reserve services from PHEV charg-

ing stations within an energy community [48]. The study aimed to minimise OC while addressing prosumer objectives, employing the alternating direction method of multipliers (ADMM). The model considered various components, including BESS, PHEVs, and SPG. Ghasemnejad et al. proposed a robust optimisation model employing flexibility constraints to limit the ramp rate of power trading between citizen energy communities and the upstream grid [49]. The objective was to minimise energy costs and prosumer discomfort. Energy price uncertainties were incorporated into the model. However, the above papers did not deal with DC μ G architecture.

Only a few studies on DC μ G EMS have considered input uncertainties in the scheduling strategy. Some researchers used forecasting techniques to predict renewable generation and load demand. Pascual et al. [50] considered a grid-connected residential DC μ G comprising SPG, small WPG, and BESS and proposed a SOC based “moving average” control strategy to smoothen the power injection profile at the point of coupling between the DC μ G and the main grid. The “persistence forecasting” approach was used for load demand, while the “numerical weather prediction” model was used for forecasting the renewable generation. Gil-González et al. proposed a semi-definite programming-based optimisation approach for optimal dispatch of DC μ G comprising WPG, SPG, and BESS [51]. The objective was to minimise the cost of energy procured from the grid. Gil-Gonzales et al. [52] formulated a convex economic-dispatch model for a DC μ G comprising dispatchable and non-dispatchable units, flexible loads, and BESS using the “second order cone programming” technique. The objective was to minimise the energy procurement cost in a day. The load and renewable generation forecasts were made using receding horizon control and an artificial neural network. Lian et al. proposed a MO optimisation framework for islanded data centre DC μ G [53]. The objective was to optimise wind power curtailment and reduce the OC. A robust MO optimisation model was used to handle uncertainties of WPG generation and load demand.

Some researchers have modelled input uncertainties using probabilistic frameworks. For instance, the researchers in [54] proposed a flexibility-constrained MO smart home EMS. SPG uncertainty was incorporated using scenarios. The three objectives were to minimise payment costs, maximise end-user satisfaction, and minimise energy exchange. Dorahaki et al. proposed a MO model for a community energy system incorporating a sharing economy to minimise energy expenditure and end-user discomfort [55]. The

model incorporated combined heat and power, thermal storage systems, and ESS, accessible by end-users based on ownership rates. SPG uncertainty was modelled using a scenario-based approach. End-user preferences were incorporated into the thermal comfort model. The authors in [56] proposed a two-stage risk-based optimisation model for energy management of energy communities. In the first stage, the energy community managers of citizen energy communities and renewable energy communities solved the problem and communicated the results to the Smart Multi-Carrier Energy Network operator. The Smart Multi-Carrier Energy Network operator maximised profits in the second stage while considering Smart Multi-Carrier Energy Network constraints. Energy demand and generation uncertainties were incorporated using a scenario-based approach. Dorahaki et al. proposed a two-stage optimisation framework for transactive networked multi-carrier energy systems [57]. The first stage dealt with the islanding operation, in which each multi-carrier energy system minimised its OC and communicated the results to the central operator. The central operator minimised the total OC of the transactive networked multi-carrier energy system, considering energy transactions and fairness constraints in the second stage. Renewable (solar and wind) uncertainties were modelled using a scenario-based approach. Gholami et al. used the reactive power injection capability of inverters of EV charging stations to improve the voltage stability of an AC distribution network comprising WPGs and diesel generators [58]. Uncertainties of RES generation, electricity demand, and EV charging demand were modelled in the probabilistic domain using “Hong’s 2m point estimate method”. The authors in [59] proposed a method to enhance the flexibility of operation of an AC system with RES generation uncertainty by coordinating BESS and thermal storage. Uncertainties were modelled in the probabilistic domain and represented by “probability serialisation”. A CCP approach was used in this paper. However, the above papers did not deal with the DC μ G architecture.

Only a few papers have dealt with uncertainty modelling of input random variable (RV) in the probabilistic framework for the DC μ G architecture. Cingoz et al. [60] used the PSO algorithm to find optimal droop settings of controllable sources in a DC μ G for minimising the DC bus voltage and current sharing accuracy degradation. The authors used “normal distribution” and the scenario-based approach to model the uncertainty of load demand. The authors in [61] simultaneously minimised the OC and emissions in a grid-connected DC μ G comprising dispatchable and non-dispatchable (SPG) DG units

and loads using PSO, wherein uncertainties of RES generation and load were incorporated by probabilistic “Hong’s 2m point estimate method”. Shuai et al. [62] used approximate dynamic programming to solve the day-ahead and intra-day economic dispatch problem of a DC μ G. Uncertainties related to renewable generation, load demand, and price were modelled using the Monte Carlo Simulation (MCS) approach. Hajiamoosha et al. proposed an EMS for a grid-connected μ G to minimise OCs and emissions. The μ G consisted of combined heat and power units, WPG, SPG, BESS, tidal stream turbines, BESS, and thermal energy storage [63]. Uncertainties of RES generation, demand, and electricity price were modelled using “Monte Carlo Simulations”. The bi-objective problem was solved using the “ ϵ -constraint method”, and the optimal solution was selected from the Pareto Front using an interactive fuzzy decision-making technique.

Reddy, Jithendranath, and Chakraborty [64] proposed a MO optimisation technique for an islanded DC μ G comprising dispatchable and non-dispatchable generation sources and BESS. The objectives were to minimise the OC, improve the voltage profile and improve current sharing. The authors used the probabilistic scenario-based analytical method to model load demand and renewable generation uncertainties. The MO problem was solved using the dragonfly algorithm. The authors in [65] developed a behavioural home EMS model based on time-driven prospect theory, incorporating ESS, distributed energy resources, and smart, flexible home appliances. Non-flexible load and SPG uncertainties were incorporated using scenario-based modelling. The problem was formulated as a MILP. Reddy et al. reported a multi-objective dragonfly algorithm (MODA)-based solution approach to simultaneously minimise the OC and emissions and maximise the small-signal stability margin of a DC μ G [66]. The input uncertainties were modelled in the probabilistic framework in [66].

The interdependence among the input RVs also impacts the optimal operating point of the network. However, in none of the above papers the authors modelled the correlation between the input RVs. Some researchers have modelled correlation among input RVs for EMS schemes of an AC μ G. For instance, the following papers have dealt with correlated input RVs in the EMS of an AC μ G. Chen et al. [67] proposed a probabilistic load flow method using a combination of Zhao’s Point estimate method (PEM) and Nataf Transformation (NT) for an AC μ G. Uncertainties related to WPG and load were considered. However, PHEV charging/discharging load uncertainties were not considered. Shargh et

al. [68] proposed a probabilistic MO optimal power flow method for an AC distribution network considering the correlation between input RVs. Uncertainties related to WPG and load demand were considered. Jithendranath, Das, and Guerrero [69] proposed a probabilistic-optimal power flow solution for AC μ G. Uncertainty related to renewable generation, load, and price were considered. Also, the correlation between input RVs was considered. However, AC systems were considered in [67, 68, 69].

Limited research has been done to model the correlation between input RVs for the EMS of a DC μ G. Recently, Reddy et al. [70] proposed an optimal operational strategy for an islanded DC μ G. Different heuristic techniques were compared to obtain the droop parameter for islanded DC μ G. The authors considered the uncertainties related to renewable generation and load, and the objective was to reduce OCs. Also, the correlation between SPG and load was considered to assess the robustness of the proposed method. However, the authors did not consider uncertainties related to PHEV. Reddy et al. used fuzzified “dragonfly algorithm” to find the optimal droop parameters of a droop-controlled DC μ G in MO optimisation problem [70]. The objectives were simultaneously minimising OC and improving the current sharing and voltage stability. The uncertainties of input RVs were modelled using an “Inverse NT” and incorporated into a “Gauss Quadrature Point Estimate Method” algorithm. Reddy et al. proposed a strategy for optimal droop-controlled standalone DC μ G operation [66]. The authors used a probabilistic approach to address load and SPG uncertainties. They solved an MO problem to improve small-signal stability, minimise OCs, and reduce emissions using a modified “dragonfly” algorithm.

1.3.3 Studies on Electricity-Hydrogen sectoral coupling

The global interest in carbon-free fuels is on the rise. Hydrogen (H_2) is emerging as a popular clean fuel. In particular, H_2 produced through water electrolysis systems, also known as electrolyser or power to hydrogen (P2H) units, has gained significant attention as a decarbonised energy carrier and a viable, clean fuel option [71]. Also, the H_2 storage system (HSS), having high energy density, possesses long-term, large-scale energy storage potential at a reduced physical size, provides arbitrage benefit, and maintains abundant energy backup during emergency conditions, thereby enhancing the resilience of system operation [72, 73]. Therefore, an integrated electricity- H_2 system, along with hybrid energy storage system (HESS) comprising HSS (high energy density and longer autonomy

at a reduced physical size) and BESS (high power density with fast transient response), has the potential to decarbonise the energy system, reduce RES curtailment, provide RES generation firming services, and offer flexibility with economic competitiveness to support resilient and islanded operation [74]. Being a controllable load, the P2H unit can provide demand-side flexibility to the DC μ GO. The green H_2 produced by RES generation can be converted back to electricity using a hydrogen to power (H2P) unit and injected into the power network during periods of low RES generation or high energy price in the energy market. Optimal H_2 production using RES power with HSS and BESS reduces the OC [75]. Therefore, several investigations have been reported on the electricity- H_2 nexus of late.

Rabiee et al. reported that the incorporation of P2H and HSS improves the flexibility of a power system and can contribute to greater security of the system in the event of line outages [76]. However, the study was carried out on an AC transmission level system. Huang et al. proposed an economic model predictive control (EMPC) framework to minimise the OC of an μ G comprising FC, HSS, BESS, and WPG [77]. The authors considered a constant efficiency model of Alkaline electrolyzer (AE) in the paper. Huang et al. proposed an EMPC strategy for an IES encompassing multiple energy vectors like electricity, H_2 , and heat [78]. The objective was to minimise the OC by controlling H2P, P2H, HSS, BESS, and combined heat and power unit using MPC. Satloo et al. proposed a fast ADMM-based solution for minimising the OC of multi- μ Gs comprising both electricity and H_2 generation facilities in a local energy market [79]. The uncertainty of input RVs was modelled in a hybrid stochastic-robust-information gap decision theory framework. Li et al. considered a coupled transportation-electricity system model comprising BESS, P2H, H2P, HSS, and PHEV parking lots and applied MPC technique to minimise OC of net zero buildings [80]. MPC was also used to minimise the operation and maintenance cost of a μ G comprising RES, BESS, PHEVs, P2H and H2P units, and HSS [81]. The above studies on electricity- H_2 sectoral coupling either dealt with IES or AC systems, in which either the power system network model and operating constraints were not considered, or AC power system models were used [76, 77, 78, 79, 80, 81].

Li et al. proposed a framework for the economic load dispatch problem for a grid-connected DC μ G comprising a WPG, SPG, FC, BESS, and loads. [82]. The virtual resistance values of the BESS, FC, and the grid-interlinking converter (GIC) were taken

as the DVss, which were optimised using GA to minimise the total OCs of the DC μ G system. Han et al. [83] proposed a two-level EMS for an islanded DC μ G consisting of SPG, BESS and FC. The objectives were to minimise H_2 consumption while maintaining system stability and the DC bus voltage. The upper-level/system-level control determined the distribution of the power demand among the SPG, BESS, and FC. The settings of the FC were determined based on the SOC of the BESS and the load-SPG generation mismatch. The lower-level controller handled individual units. The authors also compared the performance of classical proportional integral (PI) control and the “state machine control strategy” for the EMS in this paper. Han et al. [84] reported an optimal component sizing strategy for a DC μ G comprising SPG, BESS, FC, electrolyser, hydrogen tank, PHEV and building loads. The component optimisation was carried out using the HOMER software. The authors also suggested optimal local (sliding mode) and system-level controllers for enhancing the system economy by distributing the power demand among the FC and BESS units. The controller design was validated using hardware-in-the-loop simulations. Han et al. proposed a two-level EMS for a SPG-FC-BESS-based DC μ G [85]. The authors defined two control levels in the paper: the device and system control levels. Droop control and maximum power point tracking (MPPT)-droop dual-mode control were employed at the device control level to improve reliability. At the system control level, BESS and FC were coordinated to reduce consumption. Further, HSS was used to improve the DC μ G stability under high SPG penetration. Most of the above investigations on coupled electricity- H_2 systems pertain to integrated energy hubs. The investigation reported in [85] dealt with a DC μ G. However, the authors did not consider the DC μ G network model and network operating constraints in the EMS formulation.

Vivas, Segura, and Andujar proposed an EMS for a grid-tied residential DC μ G comprising BESS and FC [86]. The MO problem was formulated using the fuzzy logic approach to minimise the OC and losses. Ferahtia et al. [87] proposed an EMS for a grid-connected DC μ G comprising building loads, SPG, BESS, and FCs to minimise the fuel cost while maintaining power quality and efficiency using the “salp swarm optimisation”. Hafsi et al. proposed an EMS for an isolated DC μ G comprising SPG, WPG, and FC using two optimisation strategies, namely, “equivalent consumption minimisation strategy”, and “external energy maximisation strategy EEMS” [88]. The work aimed to coordinate all units efficiently, enforce DC bus stability, and minimise the H_2 consump-

tion. Performance comparison of classical PI controllers, state machine strategy, fuzzy logic controller and artificial neural network-based controller was reported. Studies reported in [84] and [88] focused on controlling different flexible resources within the DC μ G to ensure a stable operation with improved dynamic response rather than on improving the operating economy. Also, DR programs, the impact of HSS on the operating economy, and DC μ G network operating constraints were not included in the studies. Fathy et al. [89] considered a DC μ G comprising FC, SPG, and BESS. Both grid-connected and island modes of operation were considered. The authors used a “Mamdani fuzzy logic control”-based EMS to determine optimal setpoints of the GIC, BESS, and the FC to maximise the power saving. The “fuzzy logic controller” was tuned using multiple metaheuristic algorithms.

Gugulothu, Nagu, and Pullaguram proposed an EMS for a standalone DC μ G comprising SPG, BESS, and FC [90]. The authors proposed a de-rating strategy for the SPG to overcome the deep charging issues of the BESS. Further, H_2 and oxygen mixture are controlled to enhance the FC efficiency. The objective was to reduce the fuel consumption of FC using a rule-based strategy. Alharbi et al. proposed an EMS for a DC μ G comprising BESS, FC, SPG, and supercapacitors. The objective was to reduce fuel consumption while improving the overall system efficiency using “Beluga Whale optimisation” [91]. The studies in [90] and [91] mainly dealt with controlling PE converters to achieve stable operation with good dynamic and steady-state performance rather than achieving an operating economy. Also, DR participation, the effect of HSS on operating economy, and network operating constraints were not envisaged in the study.

Most papers considering electricity- H_2 sectoral coupling with a DC μ G architecture have neglected the DC μ G network model and network operating constraints. In other words, the impact of the electricity- H_2 scheduling strategy on the DC μ G network constraints (feeder current, bus voltage, etc.) was not explicitly addressed in the above papers.

1.3.4 Studies on participation in frequency reserves market

Participation of an IES or a μ G in the frequency reserves market provides an additional revenue stream. Therefore, recently, some researchers have proposed using flexible resources like BESS, FC, AE, HSS, etc., belonging to a μ G or a virtual power plant to

provide ancillary automatic frequency restoration reserve (aFRR) services in the energy reserve market. In the aFRR services, flexible resources provide a part of the available capacity of active power to the upstream grid when the μ G is operating in grid-connected mode, thus creating an additional source of income for the μ G operator. Seyfi et al. proposed a scenario-based EMS for a combined cooling, heating, and power-based virtual energy hub with participation in both electricity and thermal energy and reserve markets to maintain the power balance [92]. Hemmanti et al. proposed a thermodynamics-based air energy storage system integrated with RES to participate in reserve and energy markets [93]. Wu et al. proposed an optimal offering strategy using a multi-stage stochastic programming model for the hydrogen fueling stations, considering reserve and energy markets [94]. Schlachter et al. proposed an optimisation strategy for capacity and operational set point determination. Authors considered BESS along with the power to heat module to participate in the frequency containment reserve (FCR) market for profit maximisation [95]. Kempitiya et al. proposed different bidding optimisation strategies using artificial intelligence for multi-stage frequency reserve markets. The author aimed to consider all the markets instead of sequential consideration of day-ahead markets to ensure higher profit [96]. Gonzalez et al. proposed a methodology for the combined optimisation of sizing and power management for SPG-based household prosumers. The system included PHEVs, household loads, BESS, and power converters. The authors considered multi-market participation, including day-ahead and aFRR markets, to enhance profitability and improve overall system efficiency [97]. Nitsch et al. proposed an agent-based electricity market model to harness the profitability from both energy and aFRR market participation [98]. Xie et al. proposed a robust MPC-based bidding strategy for wind-storage systems through real-time participation in the energy and aFRR market. The author aimed to leverage the arbitrage benefit provided by the BESS and aFRR market participation to improve the profit. Further, a robust optimisation approach was used to handle the uncertainties related to WPG and grid power price [99]. Khojasteh, Faria, and Vale proposed a linear optimal scheduling model for synchronous generators and BESS considering energy and aFRR market participation. The objective was to improve the system frequency and security by reducing the dependency on BESS to improve the system inertia [100]. Zeng, Huang, and Zong proposed a bidding framework for P2H system considering aFRR market participation. The authors aimed to compensate for

the higher production cost of H_2 through aFRR market participation [101]. Seo et al. proposed an EMS for the German balancing market. The author aimed to improve system stability and profit with an increase in PHEV penetration through energy and aFRR market participation [102].

However, the above works focused on energy hubs where network operating constraints of a power system were not considered. More specifically, to the author's knowledge, no reported work has explored DC μ G participation in the aFRR market. The DC μ G is interfaced with the upstream AC grid through a GIC. Therefore, during the grid-connected operation of the DC μ G, the DC μ GO can leverage the flexible resources within the DC μ G not only to meet the internal demand of the DC μ G but also to participate in the aFRR market by offering available active power capacity [103]. The power exchange between the DC μ G and the upstream AC grid, and hence the participation of the DC μ G in the aFRR market, occurs through the GIC. Such an operation will create an additional source of income for the DC μ GO and make the operation financially attractive. Therefore, incorporating aFRR participation capability in the EMS of a DC μ G is an attractive option to enhance the operating profit of the DC μ GO.

1.3.5 Studies on DC μ G EMS with islanding constraints

Apart from economic considerations, it is desirable that a DC μ G possesses islanding capability to sustain operation with minimum load curtailment after an unintended islanding event. The islanding capability can be incorporated by including additional islanding constraints in the EMS algorithm, which is under-explored for the DC μ G. Zia, Elbouchikhi, and Benbouzid [104] considered a grid-connected DC μ G comprising SPG, BESS, flexible and non-flexible loads and minimised the OC [104]. The authors considered bus voltages, BESS setpoints and the incentive-based DR participation as optimisation variables. The authors used regression-based models to assess the impact of temperature and depth of discharge (DOD) on the lifecycle and energy capacity degradation of a Li-Ion BESS. The degradation cost and an incentive-based DR program were also incorporated into the optimisation model. The authors solved the non-linear programming model using the primal-dual interior point algorithm using the GEKKO package in Python. In the mentioned work, the DR participants were modelled as flexible electrical load demand without considering the operating constraints of individual participating devices. Islanding con-

straints were also considered, but the authors considered that the schedule of islanding is predefined. Therefore, an incentive-based DR scheme was implemented to shift flexible loads from scheduled islanding to grid-connected operation periods.

1.3.6 Studies on resilient operation

In the resilient operation, the most crucial aspect is the “proactive preparation” period in which the system prepares itself for the upcoming emergencies [12]. Therefore, a suitable formulation methodology is needed to provide adaptable working ranges so that “proactive preparation” period can be incorporated. Hence, EMPC strategies have emerged as a reliable problem formulation framework where the control problem can be formulated as an optimisation problem over a finite time horizon. EMPC can incorporate logical or integer DVss and constraints such as varying working ranges and time delays [77, 78, 105, 106, 107, 108]. In [12], authors explored the resilient aspect of using BESS and HSS in a μ G. The problem was formulated considering two periods, namely “normal period” and “emergency period” using MPC. During the “normal period”, the objective was to reduce the OC, and during the “emergency period”, the objective was to reduce the shedding of electrical and H_2 loads. Another study by Tobajas et al. used MPC to ascertain optimal coordination between BESS, HSS, P2H, H2P, flexible loads, RES curtailment, and power exchange with an upstream grid to achieve resilience-oriented economic operation of a μ G [74]. In [105], authors framed the economic problem of scheduling HSS and WPG based energy system in EMPC framework. Similarly, [106] proposed an optimal control methodology for renewable energy μ Gs using EMPC. Acevedo-Arenas et al. proposed an EMPC based methodology for optimal dispatch of RES and DR in a grid-tied hybrid system [107]. In [109], the authors considered the normal and emergency operation of the system. In the normal operation, BESS and generators were coordinated to meet the demand. In the emergency operation, BESS discharging and load shedding were controlled to meet the critical load demand. In [108], the authors proposed an EMPC optimisation methodology for sizing the components of a standalone hybrid energy system. The above studies explored resilient operations in AC systems or IES.

Only a few studies have focused on the resilient operation of a DC μ G. For instance, Che and Shahidehpour proposed a hierarchical control strategy comprising primary, secondary, and tertiary control loops for the economic and resilient operation of a DC μ G

[110]. After unintended islanding, the loads were curtailed to compensate for the loss of the main grid power. After that, the natural gas turbine ramped up its generation to restore lost loads. However, the authors did not consider the electricity- H_2 sectoral coupling, RES generation uncertainties, DC μ G network constraints, and criticality levels of lost loads in the above study. Saha et al. proposed a sliding-mode observer-based approach to estimate the error due to sensor fault or cyber-attack for the resilient operation of a DC μ G [111]. Shahid et al. proposed a “distributed consensus-based control” scheme consisting of communication links between several DC μ Gs to enhance the resilience of the DC μ G operation in the face of “information islanding” [112]. The authors in [113] proposed a framework to assess the resilience and “fault ride-through capability” of a DC μ G and developed scalable resilience guarantees during a time-varying unknown “denial of service attack (DoSA)”. Cheng and Chow proposed a “resilient collaborative distributed” consensus-based EMS for a DC μ G to circumvent the impact of colluding and non-colluding false data injection attack (FDIA) [114]. A primal-dual subgradient algorithm was used to derive the consensus criteria for economic operation, and attack identification was modelled using a majority voting technique. However, the authors did not consider a detailed network model of the DC μ G in the problem formulation.

The researchers in [115] proposed a resilient control strategy using “adaptive discord element” to detect cyber-attacks on sensors and cyber-link and mitigation by an event-driven approach. Hu et al. proposed an “event-triggered” controller to detect and mitigate the impacts of intermittent DoSA in a DC μ G serving constant power loads [116]. Jena, Padhy, and Guerrero proposed a cyber-attack detection approach based on disagreement Laplacian potential in an interconnected DC μ G cluster to achieve fault-ride through operation during a cyber attack, thereby enhancing the resilience of the DC μ G cluster [117]. Liu et al. reported a resilient controller design for a DC μ G for current sharing and voltage restoration to mitigate impacts of FDIA and DoSA [118]. The authors in [119] proposed a “diverging factor-based” algorithm to detect man-in-the-middle attack (MITM) attacks and designed a “multi-layer resilient controller” to mitigate the impacts and ensure stability of DC μ G operation. Ni et al. proposed a MPC-based control strategy for HESS comprising BESS and superconducting magnetic energy storage to ensure resilient operation for maintaining DC link voltage and power allocation among HESS elements in the face of load and RES generation fluctuation and accidental disconnection

from the main grid [120]. Using a residual-capacity-based algorithm, the authors allocated the power-sharing responsibility among the HESS elements. Lian et al. reported a resilient distributed secondary control scheme to ensure stability, voltage restoration, and current sharing for a DC μ G considering DoSA in the distributed secondary control layer [121]. The authors used a “resilient sampling mechanism” for the secondary control layer and presented theoretical analysis for controller parameter selection. Sadabadi et al. proposed a graph-theoretical and Lyapunov-based framework resilient distributed control scheme for maintaining stability, voltage restoration, and proportional current-sharing in a DC μ G during an unknown cyber-attack [122]. Chen et al. introduced an average consensus-based distributed control scheme with PI controllers by casting the average voltage restoration problem as an optimisation problem to improve the resilience of DC μ G operation during a DoSA [123]. The authors in [124] proposed a “gated recurrent unit”-based scheme to enhance the resilience of a DC μ G in the face of “FDIA”. The scheme proposed in [124] estimated the actual bus voltages using an “offline-trained” “gated recurrent unit” followed by mitigation of the cyber-attack using a combination of the “gated recurrent unit” and PI controllers.

However, all the above studies on the resilient operation of a DC μ G dealt with the primary and secondary-level control schemes to maintain DC link voltage stability, voltage restoration, and current sharing during cyber-attacks. The above studies did not address tertiary level control focusing on resilience-oriented optimal scheduling of a DC μ G for maintaining the uninterrupted power supply to critical loads and minimise curtailment of non-critical loads after the loss of the main grid following a natural disaster like a hurricane, windstorm, etc., considering DC μ G network model and network operating constraints [111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124].

1.4 Research gaps and motivation

The following research gaps are identified from the literature survey:

- Most researchers have not modelled the correlation between input RVs while formulating the EMS of a DC μ G.
- Most Inverse NT based approaches for modelling correlation between input RVs used empirical relations to map correlation coefficients between the marginal distributions

to the normal space. However, the empirical formulae are defined for only a few marginal distributions. Therefore, the above technique can be applied if the input RVs are modelled using marginal distributions for which empirical formulae have been defined. For instance, no empirical relationship is defined to map correlations between load (modelled using a normal distribution) and wind speed (modelled by a Weibull distribution). This limitation restricts the Inverse-NT-based approach to a few probability density functions (pdfs).

- The performance of a DC μ G can be best optimised with proper coordination between power procurements from RES, implementation of DR program, scheduling of BESS and DCSOPs. To the author’s knowledge, no reported EMS scheme on DC μ G has encompassed all the above flexibility resources.
- Most studies involving DR in a DC μ G considered aggregated models of DR participants without dealing with equipment-level constraints. However, an aggregated model of flexible loads cannot accurately account for consumer preferences and requirements like thermal comfort (e.g., the indoor temperature of a room cooled by a TCL/air-conditioner (air con)), time-dependent energy requirement of a PHEV participating in the DR program, depth of charge (DOC) and DOD constraints of PHEV batteries, etc. Hence, apparatus-level models of flexible loads participating in the DR program should be considered for more accurate and practical representation.
- The DR can be implemented either using direct load control (DLC) or by guiding the consumption pattern of the flexible consumers by setting incentive or price-based (retail power price (RPP)) schemes. The DLC scheme is often not preferred since it compromises consumer privacy [125, 126]. The DR scheme should preferably be privacy-preserving with minimum information exchange between the DC μ GO and the flexible consumers. Most studies used incentive-based schemes with pre-decided incentive values without addressing the theoretical basis for setting the incentive price of DR participation in a DC μ G network. Similarly, to the author’s knowledge, the theoretical basis for setting the RPP for implementing a DR scheme considering the DC μ G network model and constraints has not been explicitly reported. Therefore, a theoretical basis for setting the RPP considering DC μ G network model

and associated constraints for DR implementation in a decentralised framework (for privacy preservation) needs investigation.

- While several studies on the EMS of coupled electricity- H_2 systems have been carried out, most studies pertained to AC networks or IESs, where either AC network model and constraints were considered, or network model and constraints (line ratings, bus voltage limits, etc.) were not considered at all. Therefore, the above studies on multi-energy vector coordination cannot be directly extended to an DC μ G network. In other words, a separate comprehensive investigation on the EMS for a coupled electricity- H_2 DC μ G considering DC μ G network model and network constraints is required not only to assess the impact of the sectoral coupling on operating economy but also to guarantee that network operating constraints like line ratings and bus voltage limits are not violated.
- To the author's knowledge, most researchers have not considered islanding constraints with uncertain islanding instant in the scheduling model of a DC μ G. Under normal operating conditions, a DC μ G operates in grid-connected mode. The DC μ G switches to island operation when there is a disturbance in the upstream grid. The unintended islanding instant is uncertain and not known a priori. In other words, the islanding can occur at any time within the scheduling horizon. The line flows, bus voltages, generator, and storage system outputs in island mode will deviate from the pre-islanding values. However, no network and equipment constraints should be violated after unintended islanding for a secure and reliable operation. Moreover, the existing storage units and available generation within the DC μ G should continue to supply the DC μ G load with minimum load curtailment after islanding. The same can be guaranteed by formulating and incorporating islanding constraints in the EMS of a grid-connected DC μ G. However, to the author's knowledge, this aspect has not received sufficient attention so far.
- To the best of the author's knowledge, investigation of interaction and interdependence between electricity and H_2 systems with proper co-optimal scheduling for participation in the aFRR market has not been reported for a DC μ G system considering DC μ G network model and network operating constraints.
- Most studies on resilient operation of DC μ Gs focused on cyber-security aspects and

not on energy scheduling of DC μ Gs for uninterrupted electricity supply to critical loads and minimisation of curtailment of non-critical loads after islanding due to an extreme event. On the other hand, studies dealing with resilience-oriented EMS pertain to either AC networks or integrated energy hubs, in which either AC power system models or lumped power system models (network equations and network operating security constraints ignored) have been considered. Therefore, the above-mentioned resilience-oriented scheduling schemes cannot be extended directly to DC μ Gs because of the risk of violation of DC μ G network operating security constraints (line current and bus voltage limits).

The motivation of this thesis is to extend the reported threads of work on DC μ G EMS and bridge the identified gaps.

1.5 Thesis Organisation

The thesis is structured into six chapters, as detailed below:

- **Chapter-1:** Chapter 1 provides a brief overview, the literature review, and identification of the research gaps.
- **Chapter-2:** This chapter deals with a MO day-ahead EMS for a DC μ G by leveraging different flexibilities available to the DC μ GO. Uncertainties of input RVs are modelled using a probabilistic PEM approach. The correlation between the RVs is incorporated using a Newton's Interpolation method embedded inverse NT technique.
- **Chapter-3:** In Chapter 2, a centralised EMS was proposed, leveraging incentive-based DR schemes with predetermined incentive values. In Chapter 3, a decentralised EMS with a RPP-based DR implementation is proposed for an electricity- H_2 DC μ G. Also, the theoretical basis for setting the RPP is derived using the concept of competitive equilibrium, considering operating and network constraints.
- **Chapter-4:** The challenges related to unintended islanding events and their associated constraints remain unexplored in Chapters 2 and 3. The objective of Chapter 4 is to develop an EMS for an electricity- H_2 grid-connected DC μ G with a HESS

incorporating islanding constraints and DR implementation in an uncertain environment with correlated and uncorrelated input uncertainties to maximise the profit of the DC μ GO, minimise the electricity usage cost of consumers, and ensure secure operation after unintended islanding using leader-follower Stackelberg game.

- **Chapter-5:** Chapter 5 proposes a resilience-oriented EMS for an electricity- H_2 DC μ G considering participation of the DC μ G in the aFRR market.
- **Chapter-6:** Chapter 6 concludes the thesis by summarising the key findings and providing a brief overview of potential future research.

1.6 Assumptions

- In practical scenarios, generating pdf involves preprocessing real-world data, often containing noise, outliers, or missing values. To address this, techniques such as kernel density estimation, parametric distribution fitting, and non-parametric methods are employed to approximate the underlying distributions. In Chapter 2 of this thesis, it is assumed that the DC μ GO receives pdf after such preprocessing. In Chapters 3 to 5, the raw data are analysed for distribution characteristics, such as skewness, kurtosis, and outlier behaviour, to determine the most appropriate best-fit pdf for uncertainty representation.
- In this thesis, it is assumed that sufficient historical data is available to construct pdf, enabling the use of a probabilistic approach for uncertainty modelling. This choice allows for a more detailed and realistic representation of system behaviour, as it captures the likelihood of different outcomes based on actual data patterns. While other methods like interval or affine arithmetic are commonly used when data is scarce, this work focuses on leveraging the richness of available data to quantify uncertainty through well-fitted pdf, supporting more accurate and informed decision-making.
- In this thesis, it is assumed that the marginal probability distributions of the input random variables are known. Based on this assumption, the Nataf transformation is adopted in Chapter 2 as it effectively models dependencies among correlated

variables while requiring only the marginals, offering a practical balance between accuracy and computational efficiency. Although other approaches like the maximum entropy principle or polynomial chaos expansion (PCE) exist, they were not pursued in this thesis. While maximum entropy-based transformations are more general and do not require marginals, they rely on assumptions about moment constraints and often involve iterative numerical procedures [127]. Similarly, polynomial chaos expansion (PCE) based methods offer powerful uncertainty propagation capabilities but may suffer from convergence issues in high-dimensional problems or when the input space is poorly characterised [128].

- In this thesis, the Stackelberg game framework is developed in Chapter 4 under the assumptions of perfect rationality and complete information for both the leader and the followers. These assumptions ensure that the game is well-defined and analytically tractable, allowing for a clear formulation of strategies and outcomes. While such idealised conditions may not always reflect real-world complexities, they provide a structured foundation for the proposed model.
- The linearisation applied in this thesis is specifically targeted at the power flow equations, based on the assumption that bus voltages in the DC μ G remain within $\pm 10\%$ of their nominal values. This is a reasonable condition during normal operation and ensures that the approximation error introduced by linearisation remains negligible. As a result, the linearised model offers a practical and sufficiently accurate representation for most operating scenarios.