Chapter 4

A Non-Cooperative Game Analysis of Peer-to-Peer Energy Transactions Involving Multi-Communities

4.1 Introduction

The previous chapter dealt with P2P energy sharing among a group of buildings in close proximity involved in a cooperative game. This chapter proposes a novel framework for economic benefit through P2P trading among buildings at different geographical locations using a non-cooperative game. The proposed framework has been realised as P2P energy sharing between nearby buildings and between buildings located geographically far away to account for energy losses. A charge is also added for long-distance energy sharing. The proposed decentralized P2P energy-sharing framework uses the concept of virtual communities (VCs) to tackle the scalability issue. The number of transactions is reduced by grouping the buildings into VCs based on their geographical locations. A non-cooperative game is formulated and solved in a decentralised manner for the following (i) energy management of individual buildings, (ii) building-to-building (B2B) energy exchange, (iii) building-to-community (B2C) energy exchange, (iv) energy management of the respective VC, and (v) community-to-community (C2C) energy exchange. Load shifting is used to incorporate demand-side management. The proposed two-level P2P framework considers a realistic model of buildings that can endogenously decide their role (buyer or seller or both) to maximise their profits. Cloud computing-based algorithm is proposed for determining the energy profile and prices for each internal transaction (B2B, B2C, and C2C) separately. Cloud computing is used to avoid privacy/security issues normally arising in any data-centric framework. A shareable battery energy storage system (BESS) is also assumed to be present in each VC. The contribution of this chapter can be summarised as follows.

- Cloud computing-based decentralised P2P energy sharing framework is developed to reduce the energy cost of all buildings. The power exchanges and prices corresponding to the P2P transactions are determined in a decentralised way using the proposed algorithm based on a non-cooperative game.
- The proposed framework uses the concept of virtual communities connected through a computing cloud for P2P energy trading between nearby and geographically distant buildings. The network loss is incorporated in th prices by adding penalty charges based on inter-community distances.
- The proposed framework does not require the surplus or the deficit energy status of each participant in advance for P2P energy sharing. The roles of prosumers are decided endogenously.
- The data privacy of each prosumer is preserved by cloud computing-based decentralised framework without any need of prior information about their energy status.

4.2 Problem Formulation

4.2.1 System Model

As shown in Figure 4.1, several buildings are clustered based on geographical distances to form virtual communities (VCs). Each VC has a BESS shared with the buildings of that VC. Each VC comprising of a group of buildings and BESS is connected through a computing cloud. A VC in itself is a non-profit based entity; however, in relation to each other, each VC is competing with others for profit. The concept of VCs helps in taking advantage of P2P energy sharing among buildings with a less computational and communication burden. Also, a shareable BESS is easy to manage with a VC cloud. Each building in a VC consists of RESs, i.e. solar power generations (SPG) and wind power generations (WPG), flexible load, a local energy management system, and advanced metering infrastructure. The buildings are connected to the utility through the local distribution network. In addition, in each VC there is a cloud platform enabling secure information and computing for B2B and B2C energy and price sharing. Similarly, a secure platform is used for C2C energy and price sharing. The communication infrastructure is assumed to be robust in this model.

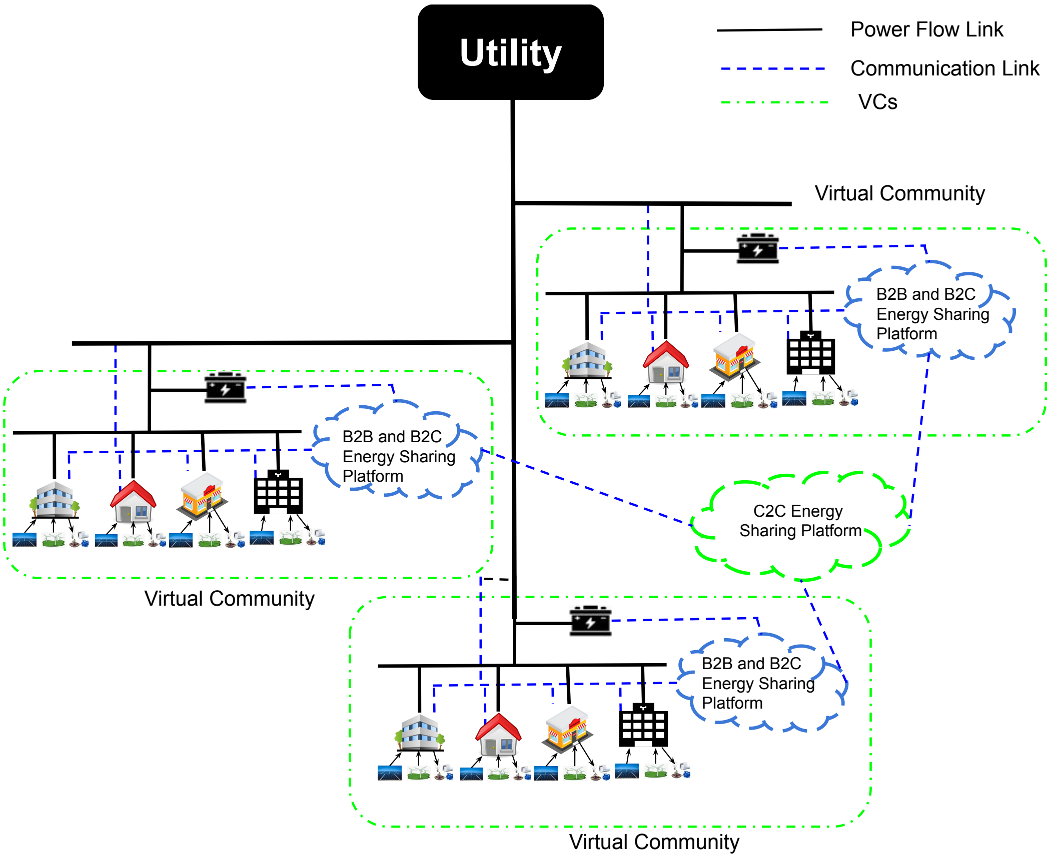


Figure 4.1: PEER-TO-PEER ENERGY SHARING FRAMEWORK

4.2.2 Cost Model of Buildings

The cost function for i_{th} building of n_{th} VC i.e., $C^b_{n,i}$, comprises the cost of power exchange with the utility ($C^U_{n,i,t}$), the cost of discomfort experienced by the consumer in shifting load ($C^{ls}_{n,i}$), cost of B2B energy sharing ($C^{b2b}_{n,i,t}$) and cost of B2C energy sharing ($C^{b2c}_{n,i,t}$). And $C^b_{n,i}$ can be expressed as,

$$C^b_{n,i} = \sum_{t=1}^{T}(C^U_{n,i,t} + C^{ls}_{n,i,t} + C^{b2b}_{n,i,t} + C^{b2c}_{n,i,t}) \tag{4.1}$$

4.2.2.1 Exchange with the Utility

Each i_{th} building of n_{th} VC can trade with the utility. The respective cost function corresponding to utility usage at t_{th} hour is given by

$$C^{U}_{n,i,t} = (\Lambda^{im}_{t} + \epsilon^{CE})P^{b}_{n,i,t} - \Lambda^{ex}_{t}P^{s}_{n,i,t}. \tag{4.2}$$

4.2.2.2 Load Shifting

The consumer's discomfort due to load shifting [66] can be expressed as,

$$C^{ls}_{n,i} = \Lambda^{dis}_{t}(P^{ls}_{n,i,t} - L_{n,i,t})^2, \tag{4.3}$$

which is added as an additional cost (penalty) to the cost function. The quantum of load shift is constrained as follows.

$$L^{min}_{n,i,t} \leq P^{ls}_{n,i,t} \leq L^{max}_{n,i,t}, \qquad and \tag{4.4}$$

$$\sum_{t=1}^{T} P^{ls}_{n,i,t} = \epsilon^{LS} \sum_{t=1}^{T} L_{n,i,t}. \tag{4.5}$$

Here, $\epsilon^{LS} = 1$ for no load-curtailment and $\epsilon^{LS} < 1$ for load curtailment.

4.2.2.3 B2B and B2C Energy Exchange

The cost of B2B energy exchange within n_{th} VC, is given by

$$C^{b2b}_{n,i,t} = \sum_{\substack{j=1 \\ j \neq i}}^{N^b_n} \rho^{b2b}_{n,i,j,t} P^{b2b}_{n,i,j,t}. \tag{4.6}$$

The cost of B2C energy exchange is given by,

$$C^{b2c}_{n,i,t} = \rho^{b2c}_{n,i,t} P^{b2c}_{n,i,t}. \tag{4.7}$$

Here, $P^{b2b}_{n,i,j,t} > 0$ when i^{th} building is importing from j^{th} building, while $P^{b2b}_{n,i,j,t} < 0$ when exporting to j^{th} building. Similarly, $P^{b2c}_{n,i,t} > 0$ when i^{th} building is importing from its own VC and $P^{b2c}_{n,i,t} < 0$ when it is exporting to own VC. Constraints related to B2B and B2C sharing are as follows.

$$P^{b2b}_{n,i,j,t} + P^{b2b}_{n,j,i,t} = 0, \tag{4.8}$$

$$\rho^{b2b}_{n,i,j,t} = \rho^{b2b}_{n,j,i,t}, \tag{4.9}$$

$$\Lambda^{ex}_{t} \leq \rho^{b2b}_{n,i,j,t} \leq \Lambda^{im}_{t}, \qquad and \tag{4.10}$$

$$\Lambda_t^{ex} \leq \rho_{n,i,t}^{b2c} \leq \Lambda_t^{im}. \tag{4.11}$$

Here, (4.8) and (4.9) are constraints for B2B transactions. While (4.10) and (4.11) are used to encourage more B2B and B2C transactions than transactions with utility. The load balance is given as follows.

$$P_{n,i,t}^{s} - P_{n,i,t}^{b} - \sum_{\substack{j=1 \\ j\neq i}}^{N_n^b} P_{n,i,j,t}^{b2b} - P_{n,i,t}^{b2c} = R_{n,i,t} - P_{n,i,t}^{ls}, \tag{4.12}$$

$$P_{n,i}^{min} \leq P_{n,i,t}^{s} - P_{n,i,t}^{b} - \sum_{\substack{j=1 \\ j\neq i}}^{N_n^b} P_{n,i,j,t}^{b2b} - P_{n,i,t}^{b2c} \leq P_{n,i}^{max}. \tag{4.13}$$

Here, (4.13) ensures the power exchange limits of the buildings in view of distribution line limits. Let γ_b :=$\{P^b \geq 0$, $P^s \geq 0$, (4.4), (4.5), (4.8) to (4.13)$\}$. γ_b is a set of constraints for buildings which is to be used in later sections. This framework doesn't need any separate constraints to prevent simultaneous buying and selling to the grid, as explained in Appendix III.

4.2.3 Cost Model of Virtual Community

The VC charges/discharges the common BESS and participates in C2C transactions. The cost function for the n_{th} VC is given by,

$$C_n^c = \sum_{t=1}^{T}(C_{n,t}^{Bat} + C_{n,t}^{c2c}). \tag{4.14}$$

4.2.3.1 Battery Energy Storage System (BESS)

In this work, the utilization cost of BESS is considered, while ignoring the installation cost of the BESS. Thus, the cost of the BESS is given by,

$$C_{n,t}^{Bat} = \Lambda^{UTI}(P_{n,t}^{ch} + P_{n,t}^{dis}), \tag{4.15}$$

subject to the following constraints,

$$P_n^{ch_{min}} \leq P_{n,t}^{ch} \leq P_n^{ch_{max}}, \tag{4.16}$$

$$P_n^{dis_{min}} \leq P_{n,t}^{dis} \leq P_n^{dis_{max}}, \tag{4.17}$$

Constraints (4.16) and (4.17) impose maximum and minimum limits on the charging and discharging power of the BESS. The state of charge (SOC) update equation is as follows.

$$SOC_{n,t} = (1-\eta_n^{loss})SOC_{n,t-1} + \eta_n^{ch} P_{n,t}^{ch} - \frac{1}{\eta^{dis}{}_n} P_{n,t}^{dis}, \qquad and \tag{4.18}$$

the SOC level is constrained by the following equation.

$$SOC_n^{min} \leq SOC_{n,t} \leq SOC_n^{max}. \tag{4.19}$$

4.2.3.2 C2C Energy Exchange

The C2C energy exchange cost includes the cost of inter-VC energy exchanges and penalties associated with inter-VC distances. The penalty only applies to VCs exporting energy to other VCs. Thus, the cost function can be expressed as follows.

$$C_{n,t}^{c2c} = \sum_{\substack{m=1 \\ m\neq n}}^{N^c} \rho_{n,m,t}^{c2c} P_{n,m,t}^{c2c} + \sum_{\substack{m=1 \\ m\neq n}}^{N^c} K_d d_{n,m}^{c2c} P_{n,m,t}^{c2c^{aux}}. \tag{4.20}$$

Here, the auxiliary variable, $P_{n,m,t}^{c2c^{aux}}$, is used to penalise only power export i.e. when $P_{n,m,t}^{c2c^{aux}} \geq -P_{n,m,t}^{c2c}$ and $P_{n,m,t}^{c2c^{aux}} \geq 0$. The cost function $C_{n,t}^{c2c}$ is subjected to the following constraints.

$$P_{n,m,t}^{c2c} + P_{m,n,t}^{c2c} = 0, \tag{4.21}$$

$$\rho_{n,m,t}^{c2c} = \rho_{m,n,t}^{c2c}, \tag{4.22}$$

$$\Lambda_t^{ex} \leq \rho_{n,m,t}^{c2c} \leq \Lambda_t^{im}, \qquad and \tag{4.23}$$

$$P_{n,t}^{dis} - P_{n,t}^{ch} + \sum_{\substack{m=1 \\ m\neq n}}^{N^c} P_{n,m,t}^{c2c} = \sum_{i=1}^{N_n^b} P_{n,i,t}^{b2c}. \tag{4.24}$$

Here, (4.21)-(4.23) are C2C power exchange and price related constraints. In (4.24), network losses and other losses are neglected. If $P_{n,m,t}^{c2c} > 0$ then n^{th} VC is importing from m^{th} VC and if $P_{n,m,t}^{c2c} < 0$ then n^{th} VC is exporting to m^{th} VC.

A VC is considered a non-profit entity for its buildings, payments received or made by a VC will be distributed among the buildings within the VC as follows.

$$\sum_{t=1}^{T}(C_{n,t}^{Bat} + C_{n,t}^{c2c}) = \sum_{t=1}^{T}\sum_{i=1}^{N_n^b} \rho_{n,i,t}^{b2c} P_{n,i,t}^{b2c}. \tag{4.25}$$

Let γ_c :={(4.16) to (4.19), (4.21) to (4.25)}. γ_c is a set of constraints for VC, which is to be used in later sections. This framework doesn't need any separate constraints to prevent simultaneous charging and discharging, as explained in Appendix III.

4.3 Methodology

The methodology described in this section consists of day-ahead hourly scheduling of energy and the method for distributing the corresponding payments among the participants.

4.3.1 Scheduling Energy Profiles

The energy profiles of the participants are scheduled with the help of a non-cooperative game which is solved in a decentralized manner through ADMM.

4.3.1.1 Non-cooperative Game

A non-cooperative game is used to schedule the energies of various buildings and VCs. Let ψ denote this non-cooperative game whose ultimate goal is to minimize the cost function (C_F) of all the buildings. However, due to the presence of B2C coupled variables, this game is played in two levels, as given in Table 4.1.

Table 4.1: DESCRIPTION OF THE NON-COOPERATIVE GAME

Levels	Players	Strategies	Objectives
1	Buildings	$x^b_{n,i,t} := \{P^b_{n,i,t}, P^s_{n,i,t}, P^{ls}_{n,i,t}, P^{b2b}_{n,i,j,t}, P^{b2c}_{n,i,t}\}$ and $\rho^b_{n,i,t}{:=}\{\rho^{b2c}_{n,i,t},\rho^{b2b}_{n,i,j,t}\}$ such that both are within the feasible set γ_b	$C^b_{n,i}$
2	VCs	$x^c_{n,t} := \{P^{ch}_{n,t}, P^{dis}_{n,t}, P^{c2c}_{n,m,t}\}$ and $\rho^c_{n,t} := \{\rho^{c2c}_{n,m,t}\}$ such that both are within the feasible set γ_c	C^c_n

For the game ψ, let z^*_p be the generalised Nash equilibrium (GNE) of player ‘p’ where z_p is the strategy of the player ‘p’ such that: $z_p{\in}(x^b_{n,i,t},\rho^b_{n,i,t},x^c_{n,t},\rho^c_{n,t})$. This is only possible if $C_F(z^*_p, z^*_{-p}) \leq C_F(z_p, z^*_{-p})$ where z_{-p} is the strategy of other players except the player ‘p’. To find the GNE, a regularised Nikaido-Isoda-function (also known as NI-function) is used [80]. The NI function is denoted by $\phi(z, q)$, where q has the same dimensions as z. The NI function is a fundamental tool for solving generalized Nash equilibrium problems (GNEPs) in game theory. It quantifies the improvement a player could achieve by unilaterally changing its strategy while keeping other players' strategies fixed. Thus, the regularised NI function is given by,

$$\phi(z,q) = \sum_{p}^{players} [C_F(z_p, z_{-p}) - C_F(q_p, z_{-p})] - \frac{\delta}{2}||z-q||_2^2. \tag{4.26}$$

The above function denotes the total gains of player p if it switches from its current strategy z_p to q_p regardless of other players' strategies. Here δ is a known parameter. Let τ be the gain maximizer function such that,

$$\tau(z,q) = \arg max_{q\in(\gamma_b,\gamma_c)}\phi(z,q). \tag{4.27}$$

This maximizes the deviation potential of each participant in the game by checking whether a better strategy (q) exists compared to the current one (z). At equilibrium, the function value approaches zero, meaning that no participant can improve its objective function unilaterally, indicating that the Generalized Nash Equilibrium (GNE) is reached. Thus, Equation (4.27) is crucial for reformulating and solving the GNEP, ensuring the stability and convergence of the non-cooperative game. Substituting ϕ from equation (4.26) in (4.27) ,

$$\tau(z,q) = \underset{q\in(\gamma_b,\gamma_c)}{\arg\max} \sum_{p}^{players} [C_F(z_p, z_{-p}) - C_F(q_p, z_{-p})] - \frac{\delta}{2}||z-q||_2^2. \tag{4.28}$$

The first term in equation (4.28) represents the current strategy which can be ignored. Therefore,

$$\tau(z,q) = \underset{q\in(\gamma_b,\gamma_c)}{\arg\min} \sum_{p}^{players} [C_F(q_p, z_{-p})] + \frac{\delta}{2}||z-q||_2^2. \tag{4.29}$$

The cost function is $C^b_{n,i}$. So, using (4.1), we can write,

$$\tau(z,q) = \underset{q\in(\gamma_b,\gamma_c)}{\arg\min} \sum_{t=1}^{T}\sum_{n=1}^{N^c}\sum_{i=1}^{N^b_n}[C^U_{n,i,t} + C^{ls}_{n,i,t} + C^{b2b}_{n,i,t} + C^{b2c}_{n,i,t}] + \frac{\delta}{2}||z-q||_2^2. \tag{4.30}$$

According to (4.8) and (4.9), there will be no effect on the total cost of the B2B ($C^{b2b}_{n,i,t}$) in the above equation. Thus,

$$\tau(z,q) = \underset{q\in(\gamma_b,\gamma_c)}{\arg\min} \sum_{t=1}^{T}\sum_{n=1}^{N^c}\left[\sum_{i=1}^{N^b_n}(C^U_{n,i,t} + C^{ls}_{n,i,t}) + \sum_{i=1}^{N^b_n} C^{b2c}_{n,i,t}\right] + \frac{\delta}{2}||z-q||_2^2. \tag{4.31}$$

According to (4.25), $\sum_{i=1}^{N^b_n} C^{b2c}_{n,i,t}$ in the above equation can be substituted to obtain,

$$\tau(z,q) = \underset{q\in(\gamma_b,\gamma_c)}{\arg\min} \sum_{t=1}^{T}\sum_{n=1}^{N^c}\left[\sum_{i=1}^{N^b_n}(C^U_{n,i,t} + C^{ls}_{n,i,t}) + (C^{Bat}_{n,t} + C^{c2c}_{n,t})\right] + \frac{\delta}{2}||z-q||_2^2. \tag{4.32}$$

Here, $C_{n,t}^{c2c}$ will consist only of the penalty charged based on the distance between the VCs. There will be no effect on the actual cost of importing/exporting power because of (4.21) and (4.22). Thus,

$$\tau(z,q) = \arg\min_{q\in(\gamma_b,\gamma_c)} \sum_{t=1}^{T}\sum_{n=1}^{N^c}\Bigg[\sum_{i=1}^{N_n^b}(C_{n,i,t}^{U} + C_{n,i,t}^{ls}) + C_{n,t}^{Bat} + C_{n,t}^{c2c^{distance}}\Bigg] + \frac{\delta}{2}||z-q||_2^2. \quad (4.33)$$

It is observed from (4.33), that there is no reasonable solution for $\rho_{n,i,t}^{b}$ and $\rho_{n,t}^{c}$ using above method. Therefore, the solutions will be found separately in section 4.3.2, while only the energy profiles are found as follows.

$$\tau(z,q) = \arg\min_{q\in(\gamma_b,\gamma_c)} \sum_{t=1}^{T}\sum_{n=1}^{N^c}\Bigg[\sum_{i=1}^{N_n^b}(C_{n,i,t}^{U} + C_{n,i,t}^{ls}) + C_{n,t}^{Bat} + C_{n,t}^{c2c^{distance}}\Bigg] + \frac{\delta}{2}||z^{xb}-q^{xb}||_2^2 + \frac{\delta}{2}||z^{xc}-q^{xc}||_2^2. \quad (4.34)$$

To find the GNE of the game, $\tau(z,q)$ is solved iteratively to get the best response z_p^* such that $\phi(z^*,q) = 0$. As explained earlier, due to the presence of B2C variables, this game is completed in two levels, as shown in the flowchart given in Figure 4.2. Level 1 deals with the building problems, and level 2 deals with the problems of VCs. After completing both levels, the convergence due to the presence of coupled variables is checked. Once all the variables corresponding to the current strategy reach their optimum value, the convergence of strategies is checked with respect to strategies of previous iteration. If it converges, results are obtained, or it will start the iteration again. The decentralization is used handle the coupled variables, as discussed below.

4.3.1.2 Decentralization

In the proposed framework, finding the GNE of the non-cooperative game, described in section 4.3.1.1, involves building level and VC level problems with their coupling constraints. The building level and VC level are coupled along with that B2C variables. Algorithm 2 uses the Alternating Direction Method of Multipliers (ADMM) in a nested manner to solve the problem described in (4.34). Based on nested ADMM, Algorithm 2 is proposed for a decentralised solution by decoupling the coupled constraints using the auxiliary variables ($\sigma_{n,i,j,t}^{b2b}$,$\sigma_{n,i,t}^{b2c}$,$\sigma_{n,m,t}^{c2c}$) as follows.

$$\sigma_{n,i,j,t}^{b2b} = P_{n,i,j,t}^{b2b}, \quad (4.35)$$

$$\sigma_{n,i,j,t}^{b2b} + \sigma_{n,j,i,t}^{b2b} = 0, \quad (4.36)$$

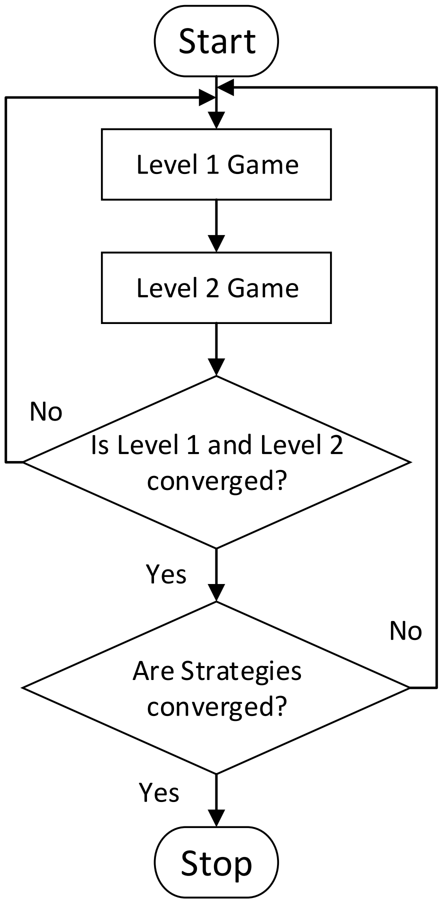


Figure 4.2: Simplified Flowchart Overview

$$\sigma^{b2c}_{n,i,t} = P^{b2c}_{n,i,t}, \tag{4.37}$$

$$P^{dis}_{n,t} - P^{ch}_{n,t} + \sum_{\substack{m=1 \\ m\neq n}}^{N^c} P^{c2c}_{n,m,t} = \sum_{i=1}^{N^b_n} \sigma^{b2c}_{n,i,t}, \tag{4.38}$$

$$\sigma^{c2c}_{n,m,t} = P^{c2c}_{n,m,t}, \qquad and \tag{4.39}$$

$$\sigma^{c2c}_{n,m,t} + \sigma^{c2c}_{m,n,t} = 0. \tag{4.40}$$

Thus, constraints (4.8), (4.24), and (4.21) are replaced by constraints (4.35, 4.36), (4.37, 4.38), and (4.39, 4.40), respectively. The augmented Lagrangian becomes:

$$\mathcal{L}(q,\sigma,\omega) = \sum_{t=1}^{T}\sum_{n=1}^{N^c}\Bigg[\sum_{i=1}^{N^b_n}(C^U_{n,i,t}+C^{ls}_{n,i,t}+\sum_{\substack{j=1 \\ j\neq i}}^{N^b_n}\frac{\delta_1}{2}(q^{xb^{b2b}}_{n,i,j,t}-\sigma^{b2b}_{n,i,j,t}+\frac{\omega^{b2b}_{n,i,j,t}}{\delta_1})^2+\frac{\delta_2}{2}(q^{xb^{b2c}}_{n,i,t}-\sigma^{b2c}_{n,i,t}+ \frac{\omega^{b2c}_{n,i,t}}{\delta_2})^2)+C^{Bat}_{n,t}+C^{c2c^{distance}}_{n,t}+\frac{\delta_3}{2}(q^{xc^{c2c}}_{n,m,t}-\sigma^{c2c}_{n,m,t}+\frac{\omega^{c2c}_{n,m,t}}{\delta_3})^2\Bigg]+\frac{\delta}{2}||z^{xb}-q^{xb}||^2_2+\frac{\delta}{2}||z^{xc}-q^{xc}||^2_2. \tag{4.41}$$

Equation (4.41) is split into four parts as follows.

- For building i of VC n,

$$F_1(q^{xb},\sigma^{b2b},\sigma^{b2c}) = \sum_{t=1}^{T}[C^U_{n,i,t} + C^{ls}_{n,i,t} + \sum_{\substack{j=1 \\ j\neq i}}^{N^b_n}\frac{\delta_1}{2}(q^{xb^{b2b}}_{n,i,j,t} - \sigma^{b2b}_{n,i,j,t} + \frac{\omega^{b2b}_{n,i,j,t}}{\delta_1})^2 + \frac{\delta_2}{2}(q^{xb^{b2c}}_{n,i,t} - \sigma^{b2c}_{n,i,t} + \frac{\omega^{b2c}_{n,i,t}}{\delta_2})^2] + \frac{\delta}{2}||z^{xb} - q^{xb}||^2_2. \tag{4.42}$$

- For B2B auxiliary variable update of VC n,

$$F_2(q^{xb},\sigma^{b2b}) = \sum_{t=1}^{T}\sum_{i=1}^{N^b_n}\sum_{\substack{j=1 \\ j\neq i}}^{N^b_n}\frac{\delta_1}{2}(q^{xb^{b2b}}_{n,i,j,t} - \sigma^{b2b}_{n,i,j,t} + \frac{\omega^{b2b}_{n,i,j,t}}{\delta_1})^2. \tag{4.43}$$

- For VC n,

$$F_3(q^{xb},q^{xc},\sigma^{b2c},\sigma^{c2c}) = \sum_{t=1}^{T}[\sum_{i=1}^{N^b_n}(\frac{\delta_2}{2}(q^{xb^{b2c}}_{n,i,t} - \sigma^{b2c}_{n,i,t} + \frac{\omega^{b2c}_{n,i,t}}{\delta_2})^2) + C^{Bat}_{n,t} + C^{c2c^{distance}}_{n,t} + \sum_{\substack{m=1 \\ m\neq n}}^{N^c}\frac{\delta_3}{2}(q^{xc^{c2c}}_{n,m,t} - \sigma^{c2c}_{n,m,t} + \frac{\omega^{c2c}_{n,m,t}}{\delta_3})^2] + \frac{\delta}{2}||z^{xc} - q^{xc}||^2_2. \tag{4.44}$$

- For C2C auxiliary variable update,

$$F_4(q^{xc}, \sigma^{c2c}) = \sum_{t=1}^{T} \sum_{n=1}^{N^c} \sum_{\substack{m=1 \\ m \neq n}}^{N^c} [\frac{\delta_3}{2}(q^{xc^{c2c}}_{n,m,t} - \sigma^{c2c}_{n,m,t} + \frac{\omega^{c2c}_{n,m,t}}{\delta_3})^2]. \quad (4.45)$$

The entire process is explained in Algorithm 2. Level 1 comprises of building level and intra VC level game, and Level 2 comprises of VC level and inter VC level game. First, the base strategies are initialized, followed by the minimization of objective functions and updating of the decoupling variables in steps. At the beginning of the iteration, buildings optimise their strategy (step 14), and then their B2B decoupling variables are updated (step 16). After this, each VC will optimise its strategy and update the B2C decoupling variables (step 20). At the end of the iteration, C2C decoupling variables are updated (step 25). The penalty parameters are updated with the help of their respective primal residuals (λ^P) and dual residuals (λ^D). In the end, the base strategies are updated using the relaxation method in step 37. The process is repeated until the tolerance is achieved, as shown in step 39. The results of this program will be $x^{b*}_{n,i,t} := (P^{b*}_{n,i,t}, P^{s*}_{n,i,t}, P^{ls*}_{n,i,t}, P^{b2b*}_{n,i,j,t}, P^{b2c*}_{n,i,t})$ and $x^{c*}_{n,t} := (P^{ch*}_{n,t}, P^{dis*}_{n,t}, P^{c2c*}_{n,m,t})$, which will be used for distribution of payments as described in the next section.

Algorithm 2: Algorithm for P2P energy sharing using a non-cooperative game and ADMM, where buildings in virtual communities optimize their energy trading strategies independently.

1 Initialize ρ, $toll_1$, $toll_2$, d=1
2 For each VC,
3 For each building,
4 Initialize $z^{xb}(1) := (P^b(1), P^s(1), P^{ls}(1), P^{b2b}(1), P^{b2c}(1))$
5 End for.
6 Initialize $z^{xc}(1) := (P^{ch}(1), P^{dis}(1), P^{c2c}(1))$
7 End for.
8 Repeat
9 Intialise k=1, $\sigma(1) = 0$, and $\omega(1) = 0$
10 Set $\delta(1)$, μ, ν
11 Repeat
12 For each VC,

———-*Building level*———

13 For each building,

$\underset{q^{xb}\in\gamma_b}{min} F_1(q^{xb}, \sigma^{b2b}(k), \sigma^{b2c}(k))$

End for.

———-*Intra-VC level*———-

Update σ^{b2b}: $\underset{\sigma^{b2b}\in\{(4.35),(4.36)\}}{min} F_2(q^{xb}(k+1), \sigma^{b2b})$

$\omega_1(k+1) = \omega_1(k) + \delta_1(k)(q^{xb^{b2b}}(k+1) - \sigma^{b2b}(k+1))$

$\lambda_1^P = ||q^{xb^{b2b}}(k+1) - \sigma^{b2b}(k+1)||$

$\lambda_1^D = ||\sigma^{b2b}(k+1) - \sigma^{b2b}(k)||$

———-*VC level*———-

$\underset{\substack{\sigma^{b2c}\in\{(4.37),(4.38)\}\\ q^{xc}\in\gamma_c}}{min} F_3(q^{xb}(k+1), q^{xc}, \sigma^{b2c}, \sigma^{c2c}(k))$

$\omega_2(k+1) = \omega_2(k) + \delta_2(k)(q^{xb^{b2c}}(k+1) - \sigma^{b2c}(k+1))$

$\lambda_2^P = ||q^{xb^{b2c}}(k+1) - \sigma^{b2c}(k+1)||$

$\lambda_2^D = ||\sigma^{b2c}(k+1) - \sigma^{b2c}(k)||$

End for.

———-*Inter-VC level*———

Update σ^{c2c}: $\underset{\sigma^{c2c}\in\{(4.39),(4.40)\}}{min} F_4(q^{xc}(k+1), \sigma^{c2c})$

$\omega_3(k+1) = \omega_3(k) + \delta_3(k)(q^{xc^{c2c}}(k+1) - \sigma^{c2c}(k+1))$

$\lambda_3^P = ||q^{xc^{c2c}}(k+1) - \sigma^{c2c}(k+1)||$

$\lambda_3^D = ||\sigma^{c2c}(k+1) - \sigma^{c2c}(k)||$

———-*Penalty-parameter update*———-

Do for ($\lambda_1^P, \lambda_1^D, \delta_1$),($\lambda_2^P, \lambda_2^D, \delta_2$), and ($\lambda_3^P, \lambda_3^D, \delta_3$)

If $\lambda^P < \tau\lambda^D$, then $\delta(k+1) = \frac{\delta(k)}{\nu}$

elseif $\lambda^P > \frac{\lambda^D}{\tau}$ then $\delta(k+1) = \nu\delta(k)$

else $\delta(k+1) = \delta(k)$

End do.

k=k+1

Until $||\{||\omega_1(k) - \omega_1(k-1)|| + ||\omega_2(k) - \omega_2(k-1)||$
$+||\omega_3(k) - \omega_3(k-1)||\}|| < toll_2$

Updating the strategies:

$z(d+1) = \frac{1}{k+1} z(d) + \frac{k}{k+1} q(k)$

d=d+1

Until $||z(d) - q(k)|| < toll_1$

4.3.2 Distribution of Payments

The C2C price refers to the P2P energy exchange price between different virtual communities, whereas B2B prices correspond to P2P energy trading at the building level. The key differences are as follows.

- C2C Price (Community-to-Community Price): This is the price determined for energy trading between different virtual communities. It is calculated first to establish the cost of energy exchange at the community level.
- B2B Price (Building-to-Building Price): This is the energy trading price between individual buildings within the same community. It governs the direct exchange of energy between prosumer buildings.
- P2P Price (Peer-to-Peer Price): This is a general term encompassing all decentralized energy trading interactions, including B2B, B2C, and C2C.

The pricing structure follows a hierarchical approach. C2C Price is calculated first to determine the cost of energy exchange between virtual communities. The total cost of each community is then divided among its buildings using the B2C price (Building-to-Community Price). The B2B price is incorporated along with other cost factors to determine the total cost of each building. This pricing mechanism ensures fair and efficient cost distribution across different levels of energy exchange.

For the calculation of price corresponding to B2B and B2C energy exchange, $\rho^b_{n,i,t} := (\rho^{b2c}_{n,i,t}, \rho^{b2b}_{n,i,j,t})$, and calculation of price corresponding C2C energy exchange, $\rho^c_{n,t} := (\rho^{c2c}_{n,m,t})$, in this work, Algorithm 2 is modified as described in the section below.

4.3.2.1 C2C Price

First of all, the price for each inter-VC energy sharing is computed using only the level 2 game. The computed price is distributed among the buildings within the VC as described in section 4.3.2.2. The game ψ is redefined as follows.

(i) Players: All VCs will be players in the game.

(ii) Strategies: The strategies will include only $\rho^c_{n,t} := (\rho^{c2c}_{n,m,t})$ such that they are within the feasible set γ_c.

(iii) Cost function (C_F): Here, every VC will try to reduce its cost as mentioned in (4.14). All the other parameters are either given or calculated from this equation, except for $\rho^{c2c}_{n,m,t}$. Therefore, the cost function for n^{th} VC becomes:

$$C_{Fn} = \sum_{t=1}^{T} \sum_{\substack{m=1 \\ m \neq n}}^{N^c} \rho^{c2c}_{n,m,t} P^{c2c}_{n,m,t}. \tag{4.46}$$

The price, $\rho^{c2c}_{n,m,t}$, should be such that

$$C^b_{n,i} + C^c_n \leq \underset{\substack{assuming \\ no\ c2c}}{(C^b_{n,i} + C^c_n)}. \tag{4.47}$$

Therefore, the profit would be greater if C2C transactions were to take place. In this Case, equation (4.29) gets modified as follows.

$$\tau(z,q) = \underset{q \in \{(4.22),(4.23),(4.47)\}}{\arg\min} \sum_{p}^{players} [C_{Fp}(q_p, z_{-p})] + \frac{\delta}{2} ||z - q||_2^2. \tag{4.48}$$

Substituting from equation (4.46) for C_{Fp}, we obtain,

$$\tau(z,q) = \underset{q \in \{(4.22),(4.23),(4.47)\}}{\arg\min} \sum_{t=1}^{T} \sum_{n=1}^{N^c} \sum_{\substack{m=1 \\ m \neq n}}^{N^c} [q^{\rho_c}_{n,m,t} P^{c2c}_{n,m,t}] + \frac{\delta}{2} ||z^{\rho_c} - q^{\rho_c}||_2^2. \tag{4.49}$$

Here, $z_p \in (\rho^{c2c}_{n,m,t})$ and q have the same dimensions as that of z. An auxiliary variable $\sigma^{c2c}_{n,m,t}$ is used to deal with the coupled constraint (4.22), such that,

$$\sigma^{c2c}_{n,m,t} = \rho^{c2c}_{n,m,t}, \quad and \tag{4.50}$$

$$\sigma^{c2c}_{n,m,t} = \sigma^{c2c}_{m,n,t}. \tag{4.51}$$

The modified augmented Lagrangian becomes:

$$\mathcal{L}^{\rho^c}(q,\sigma,\omega) = \sum_{t=1}^{T} \sum_{n=1}^{N^c} \sum_{\substack{m=1 \\ m \neq n}}^{N^c} [q^{\rho_c}_{n,m,t} P^{c2c}_{n,m,t} + \frac{\delta_3}{2} (q^{\rho_c}_{n,m,t} - \sigma^{c2c}_{n,m,t} + \frac{\omega^{c2c}_{n,m,t}}{\delta_3})^2] + \frac{\delta}{2} ||z^{\rho_c} - q^{\rho_c}||_2^2. \tag{4.52}$$

To solve for $\mathcal{L}^{\rho^c}(q,\sigma,\omega)$, the Algorithm 2 is modified in the following way.

(i) Remove steps (3-5, 13-19, and 21-23).

(ii) In step 6, initialize $z^{\rho_c}(1)$.

(iii) In step 20, for VC n, do

$$\underset{q^{\rho_c}\in\{(4.22),(4.23),(4.47)\}}{min} F_3(q^{\rho_c}, \sigma^{c2c}(k)). \tag{4.53}$$

Here,

$$F_3 = \sum_{t=1}^{T}\sum_{\substack{m=1\\m\neq n}}^{N^c}[q^{\rho_c}_{n,m,t}P^{c2c}_{n,m,t} + \frac{\delta_3}{2}({q^{\rho_c}_{n,m,t}}^{c2c} - \sigma^{c2c}_{n,m,t} + \frac{\omega^{c2c}_{n,m,t}}{\delta_3})^2] + \frac{\delta}{2}||z^{\rho_c} - q^{\rho_c}||^2_2.$$

(iv) In step 25, update σ^{c2c} by

$$\underset{\sigma^{c2c}\in\{(4.50),(4.51)\}}{min} F_4(q^{\rho^{c2c}_c}(k+1), \sigma^{c2c}). \tag{4.54}$$

Here,

$$F_4 = \sum_{t=1}^{T}\sum_{n=1}^{N^c}\sum_{\substack{m=1\\m\neq n}}^{N^c}[\frac{\delta_3}{2}({q^{\rho_c}_{n,m,t}}^{c2c} - \sigma^{c2c}_{n,m,t} + \frac{\omega^{c2c}_{n,m,t}}{\delta_3})^2]. \tag{4.55}$$

By applying the above modification in Algorithm 2, we get a modified algorithm presented as Algorithm 3. Using Algorithm 3, we will get $\rho^{c2c}_{n,m,t}$ for each VC. The same will be distributed among the buildings within the VC.

Algorithm 3: Algorithm for determining the C2C prices involved in P2P energy sharing among the virtual communities (C2C energy trading) using ADMM

Initialize ρ, $toll_1$, $toll_2$, d=1

For each VC,

  Initialize $z^{\rho_c}(1) := \rho^{c2c}(1)$

End for.

Repeat

  Intialise k=1, $\sigma(1) = 0$, and $\omega(1) = 0$

  Set $\delta(1)$, μ, ν

  Repeat

    For each VC,

———-VC level———-

$$\underset{q^{\rho_c}\in\{(4.22),(4.23),(4.47)\}}{min} F_3(q^{\rho_c}, \sigma^{c2c}(k))$$

    End for.

———-Inter-VC level———

Update $\underset{\sigma^{c2c} \in \{(4.50),(4.51)\}}{min} F_4(q^{\rho_c^{c2c}}(k+1), \sigma^{c2c})$

$\omega_3(k+1) = \omega_3(k) + \delta_3(k)(q^{xc^{c2c}}(k+1) - \sigma^{c2c}(k+1))$

$\lambda_3^P = ||q^{xc^{c2c}}(k+1) - \sigma^{c2c}(k+1)||$

$\lambda_3^D = ||\sigma^{c2c}(k+1) - \sigma^{c2c}(k)||$

———-*Penalty-parameter update*———-

Do for ($\lambda_1^P, \lambda_1^D, \delta_1$),($\lambda_2^P, \lambda_2^D, \delta_2$), and ($\lambda_3^P, \lambda_3^D, \delta_3$)

If $\lambda^P < \tau\lambda^D$, then $\delta(k+1) = \frac{\delta(k)}{\nu}$

elseif $\lambda^P > \frac{\lambda^D}{\tau}$ then $\delta(k+1) = \nu\delta(k)$

else $\delta(k+1) = \delta(k)$

End do.

k=k+1

Until $||\{||\omega_1(k) - \omega_1(k-1)|| + ||\omega_2(k) - \omega_2(k-1)|| + ||\omega_3(k) - \omega_3(k-1)||\}|| < toll_2$

Updating the strategies:

$z(d+1) = \frac{1}{k+1}z(d) + \frac{k}{k+1}q(k)$

d=d+1

Until $||z(d) - q(k)|| < toll_1$

4.3.2.2 B2C and B2B Price

All the energy profiles and prices for C2C transactions were calculated in the previous sections 4.3.1 and 4.3.2.1, which will be used to compute B2C and B2B prices. In this Case, only the level 1 game is played. For this Case, Algorithm 2 has been modified and is presented as Algorithm 4. The game ψ is redefined in this context as follows.

(i) Players: All buildings are players in the game.

(ii) Strategies: The strategies will include only $\rho_{n,i,t}^b := (\rho_{n,i,t}^{b2c}, \rho_{n,i,j,t}^{b2b})$ such that they are within the feasible set γ_b.

(iii) Cost function (C_F): Every building will try to minimise cost. Other parameters are already calculated except $\rho_{n,i,t}^{b2c}$ and $\rho_{n,i,j,t}^{b2b}$. Thus for i^{th} building of n^{th} VC, the cost function can be written as,

$$C_{Fn,i} = \sum_{t=1}^{T}[\rho^{b2c}_{n,i,t}P^{b2c}_{n,i,t} + \sum_{\substack{j=1\\j\neq i}}^{N^b_n}\rho^{b2b}_{n,i,j,t}P^{b2b}_{n,i,j,t}]. \tag{4.56}$$

The costs, $\rho^{b2c}_{n,i,t}$ and $\rho^{b2b}_{n,i,j,t}$ should be such that,

$$C^b_{n,i} \quad \leq \quad \underset{\substack{assuming\\no\ b2c\ and\ no\ b2b}}{C^b_{n,i}} \quad . \tag{4.57}$$

Therefore, B2C and B2B transactions increase profits. Let relevant constraint sets be defined as follows, γ^ρ_1 :=[(4.10), (4.11), (4.57)] and γ^ρ_2 :=[(4.9), (4.25)]. In this Case also (4.29) is modified as follows,

$$\tau(z,q) = \underset{q\in\{\gamma^\rho_1,\gamma^\rho_2\}}{\arg\min}\sum_{p}^{players}[C_{Fp}(q_p, z_{-p})] + \frac{\delta}{2}||z-q||^2_2. \tag{4.58}$$

Substituting C_{Fp} from equation (4.56), we get

$$\tau(z,q) = \underset{q\in\{\gamma^\rho_1,\gamma^\rho_2\}}{\arg\min}\sum_{t=1}^{T}\sum_{n=1}^{N^c}\sum_{i=1}^{N^b_n}\{q^{b2c}_{n,i,t}P^{b2c}_{n,i,t} + \sum_{\substack{j=1\\j\neq i}}^{N^b_n}q^{b2b}_{n,i,j,t}$$
$$P^{b2b}_{n,i,j,t}\} + \frac{\delta}{2}||z^{\rho^{b2b}} - q^{\rho^{b2b}}||^2_2 + \frac{\delta}{2}||z^{\rho^{b2c}} - q^{\rho^{b2c}}||^2_2. \tag{4.59}$$

Here, $z^{\rho^b} \in (\rho^b_{n,i,t}, \rho^b_{n,i,j,t})$ and q have the same dimensions as that of z. Auxiliary variables $\sigma^{b2c}_{n,i,t}$ and $\sigma^{b2b}_{n,i,j,t}$ are used to handle the coupled constraint γ^ρ_2 such that,

$$\sigma^{b2b}_{n,i,j,t} = \rho^{b2b}_{n,i,j,t}, \tag{4.60}$$

$$\sigma^{b2b}_{n,i,j,t} = \sigma^{b2b}_{n,j,i,t}, \tag{4.61}$$

$$\sigma^{b2c}_{n,i,t} = \rho^{b2c}_{n,i,t}, \quad and \tag{4.62}$$

$$(C^{Bat}_{n,t} + C^{c2c}_{n,t}) = \sum_{i=1}^{N^b_n}\sigma^{b2c}_{n,i,t}P^{b2c}_{n,i,t}. \quad \forall n \tag{4.63}$$

Thus, the modified augmented Lagrangian becomes

$$\mathcal{L}^{\rho^b}(q,\sigma,\omega) = \sum_{t=1}^{T}\sum_{n=1}^{N^c}\sum_{i=1}^{N^b_n}[q^{b2c}_{n,i,t}P^{b2c}_{n,i,t} + \sum_{\substack{j=1\\j\neq i}}^{N^b_n}q^{b2b}_{n,i,j,t}P^{b2b}_{n,i,j,t}$$
$$+\sum_{\substack{j=1\\j\neq i}}^{N^b_n}\frac{\delta_1}{2}(q^{\rho^{b2b}}_{n,i,j,t} - \sigma^{b2b}_{n,i,j,t} + \frac{\omega^{b2b}_{n,i,j,t}}{\delta_1})^2 + \frac{\delta_2}{2}(q^{\rho^{b2c}}_{n,i,t} - \sigma^{b2c}_{n,i,t}$$
$$+\frac{\omega^{b2c}_{n,i,t}}{\delta_2})^2] + \frac{\delta}{2}||z^{\rho^{b2b}} - q^{\rho^{b2b}}||^2_2 + \frac{\delta}{2}||z^{\rho^{b2c}} - q^{\rho^{b2c}}||^2_2. \tag{4.64}$$

To get Algorithm 4 for this Case following changes are made in Algorithm 2.

(i) Remove steps related to the VC's optimisation, i.e. remove steps (6 and 25-28)

(ii) In Step 4, initialize z^{ρ^b}.

(iii) In step 14, for building i of VC n do

$$\underset{q^{\rho^b}\in\gamma_1^{\rho}}{min}\, F_1(q^{\rho^b}, \sigma^{b2b}(k), \sigma^{b2c}(k)). \tag{4.65}$$

Here,

$$F_1 = \sum_{t=1}^{T}[q^{b2c}_{n,i,t}P^{b2c}_{n,i,t} + \sum_{\substack{j=1\\ j\neq i}}^{N_n^b} q^{b2b}_{n,i,j,t}P^{b2b}_{n,i,j,t} + \sum_{\substack{j=1\\ j\neq i}}^{N_n^b}\frac{\delta_1}{2}(q^{\rho^{b2b}}_{n,i,j,t} - \sigma^{b2b}_{n,i,j,t} + \frac{\omega^{b2b}_{n,i,j,t}}{\delta_1})^2 + \frac{\delta_2}{2}(q^{\rho^{b2c}}_{n,i,t} - \sigma^{b2c}_{n,i,t} + \frac{\omega^{b2c}_{n,i,t}}{\delta_2})^2] + \frac{\delta}{2}||z^{\rho^{b2b}} - q^{\rho^{b2b}}||_2^2 + \frac{\delta}{2}||z^{\rho^{b2c}} - q^{\rho^{b2c}}||_2^2.$$

(iv) In step 16, for each VC n, update σ^{b2b} by

$$\underset{\sigma^{b2b}\in\{\gamma_2^{\rho}\}}{min} F_2(q^{\rho^{b2b}}(k+1), \sigma^{b2b}). \tag{4.66}$$

Here,

$$F_2 = \sum_{t=1}^{T}\sum_{i=1}^{N_n^b}\sum_{\substack{j=1\\ j\neq i}}^{N_n^b}\frac{\delta_1}{2}(q^{\rho^{b2b}}_{n,i,j,t} - \sigma^{b2b}_{n,i,j,t} + \frac{\omega^{b2b}_{n,i,j,t}}{\delta_1})^2.$$

(v) In step 20, for each VC n, do

$$\underset{\sigma^{b2c}\in\{\gamma_2^{\rho}\}}{min} F_3(q^{\rho^{b2c}}(k+1), \sigma^{b2c}). \tag{4.67}$$

where,

$$F_3 = \sum_{t=1}^{T}\sum_{i=1}^{N_n^b}[\frac{\delta_2}{2}(q^{\rho^{b2c}}_{n,i,t} - \sigma^{b2c}_{n,i,t} + \frac{\omega^{b2c}_{n,i,t}}{\delta_2})^2]. \tag{4.68}$$

Algorithm 4: Algorithm for determining the B2B and B2C prices involved in P2P energy sharing among the virtual communities and buildings (B2C and B2B energy trading) using ADMM

1 Initialize ρ, $toll_1$, $toll_2$, d=1

2 For each VC,

3 For each building,

Initialize $z^{\rho^b}(1) := (\rho^{b2c}_{n,i,t}(1), \rho^{b2b}_{n,i,j,t}(1))$

End for.

End for.

Repeat

Intialise k=1, $\sigma(1) = 0$, and $\omega(1) = 0$

Set $\delta(1)$, μ, ν

Repeat

For each VC,

———-*Building level*————

For each building,

$\underset{q^{\rho^b} \in \gamma_1^{\rho}}{min} F_1(q^{\rho^b}, \sigma^{b2b}(k), \sigma^{b2c}(k))$

End for.

————*Intra-VC level*———-

Update $\underset{\sigma^{b2b} \in \{\gamma_2^{\rho}\}}{min} F_2(q^{\rho^{b2b}}(k+1), \sigma^{b2b})$

$\omega_1(k+1) = \omega_1(k) + \delta_1(k)(q^{xb^{b2b}}(k+1) - \sigma^{b2b}(k+1))$

$\lambda_1^P = ||q^{xb^{b2b}}(k+1) - \sigma^{b2b}(k+1)||$

$\lambda_1^D = ||\sigma^{b2b}(k+1) - \sigma^{b2b}(k)||$

———-*VC level*———-

$\underset{\sigma^{b2c} \in \{\gamma_2^{\rho}\}}{min} F_3(q^{\rho^{b2c}}(k+1), \sigma^{b2c})$

$\omega_2(k+1) = \omega_2(k) + \delta_2(k)(q^{xb^{b2c}}(k+1) - \sigma^{b2c}(k+1))$

$\lambda_2^P = ||q^{xb^{b2c}}(k+1) - \sigma^{b2c}(k+1)||$

$\lambda_2^D = ||\sigma^{b2c}(k+1) - \sigma^{b2c}(k)||$

End for.

———-*Penalty-parameter update*———-

Do for $(\lambda_1^P, \lambda_1^D, \delta_1)$,$(\lambda_2^P, \lambda_2^D, \delta_2)$, and $(\lambda_3^P, \lambda_3^D, \delta_3)$

If $\lambda^P < \tau\lambda^D$, then $\delta(k+1) = \frac{\delta(k)}{\nu}$

elseif $\lambda^P > \frac{\lambda^D}{\tau}$ then $\delta(k+1) = \nu\delta(k)$

else $\delta(k+1) = \delta(k)$

End do.

k=k+1

Until $||\{||\omega_1(k) - \omega_1(k-1)|| + ||\omega_2(k) - \omega_2(k-1)||$
$+||\omega_3(k) - \omega_3(k-1)||\}|| < toll_2$

Updating the strategies:

$z(d+1) = \frac{1}{k+1} z(d) + \frac{k}{k+1} q(k)$

d=d+1

Until $||z(d) - q(k)|| < toll_1$

Thus, all the energy schedules and the respective prices for various energy-sharing schedules are computed. All the discussed steps of P2P energy sharing and the corresponding price determination are summarised in the flowchart as depicted in Figure 4.3. The P2P energy exchange framework inherently includes B2B, B2C, and C2C transactions. In Figure 4.3, the scheduling of P2P energy occurs first, encompassing all three types of exchanges—B2B, B2C, and C2C. Based on this scheduled energy exchange, the corresponding prices are determined. Since C2C energy exchange occurs between different communities, the first step is to determine the C2C price based on the exchanged C2C power. Once the C2C price is established, the B2B and B2C energy exchange prices are computed based on scheduled power transactions and the C2C price. Regarding optimization levels, Level 1 represents building-level optimization, where each building optimizes its energy transactions, while Level 2 represents community-level optimization, where communities manage their internal and external energy exchanges.

Figure 4.4 illustrates the working of the P2P energy trading which includes B2B, B2C, and C2C energy trading.

4.4 Simulation Results

In this work, nine buildings having different load profiles (Figure 4.6(a)) and renewable generations (Figure 4.6(b)) are considered. The buildings are clustered into three VCs (C1, C2, and C3), each comprising three buildings. In Figure 4.6(b), the SPV and WPG data are given for the rated capacity of 125kW and 100kW, respectively. The installed capacity of SPV, WPG and BESS present in the system is specified in Table 4.5. The SOC level is to be maintained in [20%,90%]. The charging and discharging power is limited to 50kW/h for all the BESSs. A time-varying energy price (Figure 4.11) is considered for energy exchange (import and export) with the utility. Since load curtailment is not done, the fraction of load supplied (ϵ^{LS}) is 1. The load can be shifted up to $\pm 10\%$. Table 4.6 mentions the values of other parameters used.

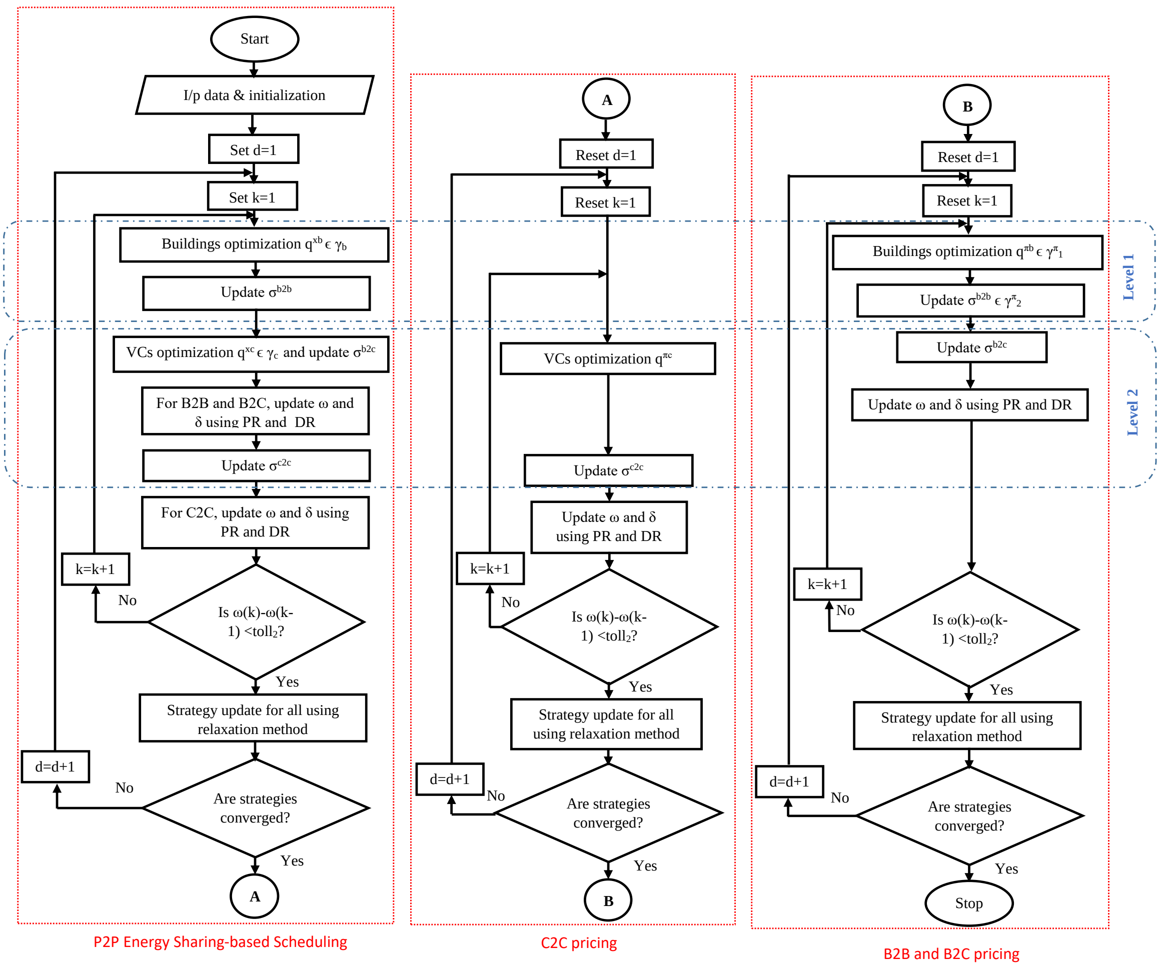


Figure 4.3: FLOWCHART OF PROPOSED ENERGY SHARING FRAMEWORK HIGHLIGHTING THE LEVELS AT WHICH THE GAMES ARE PLAYED

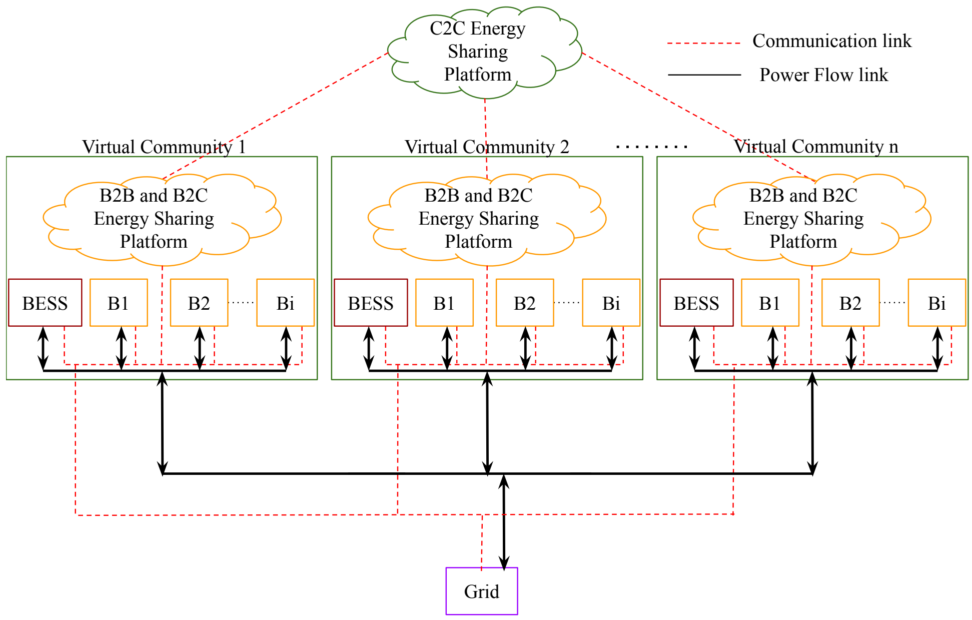


Figure 4.4: Illustration for P2P Energy Trading

The code is implemented on a laptop with a Core $i3$ 1.20 GHz processor with 4 GB RAM. GAMS/CONOPT4 solver is used for optimization.

The respective electrical network analogy of the information in Table 4.5 regarding C1, C2, C3: B1, B2, B3 with SPV, WPG, BESS in terms of B2B, P2P, C2C is given in Figure 4.5.

Each building in the B2B energy trading framework can be considered as a node in the equivalent circuit. This is because

- Each building has its own load, renewable energy generation (SPV and WPG), and potentially a battery energy storage system (BESS), making it a distinct entity in terms of power flow.
- The interaction between buildings (energy exchange) can be represented as power flows between nodes in an electrical network.

The buildings are further aggregated into communities, as shown in the figure, resulting in building-to-community(B2C) interactions. The community battery energy storage system (CBESS) acts as an additional node that interacts with all buildings through the community. The buildings interact with buildings of other communities though C2C interactions. Thus, with the help of P2P energy trading at various stages (B2B, B2C, C2C), the cost of all the participants and grid dependency decreases. This equivalent circuit representation effectively models the P2P energy trading framework, with each building considered as a node. This abstraction

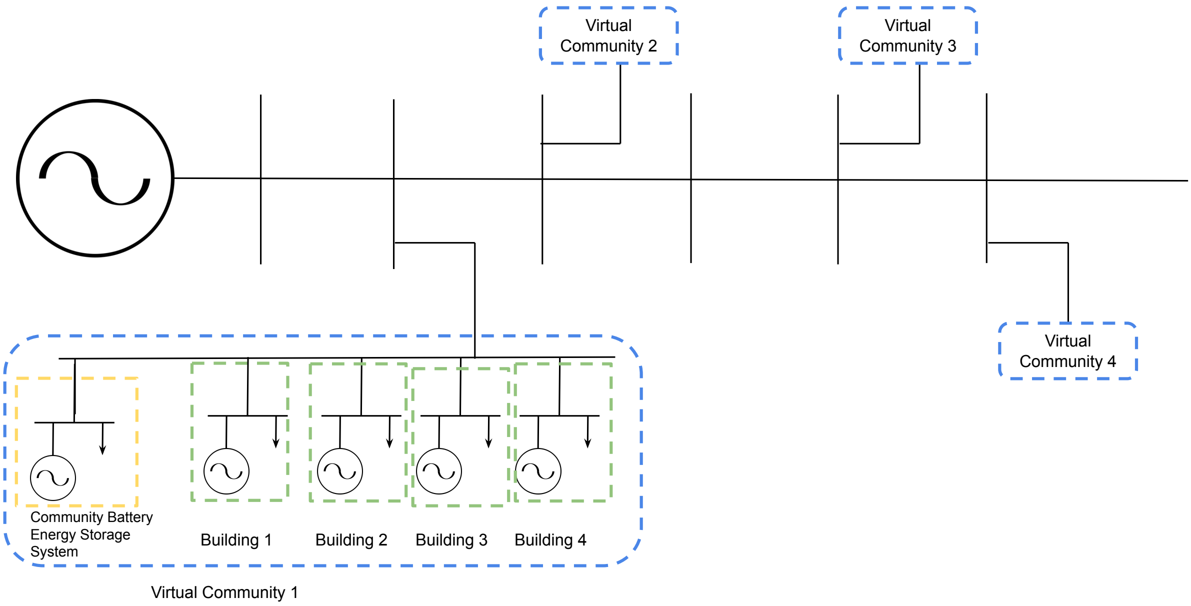


Figure 4.5: ELECTRICAL NETWORK ANALOGY

Table 4.5: INSTALLED CAPACITY OF SOLAR POWER GENERATION (SPG), WIND POWER GENERATION (WPG) AND BATTERY ENERGY STORAGE SYSTEM (BESS)

	C1			C2			C3		
	B1	**B2**	**B3**	**B1**	**B2**	**B3**	**B1**	**B2**	**B3**
SPV (kW)	125	125	125	100	125	125	100	100	100
WPG (kW)	100	100	100	50	100	100	50	50	50
BESS (kW)	200			200			200		

highlights the interactions among communities, buildings and their shared resources (CBESS), enabling detailed analysis of power flows and cost optimization.

To demonstrate the effectiveness of the proposed P2P framework, four different cases are considered:

Case I (Base case): In this case, all the buildings manage their energy to meet their load demand. Buildings can exchange energy with the utility only. In this case, BESSs are not used.

Case II: In this case, only B2B and B2C transactions are considered. In this case, a shared BESS is used.

Case III: In an extension to the second case, C2C transactions are also considered.

Case IV: This case is similar to *Case III*. In addition, penalty charges based on the distance between VCs are also considered in C2C transactions.

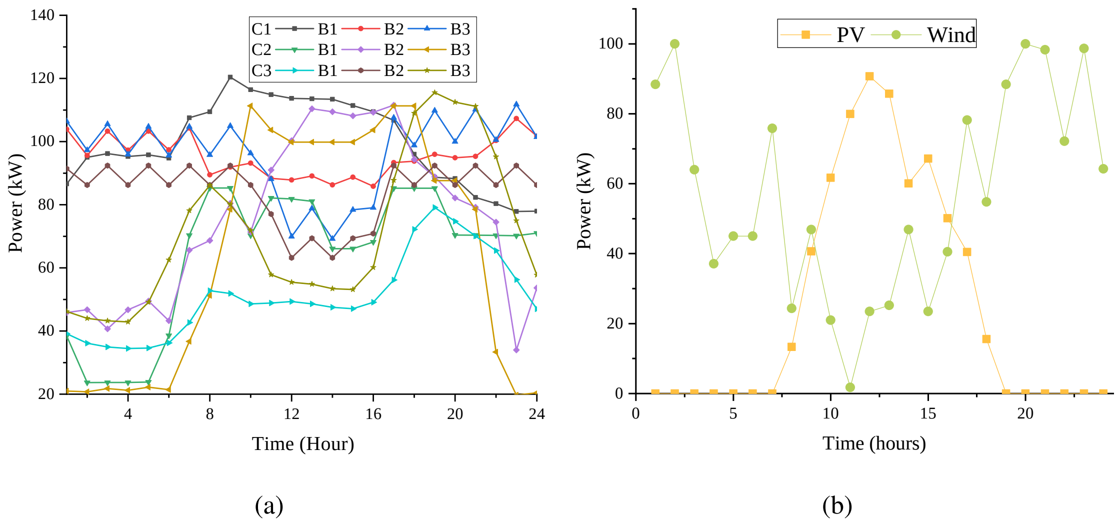


Figure 4.6: (A) LOAD PROFILE OF BUILDINGS IN ALL COMMUNITIES, (B) RES POWER GENERATION

Table 4.7 shows the summary of results for all the cases.

In *Case I*, the cost function of each building is the highest, and the energy exchange with utility is also the highest among all cases.

In *Case II*, the cost function of each building is reduced compared to the base case due to B2B transactions and BESS. As shown in Figure 4.6(a), all buildings in a VC have different load profiles. Due to this, buildings have different surplus/deficit power in a given time interval. Therefore, intra-VC B2B transactions can occur at a price lying between the utility buying and selling price. Also, BESS is used in B2C transactions. In *Case II*, the energy cost decreases from 300.18$, 126.17$, and 415.76$ to 268.40$, 56.38$, and 385.86$ for C1, C2, and C3, respectively.

In *Case III*, the cost function of each building is further reduced due to the C2C transaction introduced in this case. In fact, the cost function is the lowest in *Case III* among all the cases. In *Case III*, buildings have more opportunities to exchange energy with buildings of other VCs through C2C transactions.

In *case IV*, the cost function has increased compared to *Case III* because of the penalty charges (due to distances). But, still, the cost function is less compared to *Case I* and *Case II*.

Table 4.6: VALUES OF OTHER PARAMETERS

Parameters	**Values**
η^{ch}	96%
η^{dis}	106%
η^{loss}	0.5%
Λ^{UTI}	0.06$/kW
ϵ^{CE}	0.05$/kW
Λ^{dis}	$0.03\$/kW^2$
δ	0.02
$toll_1$	0.01
$toll_2$	0.001
τ	0.02
ν	2

Table 4.7: COMPARISON OF ALL THE CASES

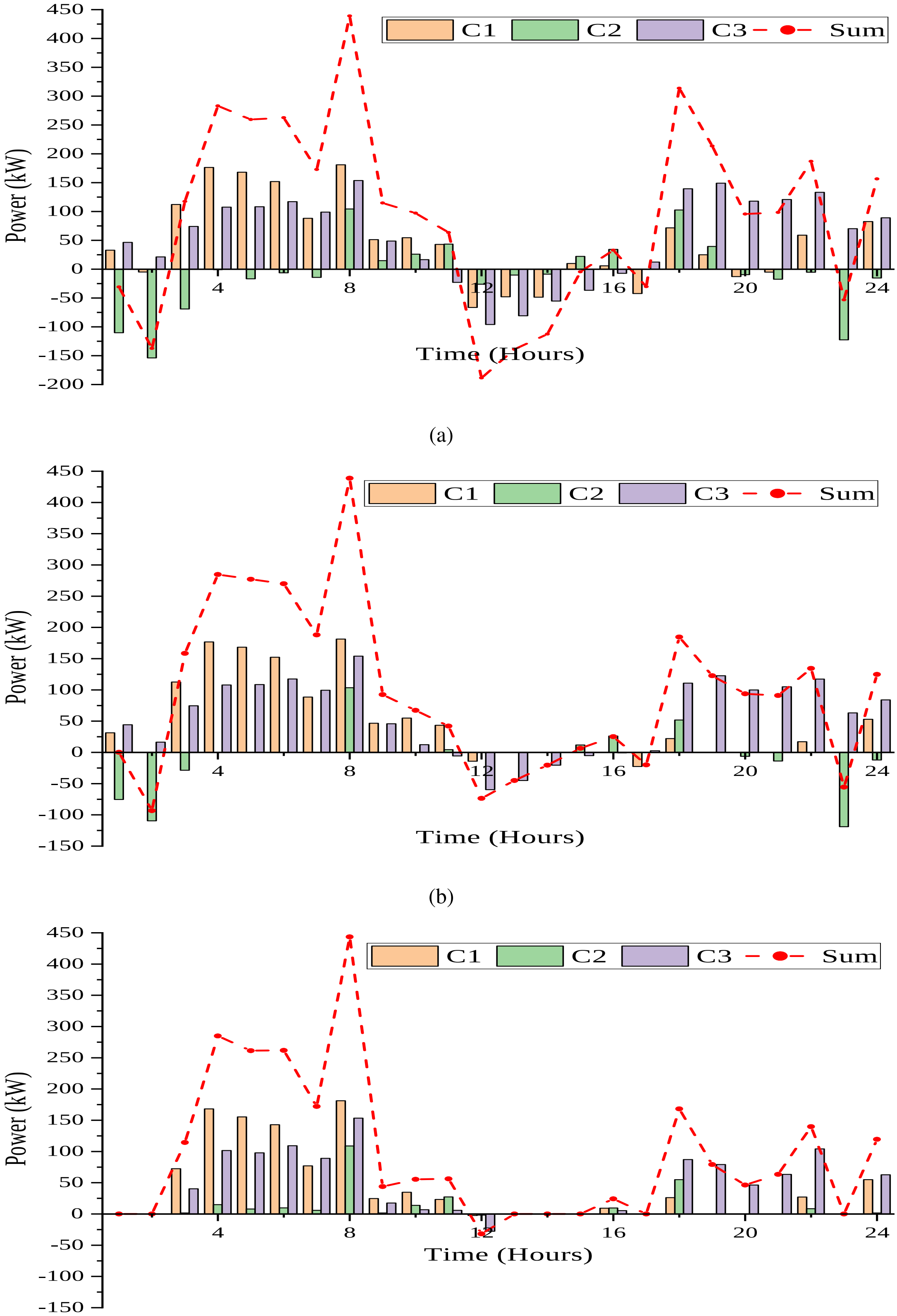
		C1				C2				C3				Total Sum
		B1	B2	B3	Sum	B1	B2	B3	Sum	B1	B2	B3	Sum	
$Cost$	$CaseI$	104.80	90.22	105.17	300.18	106.93	13.16	6.08	126.17	53.48	214.44	147.83	415.76	842.11
	$CaseII$	90.94	84.58	92.88	268.40	81.38	-3.58	-21.41	56.38	53.49	193.91	138.46	385.86	710.64
	$CaseIII$	86.68	78.65	86.21	251.54	72.30	-7.58	-35.04	29.68	36.12	178.49	119.32	333.92	615.14
	$CaseIV$	85.10	81.06	88.66	254.81	76.96	-5.89	-27.40	43.67	42.98	185.63	129.74	358.34	656.83
P^b	$CaseI$	477.38	415.17	477.89	1370.44	405.79	102.50	112.39	620.68	223.23	886.23	584.53	1693.99	3685.11
	$CaseII$	381.12	379.74	387.74	1148.60	86.64	55.40	55.64	197.68	342.67	625.56	517.98	1486.22	2832.50
	$CaseIII$	331.61	329.32	336.52	997.44	119.96	76.69	69.91	266.57	212.08	470.86	387.10	1070.04	2334.05
	$CaseIV$	320.96	317.70	326.42	965.08	81.68	52.90	53.76	188.34	263.10	529.13	446.86	1239.08	2392.50
P^s	$CaseI$	54.27	95.62	135.79	285.69	35.40	265.83	518.57	819.79	166.35	32.46	169.05	367.86	1473.34
	$CaseII$	5.76	11.53	19.34	36.63	30.09	130.57	204.15	364.81	46.64	13.72	76.46	136.83	538.26
	$CaseIII$	0.00	0.44	2.00	2.45	0.03	0.84	0.92	1.79	8.39	1.63	17.55	27.57	31.81
	$CaseIV$	0.00	0.00	0.00	0.00	0.00	5.64	23.72	29.36	22.70	6.42	39.39	68.51	97.87
P_{im}^{b2b}	$CaseII$	87.62	32.10	45.84	165.55	210.02	56.24	20.78	287.03	2.59	152.40	47.98	202.97	655.55
	$CaseIII$	92.15	35.28	47.91	175.34	257.95	68.77	19.76	346.48	5.97	177.32	49.19	232.48	754.30
	$CaseIV$	92.45	35.70	47.77	175.92	259.98	67.38	20.53	347.89	4.19	171.92	48.56	224.67	748.48
P_{ex}^{b2b}	$CaseII$	53.61	55.99	55.95	165.55	15.23	102.77	169.03	287.03	141.56	2.61	58.88	203.06	655.64
	$CaseIII$	55.27	58.80	61.26	175.33	14.64	119.42	212.46	346.52	149.61	4.63	78.21	232.45	754.30
	$CaseIV$	55.22	58.61	62.12	175.94	14.99	121.89	210.93	347.82	144.26	5.82	74.55	224.63	748.38
P_{im}^{b2c}	$CaseII$	72.07	57.83	72.86	202.77	164.23	46.58	58.87	269.68	17.32	125.11	60.22	202.65	675.09
	$CaseIII$	122.09	107.36	124.30	353.75	122.42	48.98	62.82	234.22	115.10	279.29	199.48	593.87	1181.85
	$CaseIV$	129.76	113.33	131.06	374.14	146.65	54.97	69.46	271.07	74.13	215.15	137.24	426.52	1071.73
P_{ex}^{b2c}	$CaseII$	58.33	82.61	89.06	230.00	45.17	88.20	168.29	301.66	117.51	32.96	75.35	225.82	757.47
	$CaseIII$	67.47779	93.16155	103.3708	264.01	115.27	237.51	345.29	698.07	118.27	67.44	124.53	310.24	1272.32
	$CaseIV$	64.84	88.58	101.03	254.45	102.93	211.05	315.27	629.24	117.59	50.19	103.23	271.00	1154.69

$Cost$: The total cost of the buildings, P^b: The total power bought from the utility,P^s: The total power sold to the utility,
P_{im}^{b2b} and P_{ex}^{b2b}: Power imported and exported in B2B, P_{im}^{b2c} and P_{ex}^{b2c}: Power imported and exported in B2C

Table 4.7 shows that the total power imported from the utility reduces in *case II* and *case III* compared to *case I*. The same holds for the total power exported to the utility. Around $12^{th} - 15^{th}$ hours, the power exported by the buildings to the utility in *case I* (Figure 4.7(a)) is used for B2B (Figure 4.8) and B2C (Figure 4.9) trading in *case II* and *case III*. Figure 4.10(a) shows that in *case II* and *case III*, the VC's BESS is getting charged during the time interval $12^{th} - 15^{th}$ hours, whereas in *case I*, the buildings are exporting to the utility as observed in Figure 4.7(a). In *Case II* and *Case III*, when the utility electricity price is high during $18^{th} - 21^{th}$ hours, energy stored in BESS is used for meeting the load demand. Due to the availability of C2C trading in *Case III*, the power exchange in B2C transactions has invariably increased compared to *Case II* Figure (4.9).

In Figure 4.8(a), for community C1, the power exchanged in B2B transactions is almost the same as in *Case II* and *Case III* except for $11^{th} - 13^{th}$ hours. However, for community C3, during this time interval, the variation in B2B transactions of *Case II* and *Case III* is quite significant (Figure 4.8(c)). In table 4.7, for each case, it is seen that the total power imported through B2B transactions is equal to the total power exported through B2B transactions. This validates equation (4.8). The power imported or exported during B2C transactions in *Case II* is much less than that in *Case III* or *Case IV* (Table 4.7). This is because, in *Case II*, the B2C transactions involve only BESS, whereas in *Case III* and *Case IV*, C2C is also involved in B2C transactions along with BESS. Furthermore, it can be observed that the overall B2C power is slightly less in *Case IV* compared to *Case III* due to penalty terms involved in the formulation.

Table 4.8 shows the total power imported and exported separately in C2C trading in *Case III* and *Case IV*. Due to the penalty imposed (based on distance) in *Case IV*, the imported/exported power decreases compared to *Case III*. In Figure 4.10(b), the net power exchanged in C2C transactions is depicted for *Case III* and *Case IV*. For *Case III*, the generation-load balance equation (4.24) can easily be verified from Figure 4.10(a), 4.9(b) and 4.10(b). During $12^{th} - 14^{th}$ hours, for C2, the power imported from C2C transactions (Figure 4.10(b)), and the power imported from all the buildings inside C2 through B2C transactions (Figure 4.9(b)) are used for charging the VC's BESS as shown in Figure 4.10(a). The community C3 after 18^{th} hour is importing from the other two VCs (Figure 4.10(b)) and is discharging its BESS (Figure 4.10(a)), and then this power is then transferred to buildings of C3 through B2C transactions (Figure 4.9(c)). Therefore, B2B, B2C and C2C energy sharing provides an opportunity to take advantage of the demand diversity of all buildings in all the VCs and to utilize RESs, thereby

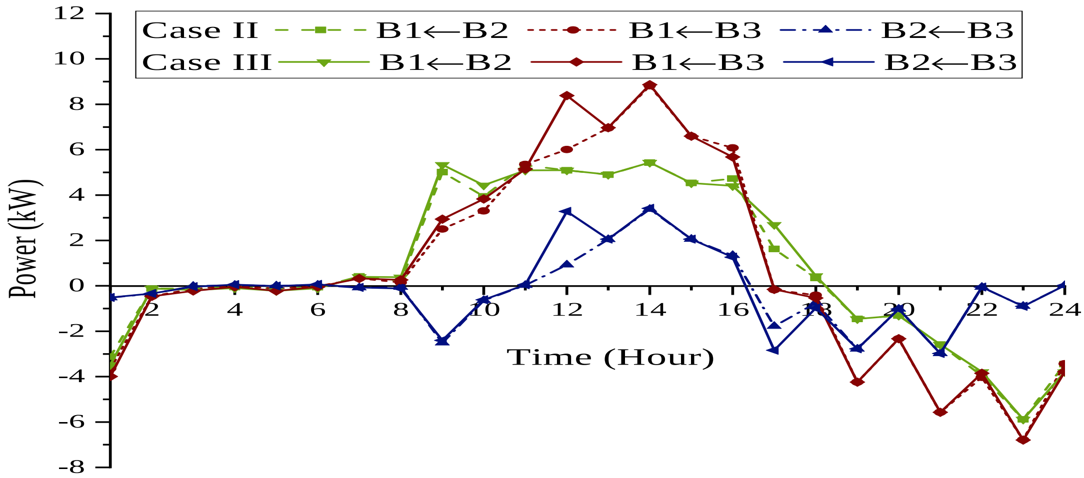


(a)

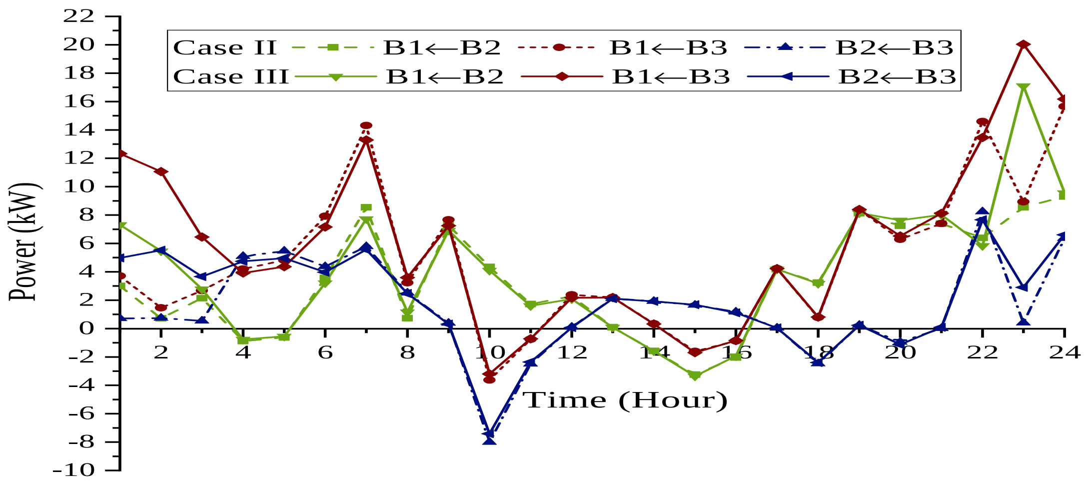
(b)

(c)

Figure 4.7: Building to Utility Power Exchange (a) *Case I*, (b) *Case II*, (c) *Case III*



(a)



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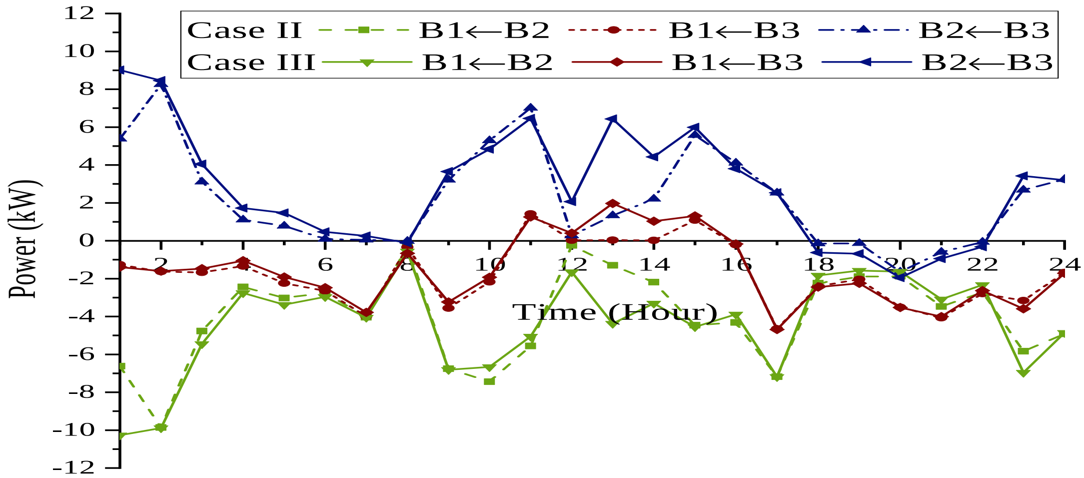
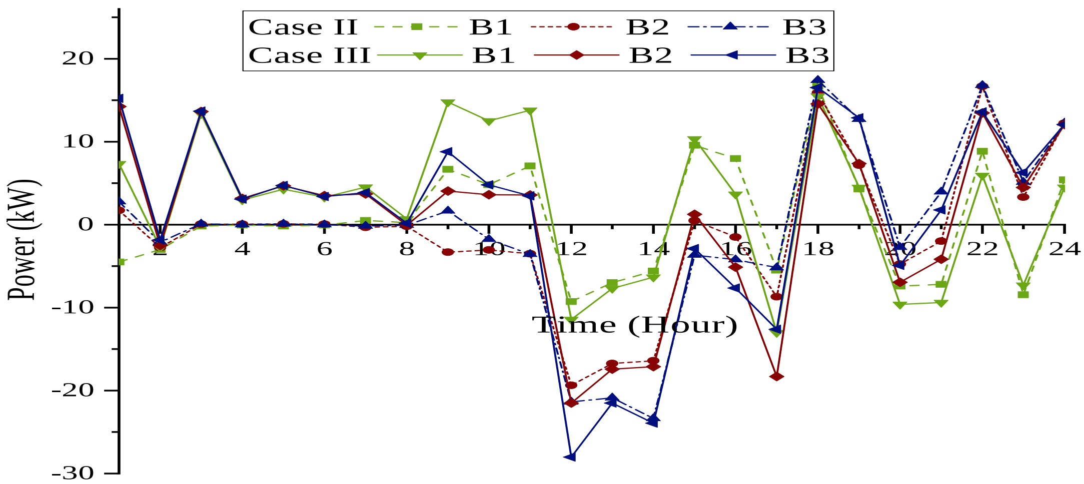
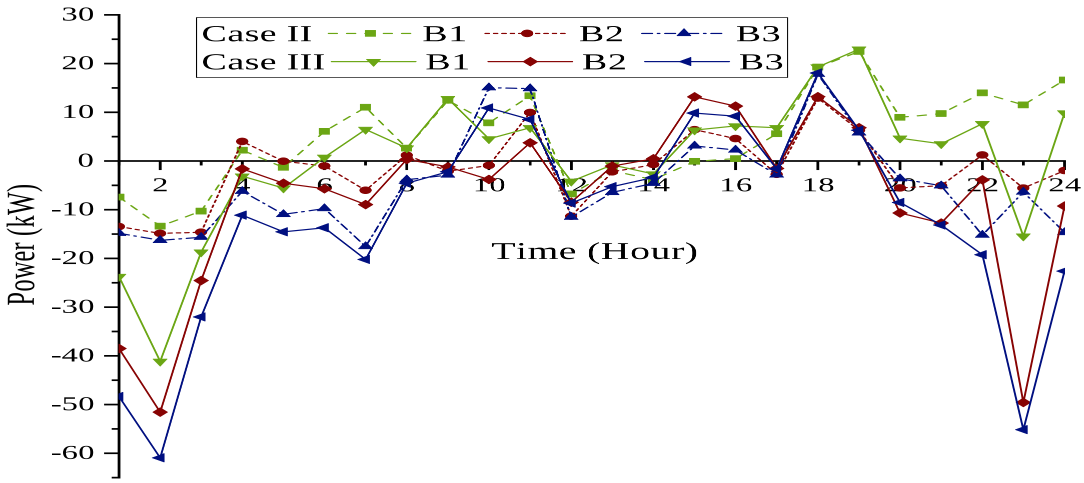


(c)

Figure 4.8: B2B Power Exchange for *Case II* and *Case III* (a) Community 1, (b) Community 2, (c) Community 3



(a)



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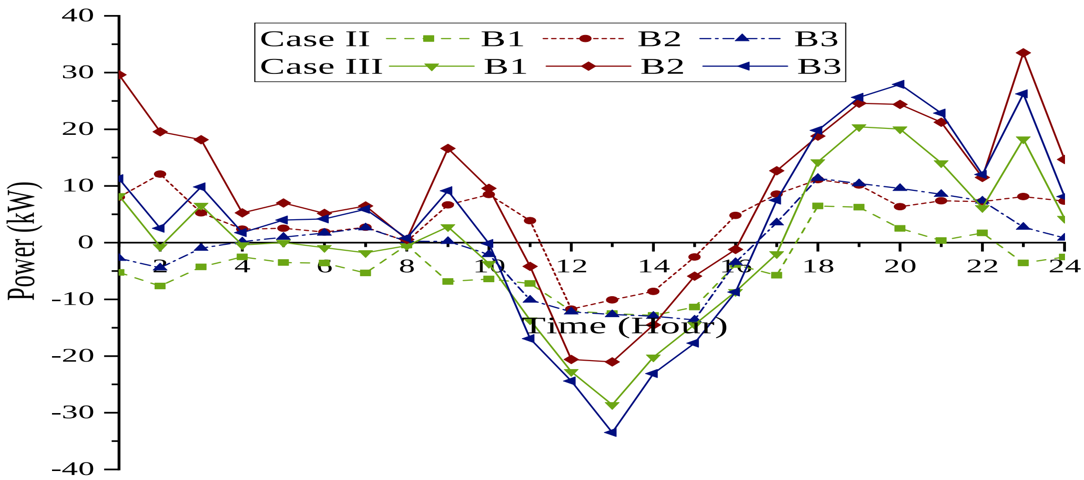
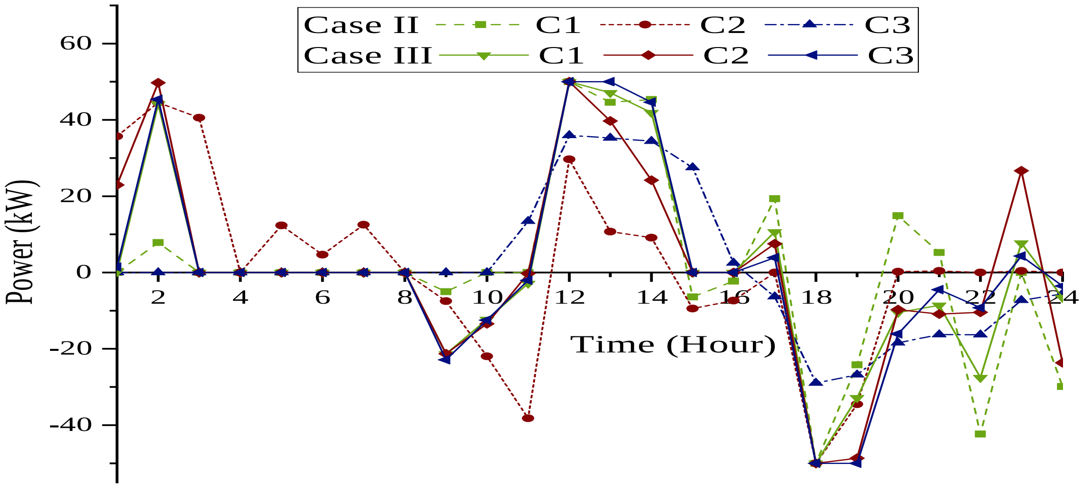
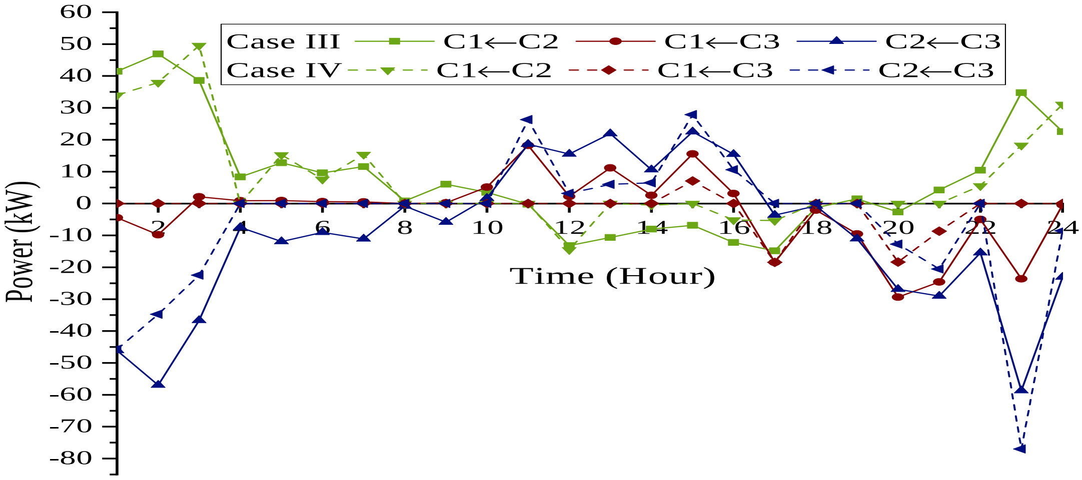


(c)

Figure 4.9: B2C Power Exchange (a) Community 1, (b) Community 2, (c) Community 3

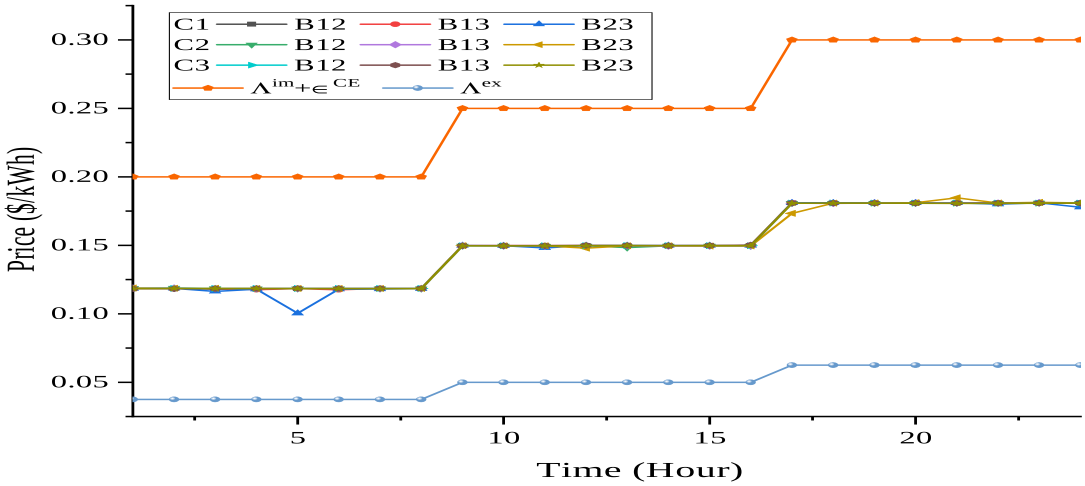


(a)

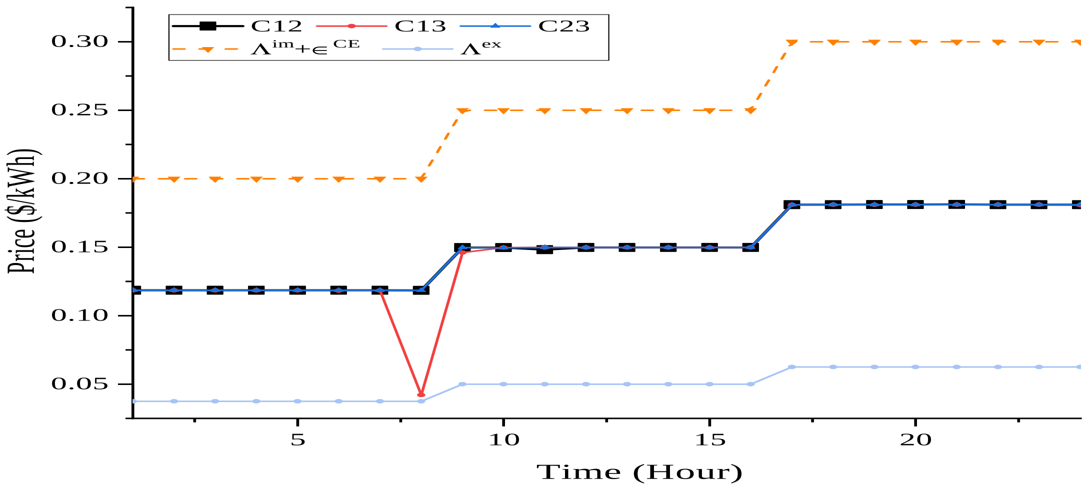


(b)

Figure 4.10: (a) The charging and discharging power of VC's battery, (b) C2C Net Power Exchange

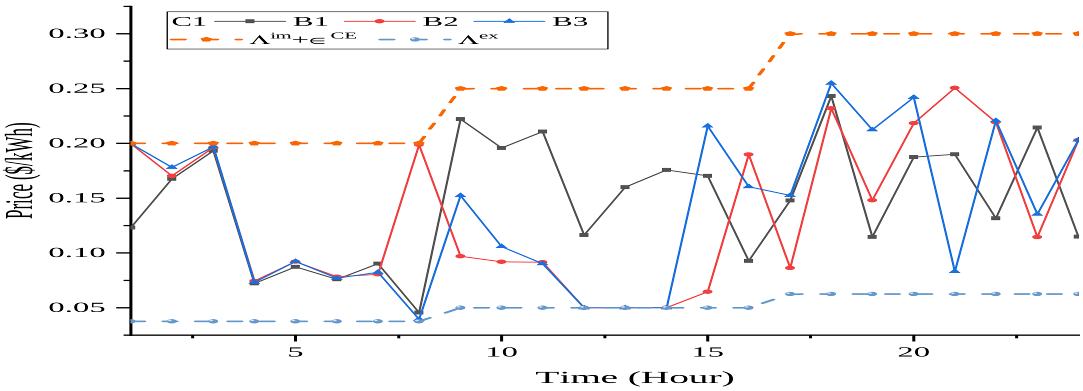


(a)

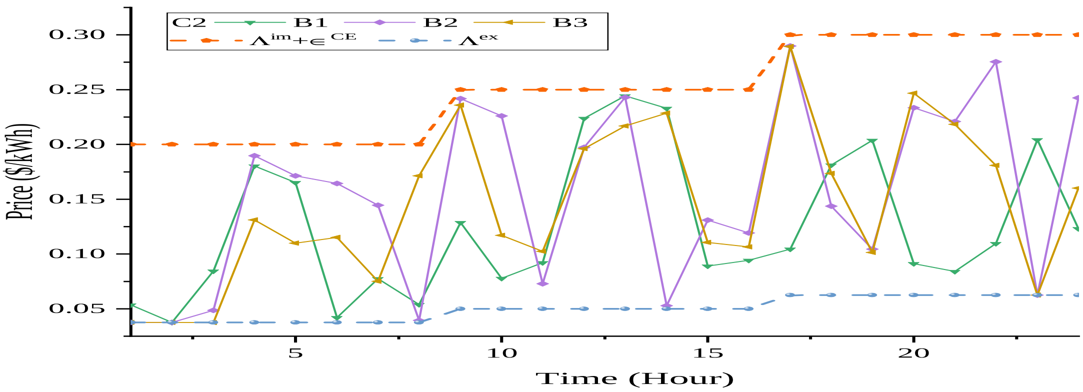


(b)

Figure 4.11: Price for internal transactions for *Case III* (a) B2B and (b) C2C



(a)



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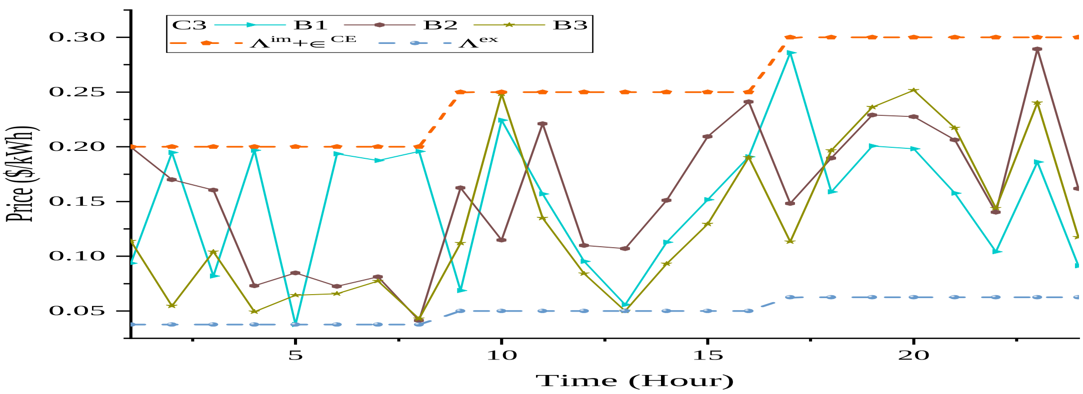

(c)

Figure 4.12: PRICE FOR B2C INTERNAL TRANSACTIONS FOR *CASE III* OF VARIOUS VCS (A) VIRTUAL COMMUNITY 1, (B) VIRTUAL COMMUNITY 2, (C) VIRTUAL COMMUNITY 3

utilizing BESSs more effectively.

Table 4.8: COMMUNITY-TO-COMMUNITY POWER EXCHANGE

		C1	**C2**	**C3**
$P_{im}^{C2C}(kW)$	$CaseIII$	316.2523	176.2641	482.0041
	$CaseIV$	221.878	105.9789	267.3148
$P_{ex}^{C2C}(kW)$	$CaseIII$	197.3159	607.631	169.5414
	$CaseIV$	71.04568	436.5601	87.52149

The energy exchange among the buildings and the VCs depends on the energy exchange prices. In $Case\ I$, the buildings can exchange energy only at utility prices. However, as shown in Figure 4.11, in $Case\ II$ and $Case\ III$, the energy exchange is possible at a price between the settled price of VCs and the utility price, due to which there is a reduction in energy exchange with the grid resulting in lower energy costs for buildings. The reduction in costs is as shown in Table 4.7. Further, the energy exchange profile and BESS charging profiles for $Case\ II$ are also different from $Case\ III$; this is due to a higher number of transactional entities (inclusion of C2C) for energy sharing, as shown in Figure 4.10 and Figure 4.9.

The prices for all the internal transactions are shown in Figure 4.11 and Figure 4.12. As expected, the prices are between the import and export utility prices, which encourages the buildings to participate in the P2P sharing framework. In Figure 4.11(a) and Figure 4.11(b), at 5^{th} hour and 8^{th} hour, respectively, a jump is observed in prices. But the respective power component is zero, as shown in 4.8(a) and Figure 4.10(b), respectively. This means that variation has no significance. Thus, the prices for B2B (Figure 4.11(a)) and C2C (Figure 4.11(b)) are almost identical for all the buildings. However, a vast variation is observed in B2C prices (Figure 4.12). This is because the VC's cost is distributed among the constituent buildings according to equation (4.25). The B2C prices depend on the contribution of buildings in B2C transactions.

The proposed framework focuses on the aggregation of buildings into virtual communities. Table 4.9 compares the total cost of buildings in the P2P energy trading framework with and without virtual communities. While the variation in individual building costs is minimal, the difference in total execution time is significant, as shown in the table. The proposed framework reduced execution time by 48 minutes compared to the existing literature. Thus, it can be concluded that aggregating buildings into virtual communities reduces computational complexity and execution time for P2P energy trading.

Table 4.9: Comparison Table

Proposed Framework			Other Framework	
C1	B1	86.6766	B1	88.1155
	B2	78.6481	B2	76.0011
	B3	86.2124	B3	85.0169
C2	B1	72.3017	B4	91.9939
	B2	-7.5833	B5	6.6372
	B3	-35.0367	B6	-19.7233
C3	B1	36.1164	B7	39.1162
	B2	178.4923	B8	166.8947
	B3	119.3154	B9	111.5659
	Sum	615.1428	Sum	645.6179
	Total Execution Time	1:31:18.826	Total Execution Time	2:19:02.387

4.5 Summary

A novel decentralized P2P energy management scheme has been proposed for virtual communities comprising several buildings equipped with RESs and shiftable loads. The proposed framework includes B2B, B2C, and C2C energy trading, optimal BESS scheduling, and demand-side management. A non-cooperative game theoretic approach is used in a decentralized manner, making the players a seller/buyer/both endogenously. Numerical results elucidate the proposed framework's effectiveness in minimizing the cost function for each building. The grouping of buildings into virtual communities reduces the number of transactions and enhances the system's scalability.

So far, the exchange among VCs has been explored for a non-cooperative game among buildings that are close to each other and that are at a certain distance away. As discussed earlier, in the non-cooperative game, the participants act independently, aiming at maximizing their benefits. However, they can work together by forming a coalition aiming to increase the benefit of all, which will be explored in the next chapter. Also, as the proposed framework lacks consideration of distribution network parameters such as voltage and line flow, a practical model has been developed in further chapters that consider all such network parameters.